

# Algebra 2 & Geometry

Core Problems — Self-Study Edition

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## PART I — ALGEBRA 2

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### QUADRATIC EQUATIONS & FUNCTIONS

Q01

How many real solutions does  $2x^2 - 4x + 5 = 0$  have?

■ *DISCRIMINANT* =  $b^2 - 4ac$

$$D = b^2 - 4ac = (-4)^2 - 4(2)(5) = 16 - 40 = -24$$

$D < 0 \rightarrow$  no real roots |  $D = 0 \rightarrow$  one root |  $D > 0 \rightarrow$  two roots

- A. Two distinct real solutions  
B. Exactly one real solution  
C. No real solutions  
D. Infinitely many solutions
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### QUADRATIC EQUATIONS & FUNCTIONS

Q02

The vertex of  $y = 3(x + 4)^2 - 7$  is:

■ *VERTEX FORM*:  $y = a(x-h)^2 + k \rightarrow$  vertex =  $(h, k)$

$$y = 3(x - (-4))^2 + (-7) \rightarrow h = -4, k = -7$$

WARNING:  $x + 4$  means  $h = -4$ , NOT  $+4$ . Always flip the sign!

- A. (4, -7)  
B. (-4, -7)  
C. (-4, 7)  
D. (4, 7)
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### POLYNOMIALS & FACTORING

Q03

When  $f(x) = x^3 - 2x^2 + 5x - 3$  is divided by  $(x - 2)$ , the remainder is:

■ *REMAINDER THEOREM*: divide  $f(x)$  by  $(x-c) \rightarrow$  remainder =  $f(c)$

$$f(2) = (2)^3 - 2(2)^2 + 5(2) - 3$$

$$= 8 - 8 + 10 - 3 = 7$$

- A. 0  
B. 3  
C. 7  
D. -3
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### POLYNOMIALS & FACTORING



SEQUENCES & SERIES

Q08

Find the sum of:  $3 + 6 + 12 + 24 + \dots + 384$

■ GEOMETRIC SERIES SUM:  $S_n = a(1 - r^n) / (1 - r)$

$$a = 3, r = 2$$

$$3 \cdot 2^{(n-1)} = 384 \rightarrow 2^{(n-1)} = 128 = 2^7 \rightarrow n = 8$$

$$S_n = 3(1 - 2^8) / (1 - 2) = 3(-255)/(-1) = 765$$

- A. 381  
C. 765  
B. 512  
D. 192
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SEQUENCES & SERIES

Q09

What is the coefficient of  $x^2y^3$  in the expansion of  $(x + y)^5$ ?

■ BINOMIAL COEFFICIENT: term with  $x^{(n-k)}y^k$  has coefficient  $C(n, k)$

$$x^2y^3: k = 3 \text{ (y-exponent)}$$

$$C(5, 3) = 5! / (3! \cdot 2!) = 120 / 12 = 10$$

- A. 5  
C. 20  
B. 10  
D. 15
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INVERSE FUNCTIONS

Q10

Find the inverse of  $f(x) = (2x + 3) / (x - 1)$

■ INVERSE: swap  $x$  &  $y$ , then solve for  $y$

$$\text{Swap: } x = (2y + 3) / (y - 1)$$

$$x(y - 1) = 2y + 3 \rightarrow xy - 2y = x + 3$$

$$y(x - 2) = x + 3 \rightarrow f^{-1}(x) = (x + 3) / (x - 2)$$

- A.  $f^{-1}(x) = (x - 3) / (x + 2)$   
C.  $f^{-1}(x) = (x + 3) / (x - 2)$   
B.  $f^{-1}(x) = (x - 2) / (x + 3)$   
D.  $f^{-1}(x) = (2x + 1) / (x - 3)$
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## PART II — GEOMETRY

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TRIANGLES — SIMILARITY & CONGRUENCE

Q11

In triangle ABC, DE is a midsegment parallel to BC. If BC = 18, find DE.

