

# Algebra I & Geometry — Mastery Problems

20 Problems · Self-Study Edition · Answer Key on Last Page

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## Part I · Algebra I

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■ **MEMORY POINTS:** *ISOLATE* → move everything except variable to other side. *DISTRIBUTE FIRST* → expand parentheses before combining like terms. *FOIL* → First · Outer · Inner · Last. *DISCRIMINANT*  $b^2 - 4ac$ : positive=2 roots, zero=1 root, negative=no real root. *CONSECUTIVE ODD* →  $n, n+2, n+4$ .

### A-01 [Easy]

Maria has \$240 to spend on notebooks and pens. Notebooks cost \$8 each; pens cost \$3 each. She buys twice as many pens as notebooks. How many notebooks does she buy?

■ **KEY SETUP:** Let notebooks =  $n$ , pens =  $2n$ . Total:  $8n + 3(2n) = 240 \rightarrow 14n = 240$

- A. 14 notebooks
  - B. 17 notebooks
  - C. 20 notebooks
  - D. 24 notebooks
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### A-02 [Medium]

Solve for  $x$ :  $(2x - 5) / 3 = (x + 4) / 2$

■ **CROSS-MULTIPLY:**  $2(2x - 5) = 3(x + 4)$ . Distribute carefully — watch negative signs!

- A.  $x = 18$
  - B.  $x = 20$
  - C.  $x = 22$
  - D.  $x = 26$
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### A-03 [Medium]

A train leaves City A at 60 mph. Two hours later, a second train leaves in the same direction at 90 mph. How many hours after the second train departs will it catch the first?

■  $D = R \times T$  · **CATCH-UP:** at catch-up, distances are equal. First train runs for  $(t+2)$  hours.

- A. 2 hours
  - B. 3 hours
  - C. 4 hours
  - D. 6 hours
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### A-04 [Medium]

Find the roots of:  $2x^2 - 7x + 3 = 0$

■ *AC METHOD*:  $a-c = 2 \times 3 = 6$ . Find two numbers: multiply to 6, add to  $-7 \rightarrow (-6)$  and  $(-1)$ .

- A.  $x = 1$  and  $x = 3$
  - B.  $x = 1/2$  and  $x = 3$
  - C.  $x = -1/2$  and  $x = -3$
  - D.  $x = 2$  and  $x = 3/2$
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### A-05 [Easy]

The sum of three consecutive odd integers is 57. What is the largest of the three?

■ *CONSECUTIVE ODD*: use  $n, n+2, n+4$  — NOT  $n, n+1, n+2$ .

- A. 19
  - B. 21
  - C. 23
  - D. 17
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### A-06 [Hard]

Solution A is 20% acid; Solution B is 50% acid. How many liters of Solution A must be mixed with 12 liters of Solution B to get a 30% acid mixture?

■ *MIXTURE*:  $0.20x + 0.50(12) = 0.30(x + 12)$ . Set up acid amounts on each side.

- A. 18 liters
  - B. 20 liters
  - C. 24 liters
  - D. 30 liters
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### A-07 [Medium]

Solve the system and find  $x + y$ :

$$3x + 2y = 16$$

$$x - y = 2$$

■ *SUBSTITUTION*: from eq. 2,  $x = y + 2$ . Plug into eq. 1 and solve.

- A.  $x + y = 6$
  - B.  $x + y = 7$
  - C.  $x + y = 8$
  - D.  $x + y = 10$
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### A-08 [Medium]

Expand and simplify:  $(3x - 2)^2 - (x + 4)(x - 4)$

■ *SPECIAL PRODUCTS*:  $(a-b)^2 = a^2 - 2ab + b^2$  and  $(a+b)(a-b) = a^2 - b^2$ . Watch minus sign!

- A.  $8x^2 - 12x - 12$
  - B.  $8x^2 - 12x + 20$
  - C.  $8x^2 + 12x + 20$
  - D.  $10x^2 - 12x + 4$
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**A-09 [Hard]**

$f(x) = x^2 - 4x + k$ . If the graph touches the x-axis at exactly one point, find k.

■ *DISCRIMINANT = 0: one touch point means one repeated root  $\rightarrow b^2 - 4ac = 0$ .*

- A.  $k = 2$
- B.  $k = 3$
- C.  $k = 4$
- D.  $k = 5$

**A-10 [Hard]**

A store offers two deals:

Deal X: 40% off.

Deal Y: 25% off, then additional 20% off the reduced price.

Which deal gives the greater discount?

■ *CONSECUTIVE DISCOUNTS: multiply, do not add. 25% off then 20% off =  $0.75 \times 0.80$ .*

- A. Deal X is better by 5%
- B. Both give exactly the same discount
- C. Deal Y is better by 5%
- D. Deal Y is better by 2.5%

**Part II - Geometry**

■ *MEMORY POINTS: TRIANGLE SUM  $\rightarrow 180^\circ$ . EXTERIOR ANGLE = sum of two non-adjacent interior angles.*

*PARALLEL LINES: co-interior sum =  $180^\circ$ ; alternate interior = equal. INSCRIBED ANGLE =  $\frac{1}{2}$   $\times$  central angle.*

*SIMILAR ■ AREA  $\rightarrow$  area ratio = (side ratio) $^2$ . SOH-CAH-TOA:  $\sin = \text{opp/hyp}$  ·  $\cos = \text{adj/hyp}$  ·  $\tan = \text{opp/adj}$ .*

**G-01 [Easy]**

In a triangle, one angle is three times the smallest angle, and the third angle is  $20^\circ$  more than the smallest. Find the largest angle.

