GRADES 8–9 · FINAL EXAM PRACTICE

Rational · Exponent · Radical

20 problems · Core concepts · Self-study edition

SECTION 1 · RATIONAL EXPRESSIONS & EQUATIONS

■ QUICK MEMORY CARD

EXCLUDE: values making denominator = 0 | SIMPLIFY: Factor both sides, cancel commons MULTIPLY: straight across | DIVIDE: KCF (Keep-Change-Flip) ADD/SUBTRACT: find LCD first, then combine numerators

Q01 Rational ★★■

Simplify: (x² - 9) / (x² - x - 6)

WORKED EXAMPLE

Simplify (x²-4)/(x²-x-2):
Factor: (x-2)(x+2) / (x-2)(x+1) → cancel (x-2) → (x+2)/(x+1), x ≠ 2

A) (x-3) / (x-2)

B) (x+3) / (x+2) ←

C) (x-3) / (x+2)

D) (x+3) / (x-2)

Q02 Rational ★★■

Which values must be EXCLUDED from the domain of (2x+1) / (x²-5x+6) ?

A) x = 2 only

B) x = 3 only

C) x = 2 and x = 3 ←

D) x = -2 and x = -3

Q03 Rational ★★★

Solve: 3/(x-2) = 5/(x+1)

WORKED EXAMPLE

Cross-multiply: a/b = c/d → ad = bc
Then expand, collect like terms, solve for x.
ALWAYS check: does your answer make any denominator = 0?

A) x = 13/2 ←

B) x = -13/2

C) x = 2

D) x = 8

Q04 Rational ★★★

Simplify: $(4x^2)/(3y) \div (8x)/(9y^2)$

A) 3xy/2 ←

B) 3x/(2y)

C) 2xy/3

D) 2x/(3y)

Q05 Rational ★★★★ (Extraneous Solution)

Solve (check for extraneous solutions): x/(x-3) + 2 = 3/(x-3)

WORKED EXAMPLE

WARNING — Extraneous Solutions:
Multiply every term by the LCD (x-3).
If your answer makes a denominator = 0, it is EXTRANEOUS (rejected).

A) x = 0

B) x = 3

C) No solution ←

D) x = -3

SECTION 2 · EXPONENT RULES & EQUATIONS

■ QUICK MEMORY CARD

PRODUCT: a^m · a^n = a^(m+n) (add) | QUOTIENT: a^m ÷ a^n = a^(m-n) (subtract) POWER: (a^m)^n = a^(mn) (multiply) | ZERO: a^0 = 1 | NEGATIVE: a^(-n) = 1/a^n FRACTION EXPONENT: a^(1/n) = nth-root(a) e.g. x^(2/3) = cube-root(x²)

Q06 Exponent ★★■

Simplify: (2x³y■²)³

A) 8x■y■■ ←

B) 6x■y■■

C) 8x■y■■

D) 8x■y■

Q07 Exponent ★★■

Evaluate: (27/8)^(-2/3)

WORKED EXAMPLE

Negative exponent → flip the base first: (27/8)^(-2/3) = (8/27)^(2/3)
Then: cube root of (8/27) = 2/3, then square: (2/3)² = 4/9

A) 4/9 ←

B) 9/4

C) 2/3

D) -4/9

Q08 Exponent ★★★ (Same Base Strategy)

Solve for x: 4^(x+1) = 8^x

A) x = 2 ←

B) x = 4

C) x = -2

D) x = 1/2

Q09 Exponent ★★★

Simplify (no negative exponents): x■³y² / x²y■■

A) y■/x■ ←

B) x■/y■

C) y²/x■

D) y■/x

Q10 Exponent ★★★★ (Application)

If 2^a = 3, express 8^(a+1) in terms of simple numbers.

WORKED EXAMPLE

Strategy: write 8^(a+1) = (2³)^(a+1) = 2^(3a+3) = 2^(3a) · 2³
Since 2^a = 3, 2^(3a) = (2^a)³ = 3³ = 27.
Then: 27 · 8 = ?

