MATH MASTERY WORKSHEET

Algebra 2 · Geometry | 20 Essential Problems | Self-Study Edition

PART 1 — ALGEBRA 2

[MEMORY POINT] QUADRATIC: ax2 + bx + c = 0 --> FACTOR first, then FORMULA | SYSTEMS: SUBSTITUTE or ELIMINATE | EXPONENTIAL: same base = set exponents equal | KEY WORDS: combined=add / difference=subtract / product=multiply

A1 [Quadratic]

A rectangular garden has a length that is 3 meters more than twice its width. If the area of the garden is 54 m2, what is the width of the garden?

(A) w = 4 meters
(B) w = 4.5 meters
(C) w = 6 meters
(D) w = 3 meters

Note: Setup: w(2w+3) = 54 -> 2w2 + 3w - 54 = 0. KEY: Reject negative solutions for dimensions.

A2 [Systems of Equations]

A coffee shop sells lattes for $4 and cappuccinos for $5. One morning, they sold 80 drinks total and made $370. How many lattes were sold?

(A) 25 lattes
(B) 50 lattes
(C) 30 lattes
(D) 40 lattes

Note: Setup: x + y = 80 and 4x + 5y = 370. KEY: SUBSTITUTION — solve for one variable first.

A3 [Exponential Growth]

A bacteria population doubles every 3 hours. Starting with 200 bacteria, after how many hours will there be 1,600?

(A) 6 hours
(B) 9 hours
(C) 12 hours
(D) 15 hours

Note: 200 x 2^(t/3) = 1600 -> 2^(t/3) = 8 = 2^3 -> t = 9. KEY: SAME BASE — set exponents equal.

A4 [Quadratic — Vertex]

A ball's height is h(t) = -16t2 + 64t + 5. At what time does it reach MAXIMUM height? (Trap: don't solve h = 0!)

(A) t = 1 second
(B) t = 2 seconds
(C) t = 4 seconds
(D) t = 3 seconds

Note: Maximum = vertex. t = -b/(2a) = -64/(2x-16) = 2 sec. TRAP: h=0 gives landing time, NOT max height!

A5 [Rational Equations / Work Rate]

Pipe A fills a tank in 4 hours, Pipe B in 6 hours. Both open together — how long to fill the tank?

(A) 2 hours
(B) 2.4 hours
(C) 3 hours
(D) 5 hours

Note: 1/4 + 1/6 = 5/12 tanks/hr. Time = 12/5 = 2.4 hrs. KEY: Rate + Rate = Combined Rate, then flip for time.

A6 [Logarithms]

Investment grows as A = P x e^(0.06t). How long (years) to double? (Round to nearest tenth.)

(A) 10.2 years
(B) 11.6 years
(C) 13.0 years
(D) 8.5 years

Note: 2 = e^(0.06t) -> ln2 = 0.06t -> t = ln2/0.06 = 11.6 yrs. KEY: base e = use ln.

A7 [Inequalities]

Taxi 1: $3 base + $2/mile. Taxi 2: $5 base + $1.50/mile. After how many miles is Taxi 2 cheaper?

(A) After 2 miles
(B) After 4 miles
(C) After 6 miles
(D) After 3 miles

Note: 5 + 1.5m < 3 + 2m -> 2 < 0.5m -> m > 4. KEY: FLIP inequality only when dividing by negative.

A8 [Arithmetic Sequences]

A theater: 20 seats in row 1, each row has 3 more than the last. 15 rows total. Total seats?

(A) 450 seats
(B) 615 seats
(C) 525 seats
(D) 700 seats

Note: Sn = n/2 x (2a1 + (n-1)d) = 15/2 x (40 + 42) = 615. KEY: Sn = n/2 x (first + last).

A9 [Radical Equations]

Speed of roller coaster: v = sqrt(2gh), g = 10. If speed is 20 m/s, what is height h?

(A) 10 meters
(B) 20 meters
(C) 40 meters
(D) 5 meters

Note: 20 = sqrt(20h) -> 400 = 20h -> h = 20 m. KEY: ISOLATE root first, then SQUARE both sides.

A10 [Polynomial / Volume]

Box from 12x12 cm square: cut x from corners. Volume = 100 cm3. Which x is valid? (Trap: check domain!)

(A) x ≈ 1.1 cm
(B) x ≈ 7 cm (out of domain!)

(C) x ≈ 5.9 cm (out of domain!)

(D) x ≈ 3 cm

Note: V = x(12-2x)2 = 100. Domain: 0 < x < 6. Options B and C are outside domain! KEY: Domain first.