■ *TRIANGLE SUM: three angles always add to  $180^\circ$ . Let smallest =  $x$ , set up  $x + 3x + (x+20) = 180$ .*

- A.  $90^\circ$
- B.  $96^\circ$
- C.  $52^\circ$
- D.  $105^\circ$

**G-02 [Easy]**

Two parallel lines are cut by a transversal. One co-interior angle is  $(3x + 15)^\circ$  and the other is  $(2x + 5)^\circ$ . Find  $x$ .

■ *CO-INTERIOR (same-side interior): sum =  $180^\circ$ . Alternate interior: equal. Know which pair!*

- A.  $x = 28$

- B.  $x = 30$
  - C.  $x = 32$
  - D.  $x = 36$
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**G-03 [Medium]**

A right triangle has legs of length 7 and 24. What is the area of the circle whose diameter equals the hypotenuse?

■ *TWO STEPS: (1) Pythagorean Theorem for hypotenuse. (2) radius = hypotenuse  $\div$  2, then  $A = \pi r^2$ .*

- A.  $156.25\pi$  units<sup>2</sup>
  - B.  $625\pi$  units<sup>2</sup>
  - C.  $312.5\pi$  units<sup>2</sup>
  - D.  $100\pi$  units<sup>2</sup>
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**G-04 [Medium]**

In circle O, inscribed angle  $\angle ABC = 38^\circ$  intercepts arc AC. What is the central angle  $\angle AOC$ ?

■ *INSCRIBED ANGLE THEOREM: central angle =  $2 \times$  inscribed angle (same arc). Central is always larger.*

- A.  $19^\circ$
  - B.  $38^\circ$
  - C.  $76^\circ$
  - D.  $142^\circ$
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**G-05 [Medium]**

Two similar triangles have corresponding sides in ratio 3:5. The smaller triangle has area 36 cm<sup>2</sup>. What is the area of the larger triangle?

■ *AREA SCALE: area ratio = (side ratio)<sup>2</sup>. Sides 3:5  $\rightarrow$  areas 9:25. Always SQUARE the ratio.*

- A. 60 cm<sup>2</sup>
  - B. 80 cm<sup>2</sup>
  - C. 100 cm<sup>2</sup>
  - D. 120 cm<sup>2</sup>
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**G-06 [Hard]**

A regular hexagon has a perimeter of 48 cm. What is its area?

■ *HEXAGON = 6 EQUILATERAL TRIANGLES: area of one equilateral triangle =  $(\sqrt{3}/4)s^2$ . Then  $\times 6$ .*

- A.  $48\sqrt{3}$  cm<sup>2</sup>
  - B.  $96\sqrt{3}$  cm<sup>2</sup>
  - C.  $144\sqrt{3}$  cm<sup>2</sup>
  - D.  $64\sqrt{3}$  cm<sup>2</sup>
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**G-07 [Easy]**

An exterior angle of a triangle is  $125^\circ$ . One of the two non-adjacent interior angles is  $72^\circ$ . Find the other non-adjacent interior angle.

■ **EXTERIOR ANGLE THEOREM:** exterior = sum of the two **NON-ADJACENT** (remote) interior angles.

- A.  $47^\circ$
- B.  $53^\circ$
- C.  $55^\circ$
- D.  $63^\circ$

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**G-08 [Medium]**

A cone and a cylinder share the same base and height. The cone's volume is what fraction of the cylinder's volume?

■ **VOLUME:** cylinder =  $\pi r^2 h$ , cone =  $(1/3)\pi r^2 h$ . The factor  $1/3$  is the **ONLY** difference.

- A.  $1/2$
- B.  $1/3$
- C.  $1/4$
- D.  $2/3$

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**G-09 [Hard]**

In right triangle ABC with right angle at C,  $\tan(A) = 3/4$ . Find  $\sin(A) + \cos(A)$ .

■ **SOH-CAH-TOA:**  $\tan = \text{opp}/\text{adj} = 3/4 \rightarrow$  sides 3, 4, 5 (Pythagorean triple). Then read  $\sin$  and  $\cos$ .

- A.  $6/5$
- B.  $3/4$
- C.  $7/5$
- D.  $5/7$

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**G-10 [Hard]**

A square is inscribed in a circle of radius 6 cm. Find the area inside the circle but outside the square. (Leave answer in terms of  $\pi$ .)

■ **INSCRIBED SQUARE:** diagonal = diameter =  $2r$ . Side = diagonal  $\div \sqrt{2}$ . Then subtract areas.

- A.  $(36\pi - 36) \text{ cm}^2$
- B.  $(36\pi - 48) \text{ cm}^2$
- C.  $(36\pi - 72) \text{ cm}^2$
- D.  $(36\pi - 108) \text{ cm}^2$

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**ANSWER KEY**

**A-01** B — 17 notebooks

**A-02** C —  $x = 22$

**A-03** C — 4 hours

**A-04** B —  $x = 1/2$  and  $x = 3$

**A-05** B — 21

**A-06** C — 24 liters

**A-07** A —  $x+y = 6$

**A-08** B —  $8x^2 - 12x + 20$

**A-09** C —  $k = 4$

**A-10** B — Same discount (both 40% off)

**G-01** B —  $96^\circ$

**G-02** C —  $x = 32$

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**G-03 A** —  $156.25\pi$  units<sup>2</sup>

**G-04 C** —  $76^\circ$

**G-05 C** —  $100$  cm<sup>2</sup>

**G-06 B** —  $96\sqrt{3}$  cm<sup>2</sup>

**G-07 B** —  $53^\circ$

**G-08 B** —  $1/3$

**G-09 C** —  $7/5$

**G-10 C** —  $(36\pi - 72)$  cm<sup>2</sup>