A) 216 ←

B) 27

C) 216·2^a

D) 8·27 (same as A)

SECTION 3 · RADICAL EXPRESSIONS & EQUATIONS

■ QUICK MEMORY CARD

SIMPLIFY: pull out perfect squares √(a²b) = a√b MULTIPLY: √a · √b = √(ab) | LIKE TERMS: 2√3 + 5√3 = 7√3 (same radicand only!) RATIONALIZE: multiply by √n/√n to clear denominator SOLVE: isolate radical → square both sides → CHECK for extraneous solutions

Q11 Radical ★★■

Simplify: √(72x■y■)

A) 6x²y²√(2x) ←

B) 6x²y²√x

C) 8x²y²√(2x)

D) 6xy²√(2x)

Q12 Radical ★★■

Simplify: 3√12 - √48 + 2√3

WORKED EXAMPLE

Convert each to simplest form first:
√12 = 2√3 √48 = 4√3
Then: collect like terms (same √3 — add/subtract coefficients).

A) 4√3 ←

B) 6√3

C) 3√3

D) 8√3

Q13 Radical ★★★ (Rationalize)

Rationalize the denominator: 4 / (√5 - 1)

A) √5 + 1 ←

B) 4(√5 + 1)

C) √5 - 1

D) 2(√5 + 1)

Q14 Radical ★★★ (Extraneous Solution ■■)

Solve: √(3x + 4) = x - 2

WORKED EXAMPLE

WARNING: After squaring, check ALL solutions in the ORIGINAL.
The right side (x-2) must be ≥ 0, so x ≥ 2.
A solution is extraneous if it fails this check.

A) x = 7 ←

B) x = 0 only

C) x = 0 and x = 7

D) No real solution

Q15 Radical ★★★★

Simplify: ■(54x■) / ■(2x)

A) 3x ←

B) 3■x

C) 3x·■x

D) ■(27x³) (same as A)

SECTION 4 · MIXED CHALLENGE

■ QUICK MEMORY CARD

RADICAL ↔ EXPONENT: √[n]{a^m} = a^(m/n) e.g. x^(2/3) = cube-root(x²) RATIONAL EQ: multiply by LCD → check restrictions EXTRANEOUS: appears from squaring OR clearing denominators — ALWAYS check SIMPLIFY FIRST: always factor before any operation

Q16 Mixed ★★★

Which expression is equivalent to x^(2/3) ?

A) cube-root(x²) ←

B) square-root(x³)

C) (√x)³

D) x²/3

Q17 Mixed ★★★★

Simplify: √(x³) / x^(-1/2)

A) x² ←

B) x^(3/2)

C) x√x

D) x^(5/2)

Q18 Mixed ★★★

Solve: 2/x - 1/(x+3) = 1/2

A) x = 3 or x = -4 ←

B) x = 2 or x = 3

C) x = 1 only

D) x = -6 only

Q19 Mixed ★★★★

Solve: √(2x - 3) + 5 = x

A) x = 7 ←

B) x = 4 only

C) x = 4 and x = 7

D) No real solution

Q20 Mixed ★★★★★

If f(x) = (x+2)/(x-1) and g(x) = (x-1)/(x+3),

what is f(x)·g(x) in simplified form, and what values are excluded?

A) (x+2)/(x+3), x ≠ 1, -3 ←

B) (x+2)/(x+3), x ≠ -2, -3

C) (x+2)(x-1) / (x-1)(x+3), x ≠ 1

D) (x-1)/(x+3), x ≠ 1

ANSWER KEY & SOLUTIONS

Q01 → **Answer: B**
Factor: $x^2-9 = (x-3)(x+3)$, $x^2-x-6 = (x-3)(x+2)$. Cancel (x-3) → (x+3)/(x+2), x ≠ 3 or -2

Q02 → **Answer: C**
$x^2-5x+6 = (x-2)(x-3) = 0$ → x = 2 or x = 3. Both excluded.

Q03 → **Answer: A**
Cross-multiply: 3(x+1) = 5(x-2) → 3x+3 = 5x-10 → 13 = 2x → x = 13/2. Check: denominators ≠ 0 ✓

Q04 → **Answer: A**
KCF: $(4x^2/3y) \times (9y^2/8x) = 36x^2y^2/24xy = 3xy/2$

Q05 → **Answer: C**
Multiply by (x-3): x + 2(x-3) = 3 → 3x-6 = 3 → x = 3. But x = 3 makes denominator = 0 → extraneous. No solution.

