

Algebra 1 & Geometry

Self-Study Worksheet · 20 Core Problems · Memory Tips Included

PART 1 — ALGEBRA 1 (Word Problems)

Topics: Linear Equations · Inequalities · Systems · Functions · Percent

Q1. [Linear Equations | FLAT + RATE × TIME = TOTAL]

Marcus rents a bike for a flat fee of \$5 plus \$3 per hour. He pays a total of \$20. Which equation correctly represents this, and how many hours did he rent?

■ *Watch out: do NOT write $3h = 20$ (forgets the flat fee).*

MEMORY
KEY

FLAT + RATE × TIME = TOTAL

(A) $3h = 20$, $h = 6.67$

(B) $5h + 3 = 20$, $h = 3.4$

(C) $3h - 5 = 20$, $h = 8.33$

(D) $3h + 5 = 20$, $h = 5$

ANSWE

R:

D | $3h + 5 = 20 \rightarrow 3h = 15 \rightarrow h = 5$ hours

Q2. [Inequalities | AT LEAST = \geq , always ROUND UP]

A school needs at least 150 chairs. They have 62. Chairs come in boxes of 8. What is the minimum number of boxes to order?

■ *Tricky: round UP for 'at least' — never round down.*

MEMORY
KEY

AT LEAST $\rightarrow \geq \rightarrow$ ROUND UP (ceiling)

(A) 10 boxes

(B) 11 boxes

(C) 11 boxes (exact division)

(D) 12 boxes

ANSWE

R:

C | Need 88 more chairs. $88 \div 8 = 11$ exactly \rightarrow 11 boxes

Q3. [Systems of Equations | SUM + DIFFERENCE \rightarrow SUBSTITUTION]

Two friends together have \$47. One friend has \$9 more than the other. How much does each friend have?

■ *Classic trap: don't add \$9 to the total. Set up a system.*

MEMORY
KEY

SUM + DIFFERENCE \rightarrow SUBSTITUTION

(A) \$18 and \$29

(B) \$19 and \$28

(C) \$20 and \$27

(D) \$23 and \$24

ANSWE

R:

B | $x + y = 47$, $x = y + 9 \rightarrow y = 19$, $x = 28$

Q4. [Percent & Proportion | SEQUENTIAL DISCOUNT: MULTIPLY]

A jacket costs \$80. First 25% off, then an additional 10% off the sale price. What is the final price?

■ $25\% + 10\%$ is NOT 35% off. Apply each discount separately!

MEMORY
KEY

SEQUENTIAL DISCOUNTS: MULTIPLY FACTORS, NOT ADD %

- (A) \$52.00 (35% off — WRONG) (B) \$48.00
(C) \$54.00 (D) \$56.00

ANSWE
R:

C | $80 \times 0.75 = 60$, then $60 \times 0.90 = \$54.00$

Q5. [Rate & Distance | HEAD-START ÷ SPEED DIFFERENCE]

Train A leaves at 60 mph. Train B leaves 1 hour later at 80 mph, same direction. After how many hours (from B's departure) does B catch A?

■ Train A has a 1-hour head start = 60 miles. Don't forget it!

MEMORY
KEY

CATCH UP = HEAD-START ÷ SPEED DIFFERENCE

- (A) 2 hours (B) 4 hours
(C) 5 hours (D) 3 hours

ANSWE
R:

D | Head start = 60 mi, gap = 20 mph, time = $60/20 = 3$ hours

Q6. [Slope & Linear Functions | DECREASING = SUBTRACT]

A candle is 12 inches tall and burns at 0.5 inches/hour. Which function gives height h after t hours? When is it burned out?

■ Burning = decreasing. The slope is negative!

MEMORY
KEY

$h = \text{START} - \text{RATE} \times t$ (decreasing = subtract)

- (A) $h = 0.5t + 12$; never burns out (B) $h = 12 - 0.5t$; 24 hours
(C) $h = 12 - 0.5t$; 6 hours (D) $h = 0.5t - 12$; 24 hours

ANSWE
R:

B | $h = 12 - 0.5t$. Set $h = 0$: $t = 24$ hours

Q7. [Age Problems | FUTURE AGE = NOW + YEARS (both!)]

