

# Algebra & Radical Expressions

Rational Functions · Exponents · Radical Expressions · Equations

20 Questions · Self-Study Edition · Print &amp; Practice

## Part I — Rational Expressions

Simplifying, multiplying, dividing, and adding/subtracting rational expressions

### MEMORY KEY

■ **FACTOR** → **CANCEL** → **SIMPLIFY** | LCD = Lowest Common Denominator

Restrictions: denominator  $\neq 0$  | Flip when **DIVIDING** ( $\div \rightarrow \times$  reciprocal)

### WORKED EXAMPLE

Simplify:  $(x^2 - 9) / (x^2 - x - 6)$

① Factor top:  $(x+3)(x-3)$

② Factor bottom:  $(x+2)(x-3)$

③ Cancel  $(x-3)$ : Answer =  $(x+3)/(x+2)$ ,  $x \neq 3$ ,  $x \neq -2$

**Q1** [Easy]

Simplify:  $(x^2 - 4) / (x^2 + 4x + 4)$

A)  $(x+2) / (x-2)$

B)  $(x-2) / (x+2)$

C)  $(x-4) / (x+4)$

D) 1

■ Factor top as  $(x+2)(x-2)$ , bottom as  $(x+2)^2$ , cancel one  $(x+2)$ .

**Q2** [Easy]

Multiply and simplify:  $[3x / (x^2-1)] \cdot [(x+1) / 6]$

A)  $3x(x+1) / [6(x^2-1)]$

B)  $x / [2(x+1)]$

C)  $x / [2(x-1)]$

D)  $3x / 2$

■ Factor  $x^2 - 1 = (x+1)(x-1)$ , cancel  $(x+1)$  and simplify  $3/6$ .

**Q3** [Medium]

Add:  $[3 / (x+2)] + [2 / (x-2)]$

A)  $5 / (x^2-4)$

B)  $(5x-2) / (x^2-4)$

C)  $(5x+2) / (x^2-4)$

D)  $5 / (2x)$

■ LCD =  $(x+2)(x-2)$ . Expand:  $3(x-2)+2(x+2) = 5x-2$ .

**Q4** [Medium]

Divide:  $[(x^2-x-6) / (x^2-9)] \div [(x+2) / (x+3)]$

A) 1

B)  $(x-3)/(x+3)$

C)  $(x+2)^2/(x^2-9)$

D)  $x-3$

■ Flip divisor, factor all, cancel  $(x+2)(x-3)(x+3) \rightarrow 1$ .

**Q5** [Medium]

Which values of  $x$  make  $(x+3) / (x^2-x-12)$  undefined?

A)  $x = 3$  only

B)  $x = -3$  only

C)  $x = 3$  and  $x = -3$

D)  $x = 4$  and  $x = -3$

■ Set denominator = 0:  $(x-4)(x+3) = 0 \rightarrow x = 4$  or  $x = -3$ .

## Part II — Rational Equations

Solving equations with rational expressions — always check for extraneous solutions

### MEMORY KEY

■ MULTIPLY both sides by LCD  $\rightarrow$  Clear denominators  $\rightarrow$  Solve  $\rightarrow$  CHECK restrictions

EXTRANEIOUS = a solution that makes the denominator = 0 (reject it!)

### WORKED EXAMPLE

Solve:  $(2/x) + (1/3) = (7/6)$

① LCD =  $6x$

② Multiply:  $12 + 2x = 7x$

③ Solve:  $12 = 5x \rightarrow x = 12/5$

④ Check:  $x \neq 0$  ✓

**Q6** [Easy]

Solve:  $(3/x) - (1/2) = (1/x)$

A)  $x = 2$

B)  $x = 3$

C)  $x = 4$

D)  $x = -4$

■ LCD =  $2x$ . Multiply:  $6 - x = 2 \rightarrow x = 4$ . Check  $x \neq 0$  ✓

**Q7** [Hard]

Solve:  $x/(x-2) = 2/(x-2) + 1$

A)  $x = 2$

B)  $x = 4$

C)  $x = 0$

D) No solution

■ Multiply by  $(x-2)$ :  $x = 2 + x - 2 \rightarrow 0 = 0$ . But  $x = 2$  is restricted. No solution.

**Q8** [Hard]

Solve:  $1/(x+1) + 1/(x-1) = 2/(x^2-1)$

A)  $x = 1$

B) No solution

C)  $x = 0$

D)  $x = \pm 1$

■ LCD =  $x^2 - 1$ . Expand:  $(x-1) + (x+1) = 2 \rightarrow x = 1$ , but  $x = 1$  is restricted  $\rightarrow$  No solution.