■ MIDSEGMENT THEOREM:  $\text{midsegment} = (1/2) \times \text{parallel base}$

$$\text{Midsegment} = (1/2) \times BC = (1/2) \times 18 = 9$$

- A. 6  
B. 9  
C. 12  
D. 36
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**TRIANGLES — SIMILARITY & CONGRUENCE**

Q12

If triangle ABC ~ triangle DEF with AB = 8, DE = 12, area(ABC) = 32. Find area(DEF).

■ SIMILAR FIGURES:  $\text{area ratio} = (\text{scale factor})^2$  NOT just  $k$

$$\text{Scale factor } k = 12/8 = 3/2$$

$$\text{Area ratio} = k^2 = (3/2)^2 = 9/4$$

$$\text{Area(DEF)} = 32 \times 9/4 = 72$$

- A. 48  
B. 72  
C. 64  
D. 54
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**CIRCLES — ARCS, CHORDS & ANGLES**

Q13

An inscribed angle intercepts an arc of  $110^\circ$ . Find the inscribed angle measure.

■ INSCRIBED ANGLE =  $(1/2) \times \text{intercepted arc}$  | Central angle = arc

$$\text{Inscribed angle} = (1/2) \times 110^\circ = 55^\circ$$

- A.  $110^\circ$   
B.  $55^\circ$   
C.  $220^\circ$   
D.  $70^\circ$
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**CIRCLES — ARCS, CHORDS & ANGLES**

Q14

Chords AB and CD intersect at E inside a circle. AE=4, EB=9, CE=6. Find ED.

■ INTERSECTING CHORDS:  $AE \times EB = CE \times ED$

$$AE \times EB = CE \times ED$$

$$4 \times 9 = 6 \times ED \rightarrow ED = 36/6 = 6$$

- A. 4  
B. 6  
C. 9  
D. 13.5
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**COORDINATE GEOMETRY & TRANSFORMATIONS**

Q15

**A line through (2, 5) is perpendicular to  $y = (3/4)x - 2$ . Find its equation.**

■ **PERPENDICULAR SLOPES:**  $m \times m = -1 \rightarrow m = -1/m$  (negative reciprocal)

Perp. slope =  $-4/3$  (negative reciprocal of  $3/4$ )

$$y - 5 = -4/3(x - 2) \rightarrow y = -(4/3)x + 8/3 + 15/3 = -(4/3)x + 23/3$$

A.  $y = (3/4)x + 7/2$

B.  $y = -(3/4)x + 5$

C.  $y = -(4/3)x + 23/3$

D.  $y = (4/3)x - 1/3$

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**AREA, VOLUME & 3D GEOMETRY**

Q16

**A square pyramid has base side 6 and height 8. Find its volume.**

■ **PYRAMID VOLUME** =  $(1/3) \times \text{Base Area} \times \text{Height}$  (don't forget  $1/3!$ )

$$\text{Base area} = 6^2 = 36$$

$$V = (1/3) \times 36 \times 8 = 96$$

A. 288

B. 96

C. 144

D. 48

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**AREA, VOLUME & 3D GEOMETRY**

Q17

**A circle has radius 10. Find the area of a sector with central angle  $72^\circ$ . (Use  $\pi = 3.14$ )**

■ **SECTOR AREA** =  $(\text{theta} / 360) \times \pi \times r^2$

$$\text{Fraction} = 72/360 = 1/5$$

$$\text{Full circle area} = 3.14 \times 100 = 314$$

$$\text{Sector} = (1/5) \times 314 = 62.8$$

A. 31.4

B. 62.8

C. 125.6

D.  $20\pi$

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**ANGLE RELATIONSHIPS & PARALLEL LINES**

Q18

**Two interior angles of a triangle are  $47^\circ$  and  $63^\circ$ . Find the adjacent exterior angle.**

■ **EXTERIOR ANGLE THEOREM:** exterior angle = sum of 2 remote interior angles

$$\text{Exterior angle} = 47^\circ + 63^\circ = 110^\circ$$

$$\text{Verify: third interior} = 180^\circ - 47^\circ - 63^\circ = 70^\circ; \text{ exterior} = 180^\circ - 70^\circ = 110^\circ \checkmark$$

A.  $70^\circ$

B.  $117^\circ$

C.  $110^\circ$

D.  $133^\circ$

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**PYTHAGOREAN THEOREM & TRIPLES**

Q19

In right triangle PQR, hypotenuse PR = 26, leg PQ = 10. Find leg QR.