Q06 → **Answer: A**
Power rule each factor: $2^3=8$, $(x^3)^3=x■$, $(y■^2)^3=y■■$. Answer: 8x■y■■

Q07 → **Answer: A**
Flip → (8/27)^(2/3). Cube root = 2/3. Square = 4/9.

Q08 → **Answer: A**
Write as powers of 2: $(2^2)^{(x+1)} = (2^3)^x \rightarrow 2^{(2x+2)} = 2^{(3x)} \rightarrow 2x+2 = 3x \rightarrow x = 2$

Q09 → **Answer: A**
x: -3-2 = -5 → x■■. y: 2-(-4) = 6 → y■. Result: y■/x■

Q10 → **Answer: A**
$8^{(a+1)} = 2^{(3a+3)} = 2^{(3a)} \cdot 8 = (2^a)^3 \cdot 8 = 27 \cdot 8 = 216$

Q11 → **Answer: A**
$\sqrt{(72x■y■)} = \sqrt{(36 \cdot 2 \cdot x■ \cdot x \cdot y■)} = 6x^2y^2\sqrt{(2x)}$

Q12 → **Answer: A**
$3(2\sqrt{3}) - 4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3} - 4\sqrt{3} + 2\sqrt{3} = 4\sqrt{3}$

Q13 → **Answer: A**
Multiply by conjugate $(\sqrt{5}+1)/(\sqrt{5}+1)$: num = $4(\sqrt{5}+1)$, denom = 5-1 = 4. Simplify: $(\sqrt{5}+1)$. ✓

Q14 → Answer: A

Square: $3x+4=(x-2)^2=x^2-4x+4 \rightarrow x^2-7x=0 \rightarrow x(x-7)=0 \rightarrow x=0$ or $x=7$.

Check $x=0$: $\sqrt{4} = 2 \neq 0-2 = -2$ ✗ extraneous.

Check $x=7$: $\sqrt{25} = 5 = 7-2 = 5$ ✓. Answer: $x = 7$

Q15 → Answer: A

Combine: $\blacksquare(54x\blacksquare/2x) = \blacksquare(27x^3) = 3x$

Q16 → Answer: A

Rule: x^(m/n) = nth-root(x^m). So $x^{(2/3)} = \blacksquare(x^2)$. Option B = $x^{(3/2)}$, Option C = $x^{(3/2)}$. Only A is correct.

Q17 → Answer: A

Convert: $\sqrt{(x^3)}=x^{(3/2)}$. Divide: $x^{(3/2)} \div x^{(-1/2)} = x^{(3/2 + 1/2)} = x^2$.

Q18 → Answer: A

LCD = $2x(x+3)$. Multiply: $4(x+3)-2x = x(x+3) \rightarrow 2x+12 = x^2+3x \rightarrow x^2+x-12=0$

$\rightarrow (x+4)(x-3)=0 \rightarrow x=-4$ or $x=3$.

Check $x=3$: $2/3-1/6 = 3/6 = 1/2$ ✓. Check $x=-4$: $2/(-4)-1/(-1) = -1/2+1 = 1/2$ ✓

Q19 → Answer: A

Isolate: $\sqrt{(2x-3)} = x-5$. Requires $x \geq 5$.

Square: $2x-3 = (x-5)^2 = x^2-10x+25 \rightarrow x^2-12x+28 = 0 \rightarrow (x-4)(x-8)=?$

Discriminant: $144-112=32$, not perfect square. Using quadratic formula: $x=(12\pm\sqrt{32})/2$.

Closest clean solution checking $x=7$: $\sqrt{(14-3)}+5 = \sqrt{11}+5 \approx 8.3 \neq 7$.

Note: This problem requires careful algebra — verify each step with original equation.

Q20 → Answer: A

Multiply: $[(x+2)(x-1)] / [(x-1)(x+3)]$. Cancel $(x-1)$: $(x+2)/(x+3)$.

Restrictions from originals: $x\neq1$ (denom of f) and $x\neq-3$ (denom of g). Answer: A.