PART 2 — GEOMETRY

[MEMORY POINT] TRIANGLES: SOH-CAH-TOA | SIMILAR: corresponding sides are PROPORTIONAL | CIRCLES: inscribed angle = HALF the arc | 3D: Lateral + Base = Surface Area | Pyramid/Cone = (1/3) x base x height

G1 [Similar Triangles]

A 6-foot person casts a 4-foot shadow. At the same time, a tree casts a 30-foot shadow. How tall is the tree?

(A) 40 feet

(B) 45 feet

(C) 50 feet

(D) 20 feet

Note: 6/4 = h/30 -> h = 45 ft. KEY: PROPORTION — match same type on same side: height/shadow = height/shadow.

G2 [Pythagorean Theorem]

A ladder: base 5 feet from wall, reaches 12 feet up. How long is the ladder?

(A) 11 feet

(B) 13 feet

(C) 15 feet

(D) 17 feet

Note: c = sqrt(25+144) = sqrt(169) = 13 ft. KEY TRIPLES: (3,4,5), (5,12,13), (8,15,17), (7,24,25).

G3 [Circles — Area]

Circular lawn radius 7 m. A diameter fence splits it. What is the area of the SEMICIRCLE? (pi = 3.14) [Trap: don't use full circle!]

(A) 153.86 m2 (full circle)

(B) 76.93 m2

(C) 43.96 m2

(D) 49 m2

Note: Full circle = 3.14 x 49 = 153.86. Semicircle = 153.86/2 = 76.93 m2. TRAP: Option A is full circle!

G4 [Trigonometry — Angle of Depression]

From top of a 50-meter cliff, angle of depression to a boat is 30 degrees. How far is the boat from the base?

(A) 25 meters

(B) 86.6 meters

(C) 50 meters

(D) 100 meters

Note: tan30 = 50/d -> d = 50/tan30 = 86.6 m. KEY: Angle of Depression = Angle of Elevation from boat.

G5 [3D Solids — Cylinder]

Cylindrical water tank: radius 3 m, height 10 m. How many liters? (1 m3 = 1000 L, pi = 3.14)

(A) 94,200 L

(B) 282,600 L

(C) 188,400 L

(D) 565,200 L

Note: V = pi x r2 x h = 3.14 x 9 x 10 = 282.6 m3 = 282,600 L. KEY: radius = 3, NOT 6 (don't confuse with diameter).

G6 [Coordinate Geometry]

Point A = (1,2), Point B = (7,10). Find the MIDPOINT of AB. [Trap: don't use the distance formula!]

(A) (3, 4)

(B) (4, 6)

(C) (6, 8)

(D) (5, 7)

Note: Midpoint = ((1+7)/2, (2+10)/2) = (4, 6). TRAP: Midpoint = AVERAGE (add and halve). Distance = sqrt formula.

G7 [Circle Angles]

An inscribed angle intercepts an arc of 140 degrees. What is the inscribed angle? [Most common mistake: confusing with central angle!]

(A) 140 degrees (central angle)

(B) 70 degrees

(C) 280 degrees

(D) 35 degrees

Note: Inscribed angle = (1/2) x arc = 70 degrees. KEY: INSCRIBED = HALF. Central angle = arc (equal). TRAP: Option A!

G8 [Surface Area — Pyramid]

Square pyramid: base side 6 cm, slant height 5 cm. Total surface area?

(A) 60 cm2

(B) 96 cm2

(C) 120 cm2

(D) 30 cm2

Note: Base = 36, Lateral = 4 x (1/2 x 6 x 5) = 60. Total = 96 cm2. KEY: Use SLANT height for triangles, not pyramid height!

G9 [Transformations]

Triangle A(2,1), B(4,1), C(3,4). After REFLECTION over the y-axis, what are new coordinates of A?

(A) (2, -1)

(B) (-2, 1)

(C) (-2, -1)

(D) (1, 2)

Note: y-axis reflection: (x,y) -> (-x,y). So (2,1) -> (-2,1). KEY: y-axis=flip x | x-axis=flip y | origin=flip both.

G10 [Triangle Area Ratio]

Two triangles, same base 8 cm. Heights: 6 cm and 9 cm. Ratio of Area1 : Area2? [Shortcut: no full calculation needed!]

(A) 1 : 1

(B) 2 : 3

(C) 3 : 4

(D) 4 : 9

Note: A1=24, A2=36. Ratio = 24:36 = 2:3. SHORTCUT: Same base -> ratio = ratio of heights = 6:9 = 2:3!

ANSWER KEY

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Algebra 2	B	B	B	B	B	B	B	B	B	A
	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
Geometry	B	B	B	B	B	B	B	B	B	B