■ PYTHAGOREAN TRIPLES: 3-4-5 | 5-12-13 | 8-15-17 (and multiples!)

$$QR^2 = 26^2 - 10^2 = 676 - 100 = 576$$

QR = 24 (this is the triple 5-12-13 scaled by 2)

- A. 20
- B. 24
- C. 22
- D. 28

**CIRCLES — POWER OF A POINT**

Q20

From point P, a secant has external segment 4 and total length 9. Tangent from P = t. Find t.

■ TANGENT-SECANT:  $t^2 = \text{external segment} \times \text{whole secant length}$

$$t^2 = \text{external} \times \text{whole} = 4 \times 9 = 36$$

$$t = 6$$

TRAP: use the WHOLE secant (9), not the chord inside (5)

- A. 3
- B. 6
- C. 36
- D. 2 root 5

## ANSWER KEY

### Algebra 2 (Q01–Q10)

Q#	Answer	Memory Key
Q01	<b>C</b>	DISCRIMINANT = $b^2 - 4ac$
Q02	<b>B</b>	VERTEX FORM: $y = a(x-h)^2 + k \rightarrow \text{vertex} = (h, k)$
Q03	<b>C</b>	REMAINDER THEOREM: divide $f(x)$ by $(x-c) \rightarrow \text{remainder} = f(c)$
Q04	<b>B</b>	RATIONAL ROOT THEOREM: $\pm (\text{factors of constant}) / (\text{factors of leading coeff})$
Q05	<b>B</b>	LOG PRODUCT RULE: $\log M + \log N = \log(MN)$   Check DOMAIN after solving!
Q06	<b>B</b>	SAME BASE: $a^x = a^y \rightarrow x = y$
Q07	<b>C</b>	i-CYCLE (length 4): $i^1=i \ i^2=-1 \ i^3=-i \ i^4=1 \rightarrow \text{divide exp by 4, use remainder}$
Q08	<b>C</b>	GEOMETRIC SERIES SUM: $S_n = a(1 - r^n) / (1 - r)$
Q09	<b>B</b>	BINOMIAL COEFFICIENT: term with $x^{(n-k)} \cdot y^k$ has coefficient $C(n, k)$
Q10	<b>C</b>	INVERSE: swap $x$ & $y$ , then solve for $y$

### Geometry (Q11–Q20)

Q#	Answer	Memory Key
Q11	B	MIDSEGMENT THEOREM: midsegment = $(1/2) \times$ parallel base
Q12	B	SIMILAR FIGURES: area ratio = (scale factor) <sup>2</sup> NOT just k
Q13	B	INSCRIBED ANGLE = $(1/2) \times$ intercepted arc   Central angle = arc
Q14	B	INTERSECTING CHORDS: $AE \times EB = CE \times ED$
Q15	C	PERPENDICULAR SLOPES: $m \blacksquare \times m \blacksquare = -1 \rightarrow m \blacksquare = -1/m \blacksquare$ (negative reciprocal)
Q16	B	PYRAMID VOLUME = $(1/3) \times$ Base Area $\times$ Height (don't forget 1/3!)
Q17	B	SECTOR AREA = $(\text{theta} / 360) \times \text{pi} \times r^2$
Q18	C	EXTERIOR ANGLE THEOREM: exterior angle = sum of 2 remote interior angles
Q19	B	PYTHAGOREAN TRIPLES: 3-4-5   5-12-13   8-15-17 (and multiples!)
Q20	B	TANGENT-SECANT: $\text{tangent}^2 = \text{external segment} \times \text{whole secant length}$