Lisa is 3 times as old as her brother. In 4 years she will be twice as old. How old is each person now?

■ Add 4 to BOTH people in the future condition.

MEMORY KEY **FUTURE AGE = PRESENT + N (add to BOTH sides)**

(A) Lisa: 9, Bro: 3

(B) Lisa: 6, Bro: 2

(C) Lisa: 12, Bro: 4

(D) Lisa: 15, Bro: 5

ANSWER R: **C | $3b + 4 = 2(b + 4) \rightarrow b = 4$, Lisa = 12**

Q8. [Mixture Problems | CONC x VOL = AMOUNT]

How many liters of 40% acid solution must be mixed with 6 L of 10% solution to get a 20% solution?

■ Set up: $0.40x + 0.10(6) = 0.20(x+6)$. Don't average percentages.

MEMORY KEY **CONCENTRATION x VOLUME = AMOUNT OF SOLUTE**

(A) 2 liters

(B) 3 liters

(C) 4 liters

(D) 5 liters

ANSWER R: **B | $0.40x + 0.60 = 0.20x + 1.20 \rightarrow 0.20x = 0.60 \rightarrow x = 3$**

Q9. [Word Inequality | MINIMUM $\rightarrow \geq \rightarrow$ ROUND UP]

Sam needs \$50 for free shipping. He has \$31.75. Items cost \$4.25 each. Fewest items to reach free shipping?

■ $4.25n \geq 18.25$ gives $n \geq 4.29\dots$, so round UP to 5.

MEMORY KEY **MINIMUM $\rightarrow \geq \rightarrow$ CEILING function (round up)**

(A) 3 items

(B) 4 items

(C) 5 items

(D) 6 items

ANSWER R: **C | $31.75 + 4.25n \geq 50 \rightarrow n \geq 4.29 \rightarrow n = 5$**

Q10. [Functions & Domain | DOMAIN = VALID INPUTS ONLY]

Parking: \$3 first hour + \$1.50 each additional. $C(h) = 3 + 1.5(h-1)$ for $h \geq 1$. Cost for 5 hours? Is $h = 0$ in the domain?

■ $h = 0$ means no parking at all. Not a valid input.

MEMORY
KEY

DOMAIN = real-world valid inputs only

(A) \$9.00; $h=0$ is valid

(B) \$9.00; $h=0$ NOT in domain

(C) \$10.50; $h=0$ not in domain

(D) \$7.50; $h=0$ is valid

ANSWE
R:

B | $C(5) = 3 + 1.5(4) = \$9.00$. $h=0$ not in domain ($h \geq 1$)

PART 2 — GEOMETRY (Core Problems)

Topics: Angles · Triangles · Circles · Volume · Coordinate Geometry

Q1. [Parallel Lines | CO-INTERIOR = SUPPLEMENTARY (180°)]

Two parallel lines cut by a transversal. Co-interior angles are $(3x + 15)^\circ$ and $(2x + 5)^\circ$. Find x .

■ Co-interior angles SUM to 180° , they are NOT equal.

MEMORY
KEY

CO-INTERIOR = SUPPLEMENTARY (C-shape = 180°)

(A) $x = 16$

(B) $x = 20$

(C) $x = 32$

(D) $x = 40$

ANSWE
R:

C | $(3x+15)+(2x+5)=180 \rightarrow 5x+20=180 \rightarrow x = 32$

Q2. [Triangle Angles | TRIANGLE SUM = 180°]

In triangle ABC: angle A = $(2x)^\circ$, B = $(x+30)^\circ$, C = $(x-6)^\circ$. Find each angle.

■ Sum must equal exactly 180. Combine all three expressions first.

