

# MATH MASTERY

Algebra 2 & Geometry — Essential Problem Set

20 Questions | Multiple Choice | Answer Key Included

## ALGEBRA 2

### A-01 Quadratic Formula — Discriminant & Nature of Roots

★ **MEMORY KEY:**  $b^2 - 4ac$ : (+) two real roots, (0) one real root, (-) no real roots

**Example:**  $x^2 - 4x + 4 = 0 \rightarrow$  discriminant  $= 16 - 16 = 0 \rightarrow$  one real (double) root:  $x = 2$

**What is the discriminant of  $2x^2 - 3x + 5 = 0$ , and how many real solutions does it have?**

- (A)  $\Delta = 31$ , two real solutions  
(B)  $\Delta = -31$ , no real solutions  
(C)  $\Delta = 31$ , no real solutions  
(D)  $\Delta = -31$ , two real solutions

### A-02 Complex Numbers — Powers of $i$

★ **MEMORY KEY:**  $i$  cycle:  $i, -1, -i, 1$  (period 4). Divide exponent by 4, use remainder.

**Example:**  $i^{13}$ :  $13 / 4 = 3$  remainder 1  $\rightarrow i^1 = i$

**Simplify  $i^{47} + i^{50}$  ( $= i^{47} + i^{50}$ )**

- (A) 0  
(B)  $-i + 1$   
(C)  $-i - 1$   
(D)  $i - 1$

### A-03 Polynomial Functions — End Behavior

★ **MEMORY KEY:** Leading term decides all. Even degree  $\rightarrow$  same ends. Odd  $\rightarrow$  opposite ends. Negative coefficient flips.

**Example:**  $f(x) = -x^4 + 3x$ : even degree, negative  $\rightarrow$  both ends go to  $-\infty$

**Describe the end behavior of  $f(x) = -3x^3 + 7x^2 - 2$**

- (A)  $x \rightarrow +\infty$ :  $f \rightarrow +\infty$ ;  $x \rightarrow -\infty$ :  $f \rightarrow -\infty$   
(B)  $x \rightarrow +\infty$ :  $f \rightarrow -\infty$ ;  $x \rightarrow -\infty$ :  $f \rightarrow +\infty$   
(C) Both ends go to  $+\infty$   
(D) Both ends go to  $-\infty$

### A-04 Logarithms — Log Properties — Common Mistake

★ **MEMORY KEY:**  $\log(a/b) = \log a - \log b$ . NOT  $\log a / \log b$  (WRONG!)

**Example:**  $\log_2(8) - \log_2(2) = \log_2(8/2) = \log_2(4) = 2$

**Which expression equals  $\log_2(27x^4)$ ?**

- (A)  $3 + 4 \cdot \log_2(x)$   
(C)  $3 \cdot \log_2(x^4)$

(B)  $27 + 4 \cdot \log \blacksquare(x)$

(D)  $4 \cdot \log \blacksquare(27x)$

**A-05 Rational Exponents — Fractional Exponent  $\leftrightarrow$  Radical**

★ **MEMORY KEY:**  $x^{(m/n)} = (n\text{-th root of } x)^m$ . Bottom = root index, Top = power.

**Example:**  $8^{(2/3)} = (\text{cube-root of } 8)^2 = 2^2 = 4$

**Evaluate  $(-27)^{(2/3)}$**

(A) 9

(C) 18

(B) -9

(D) Undefined

**A-06 Sequences & Series — Infinite Geometric Series Sum**

★ **MEMORY KEY:** Infinite geometric ( $|r| < 1$ ):  $S = a / (1 - r)$

**Example:**  $2 + 6 + 18 + 54$ :  $a=2, r=3, n=4 \rightarrow S = 2 \cdot (1-81)/(1-3) = 80$

**Find the sum of the infinite geometric series:  $12 + 4 + 4/3 + \dots$**

(A) 16

(C) 36

(B) 18

(D) The series diverges

**A-07 Radical Equations — Extraneous Solutions**

★ **MEMORY KEY:** After solving radical equation  $\rightarrow$  ALWAYS check back in the original. Squaring creates fake solutions!

**Example:**  $\sqrt{x+3} = x-3$ : square  $\rightarrow$  solutions  $x=6$  and  $x=1$ . Check  $x=1$ :  $\sqrt{4}=2$  but  $1-3=-2$ . Extraneous!

**Solve  $\sqrt{3x + 4} = x - 2$ . Which value(s) are valid solutions?**

(A)  $x = 0$  and  $x = 5$

(C)  $x = 0$  only

(B)  $x = 7$  only

(D) No real solution

**A-08 Conic Sections — Circle Equation — Standard Form**

★ **MEMORY KEY:**  $(x-h)^2 + (y-k)^2 = r^2$ . Center= $(h,k)$ , radius= $r$ . Use completing the square!

**Example:**  $x^2+y^2-4x+6y=3 \rightarrow (x-2)^2+(y+3)^2=16$ . Center  $(2, -3)$ , radius 4.

**Find the center and radius of:  $x^2 + y^2 - 6x + 8y - 11 = 0$**

(A) Center  $(3, -4)$ , radius 6

(C) Center  $(3, -4)$ , radius 36

(B) Center  $(-3, 4)$ , radius 6

(D) Center  $(-6, 8)$ , radius 11

**A-09 Exponential Equations — Solve by Matching Bases**

★ **MEMORY KEY:** Same base  $\rightarrow$  set exponents equal. Different base  $\rightarrow$  take log of both sides.