## Part III — Exponent Rules

Integer and rational exponents, negative exponents

### MEMORY KEY

■ SAME base ADD exp (multiply) | SAME base SUBTRACT exp (divide)

POWER of power  $\rightarrow$  MULTIPLY | Negative exp  $\rightarrow$  FLIP (reciprocal)

$x^{1/n} = \sqrt[n]{x}$  |  $x^{m/n} = (\sqrt[n]{x})^m$

### WORKED EXAMPLE

Simplify:  $(2x^3y^2)^2 \cdot x^{-1}y$

① Power:  $4x^6y^4 \cdot x^{-1}y$

② Add exponents:  $4x^5y^5$

③  $y^5 = 1 \rightarrow$  Answer:  $4x^5$

**Q9** [Easy]

Simplify:  $x^{-3} \cdot x^5 \cdot x^{-1}$

A)  $x^1$

B)  $x^1$

C)  $x^{-1}$

D)  $x^1$

■ Same base  $\rightarrow$  ADD exponents:  $-3 + 5 + (-1) = 1 \rightarrow x$ .



**Q13** [Easy]

Simplify:  $\sqrt{72}$

A)  $8\sqrt{3}$

B)  $6\sqrt{2}$

C)  $9\sqrt{2}$

D)  $4\sqrt{18}$

■  $72 = 36 \times 2$ .  $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$ . Use **LARGEST** perfect square.

**Q14** [Medium]

Simplify:  $3\sqrt{8} - \sqrt{32} + 2\sqrt{2}$

A)  $4\sqrt{2}$

B)  $2\sqrt{2}$

C)  $4\sqrt{2}$

D)  $8\sqrt{2}$

■  $3\sqrt{8} = 6\sqrt{2}$ ,  $\sqrt{32} = 4\sqrt{2} \rightarrow 6\sqrt{2} - 4\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$ .

**Q15** [Easy]

Rationalize:  $6 / \sqrt{3}$

A)  $2\sqrt{6}$

B)  $2\sqrt{3}$

C)  $6\sqrt{3}$

D)  $6/3$

■ Multiply by  $\sqrt{3}/\sqrt{3}$ :  $6\sqrt{3}/3 = 2\sqrt{3}$ . Remember  $\sqrt{3} \cdot \sqrt{3} = 3$ .

## Part V — Radical Equations

Solving equations with square/cube roots — check for extraneous solutions!

### MEMORY KEY

■ ISOLATE radical  $\rightarrow$  SQUARE both sides  $\rightarrow$  Solve  $\rightarrow$  CHECK (plug back in!)

Squaring can CREATE fake solutions  $\rightarrow$  always verify

$\sqrt{x}$  is only real if  $x \geq 0$

### WORKED EXAMPLE

Solve:  $\sqrt{(2x+3)} = x-1$

① Square:  $2x+3 = (x-1)^2 = x^2-2x+1$

② Rearrange:  $x^2-4x-2 = 0$

③ Solve with quadratic formula

④ CHECK both solutions

**Q16** [Easy]

Solve:  $\sqrt{x+5} = 4$

A)  $x = 3$

B)  $x = 11$

C)  $x = 9$

D)  $x = 21$

■ Square:  $x+5=16 \rightarrow x=11$ . Check:  $\sqrt{16}=4 \checkmark$

**Q17** [Hard]

Solve:  $\sqrt{3x-2} = x-2$

A)  $x = 1$  only

B)  $x = 6$  only

C)  $x = 6$  only ( $x = 1$  is extraneous)

D)  $x = 1$  and  $x = 6$

■ Square:  $x^2-7x+6=0 \rightarrow x=1$  or  $x=6$ . Check  $x=1$ :  $RHS=-1 < 0 \rightarrow$  extraneous.  $x=6 \checkmark$

**Q18** [Medium]

Solve:  $\sqrt{x+3} = 1$

A)  $x = 4$

B)  $x = -4$

C)  $x = 16$

D) No real solution

■ Isolate:  $\sqrt{x} = -2$ . But  $\sqrt{x} \geq 0$  always. No real solution.

**Q19** [Hard]

Solve:  $2\sqrt{x+1} - \sqrt{4x-2} = 0$

A)  $x = 0$

B)  $x = 2$

C)  $x = 4$

D)  $x = -1$

■  $2\sqrt{x+1} = \sqrt{4x-2}$ . Square:  $4(x+1) = 4x-2 \rightarrow 4 = -2...$  Check  $x=2$ :  $2\sqrt{3}$  vs  $\sqrt{6} \approx$  verify by substitution.

**Q20** [Hard]

Solve:  $\sqrt{x^2+5} = x+1$

A)  $x = 0$

B)  $x = 1$

C)  $x = 2$

D) No real solution

■ Square:  $x^2+5 = x^2+2x+1 \rightarrow 5 = 2x+1 \rightarrow x=2$ . Check:  $\sqrt{9}=3=2+1 \checkmark$

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### ANSWER KEY

Q1: B

Q2: C

Q3: B

Q4: A

Q5: D

<b>Q6: C</b>	<b>Q7: D</b>	<b>Q8: B</b>	<b>Q9: B</b>	<b>Q10: C</b>
<b>Q11: A</b>	<b>Q12: B</b>	<b>Q13: B</b>	<b>Q14: C</b>	<b>Q15: B</b>
<b>Q16: B</b>	<b>Q17: C</b>	<b>Q18: D</b>	<b>Q19: B</b>	<b>Q20: C</b>