MEMORY
KEY

TRIANGLE SUM = 180° (always, no exceptions)

(A) $64^\circ, 94^\circ, 22^\circ$

(B) $78^\circ, 69^\circ, 33^\circ$

(C) $80^\circ, 70^\circ, 30^\circ$

(D) $60^\circ, 90^\circ, 30^\circ$

ANSWE
R:

B | $4x + 24 = 180 \rightarrow x = 39$. A=78°, B=69°, C=33°

Q6. [Exterior Angle | = REMOTE INT. #1 + REMOTE INT. #2]

An exterior angle of a triangle is $(7x+5)^\circ$. The two non-adjacent interior angles are $(3x+10)^\circ$ and $(2x+15)^\circ$. Find x .

■ Exterior angle = SUM of the two remote interior angles (not just one!).

MEMORY
KEY

EXTERIOR = REMOTE INTERIOR 1 + REMOTE INTERIOR 2

(A) $x = 20$

(B) $x = 15$

(C) $x = 25$

(D) $x = 10$

ANSWE A | $7x+5 = (3x+10)+(2x+15) \rightarrow 7x+5 = 5x+25 \rightarrow x = 10$ Check: exterior = 75° ; remotes = $40^\circ + 35^\circ = 75^\circ$ checkmark
R:

Q7. [Cylinder Volume | $V = \pi \cdot r^2 \cdot h$]

A cylindrical tank has diameter 10 m and height 6 m. Find its volume in terms of π .

■ Diameter = 10 means radius = 5. Using $d=10$ directly doubles the answer!

MEMORY
KEY

$V = \pi \cdot r^2 \cdot h$ (base area x height)

(A) $300\pi \text{ m}^3$

(B) $150\pi \text{ m}^3$

(C) $600\pi \text{ m}^3$

(D) $60\pi \text{ m}^3$

ANSWE B | $r = 5, V = \pi \cdot (5)^2 \cdot (6) = 150\pi \text{ m}^3$
R:

Q8. [Coordinate Geometry | MIDPOINT = AVERAGE; DIST = Pythagoras]

P(2, -3) and Q(8, 5). Find the midpoint and the distance between them.

■ For distance: $\Delta x = 8-2 = 6, \Delta y = 5-(-3) = 8$ (careful with negatives).

MEMORY
KEY

MID = avg coords; DIST = $\sqrt{dx^2 + dy^2}$

(A) Mid (5,1); Dist $\sqrt{96}$

(B) Mid (4,2); Dist 10

(C) Mid (5,1); Dist 10

(D) Mid (3,4); Dist 8

ANSWE C | Mid = $((2+8)/2, (-3+5)/2) = (5,1)$; Dist = $\sqrt{36+64} = 10$
R:

Q9. [Composite Area | ADD or SUBTRACT SIMPLE SHAPES]

A 10cm x 6cm rectangle has a semicircle (diameter 6cm) cut from one end. Find the remaining area. ($\pi = 3.14$)

■ Semicircle radius = 3 (not 6). Area of semicircle = half of πr^2 .

MEMORY
KEY

COMPOSITE = ADD or SUBTRACT SIMPLE SHAPES

(A) 31.74 cm²

(B) 45.87 cm²

(C) 37.74 cm²

(D) 53.13 cm²

**ANSWE
R:**

B | Rect=60, Semicircle= $0.5 \cdot 3.14 \cdot 9 = 14.13$. $60 - 14.13 = 45.87$ cm²

Q10. [30-60-90 Triangle | Sides: x, x*sqrt(3), 2x]

In a 30-60-90 triangle, the hypotenuse is 14. Find both legs.

■ Don't mix up: $\sqrt{2}$ is for 45-45-90 triangles. $\sqrt{3}$ is for 30-60-90.

MEMORY
KEY

30-60-90: short=x, long= $x \cdot \sqrt{3}$, hyp=2x | 45-45-90: leg=x, hyp= $x \cdot \sqrt{2}$

(A) 6 and $6 \cdot \sqrt{3}$

(B) 7 and $7 \cdot \sqrt{3}$

(C) 7 and $7 \cdot \sqrt{2}$

(D) 8 and $8 \cdot \sqrt{3}$

**ANSWE
R:**

B | $2x = 14 \rightarrow x = 7$. Legs: 7 and $7 \cdot \sqrt{3} \approx 12.12$