**Example:**  $4^x = 8 \rightarrow 2^{(2x)} = 2^3 \rightarrow 2x = 3 \rightarrow x = 1.5$

**Solve for  $x$ :  $9^{(x+1)} = 27^{(x-1)}$**

(A)  $x = 5$

(C)  $x = 7$

(B)  $x = 3$

(D)  $x = 1$

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**A-10 Systems of Equations — Non-Linear System — Substitution**

★ **MEMORY KEY:** Line + Parabola can have 0, 1, or 2 intersections. Substitute linear into quadratic.

**Example:**  $y=x+2$  and  $y=x^2$ : sub  $\rightarrow x+2=x^2 \rightarrow x^2-x-2=0 \rightarrow x=2$  or  $x=-1$

**How many intersection points do  $y = x^2 - 4$  and  $y = 2x - 1$  have?**

(A) 0 intersection points

(C) 2 intersection points

(B) 1 intersection point

(D) Infinitely many

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# GEOMETRY

## G-01 Triangle Congruence — SSS / SAS / ASA / AAS / HL

★ **MEMORY KEY:** SSA is NOT a congruence rule! AAA only proves similarity, not congruence.

**Example:** Two triangles: common side, angle adjacent to it equal, another angle equal → ASA.

In  $\triangle ABC$  and  $\triangle DEF$ :  $AB = DE$ ,  $BC = EF$ , and  $\angle B = \angle E$ . Which congruence postulate applies?

- (A) SSS (C) ASA  
(B) SAS (D) AAS

## G-02 Parallel Lines & Transversal — Angle Relationships

★ **MEMORY KEY:** Co-interior (same-side interior) = supplementary (sum =  $180^\circ$ ). Alternate = equal.

**Example:** Alternate interior:  $70^\circ \rightarrow$  other side =  $70^\circ$ . Co-interior:  $70^\circ + ? = 180^\circ \rightarrow ? = 110^\circ$

Two parallel lines cut by a transversal have co-interior angles  $(3x+15)^\circ$  and  $(2x+25)^\circ$ . Find  $x$ .

- (A)  $x = 28$  (C)  $x = 16$   
(B)  $x = 29$  (D)  $x = 10$

## G-03 Pythagorean Theorem — Right Triangle — Missing Side

★ **MEMORY KEY:**  $a^2 + b^2 = c^2$  where  $c$  = hypotenuse (always the longest side). Memorize: (3,4,5), (5,12,13), (8,15,17)

**Example:** Legs 5 and 12: hypotenuse =  $\sqrt{25+144} = \sqrt{169} = 13$

In a right triangle, one leg = 7 and the hypotenuse = 25. Find the other leg.

- (A) 24 (C) 18  
(B)  $\sqrt{674}$  (D) 26

## G-04 Circle Theorems — Inscribed Angle Theorem

★ **MEMORY KEY:** Inscribed angle =  $(1/2) \times$  intercepted arc. Central angle = arc. Inscribed = half of central!

**Example:** Arc =  $140^\circ \rightarrow$  Central angle =  $140^\circ \rightarrow$  Inscribed angle =  $70^\circ$

An inscribed angle intercepts an arc of  $110^\circ$ . What is the inscribed angle measure?

- (A)  $110^\circ$  (C)  $55^\circ$   
(B)  $220^\circ$  (D)  $70^\circ$

## G-05 Triangle Similarity — Scale Factor & Area Ratio

★ **MEMORY KEY:** Similar triangles: Area ratio = (scale factor) $^2$ . Volume ratio = (scale factor) $^3$ .

**Example:** Scale factor 1:3  $\rightarrow$  Area ratio 1:9. Small area = 5  $\rightarrow$  large area = 45.

■ABC ~ ■DEF with scale factor 1:4. Area of ■ABC = 9 cm<sup>2</sup>. Find area of ■DEF.

- (A) 36 cm<sup>2</sup> (C) 72 cm<sup>2</sup>  
(B) 144 cm<sup>2</sup> (D) 18 cm<sup>2</sup>

**G-06 Volume of Solids — Sphere / Cone / Cylinder**

★ MEMORY KEY: Sphere =  $\frac{4}{3}\pi r^3$ , Cone =  $\frac{1}{3}\pi r^2 h$ , Cylinder =  $\pi r^2 h$ . Cone = 1/3 of cylinder!

Example: Cylinder  $r=3$ ,  $h=10$ :  $V = 90\pi$ . Cone same:  $V = 30\pi$ .

**A cone has radius 6 cm and height 9 cm. Find its volume. (Leave in terms of pi)**

- (A)  $324\pi$  cm<sup>3</sup> (C)  $972\pi$  cm<sup>3</sup>  
(B)  $108\pi$  cm<sup>3</sup> (D)  $162\pi$  cm<sup>3</sup>

**G-07 Coordinate Geometry — Midpoint Formula — Find Endpoint**

★ MEMORY KEY: Midpoint = average of coordinates. To find endpoint: double the midpoint then subtract the known point.

Example: Midpoint of (1,3) and (5,7):  $((1+5)/2, (3+7)/2) = (3, 5)$

**Point M(5,1) is the midpoint of AB. If A = (2, -3), find B.**

- (A) B = (8, 5) (C) B = (7, 4)  
(B) B = (3.5, -1) (D) B = (12, 8)

**G-08 Special Right Triangles — 30-60-90 and 45-45-90**

★ MEMORY KEY: 45-45-90: sides =  $x, x, x\sqrt{2}$ . 30-60-90: sides =  $x, x\sqrt{3}, 2x$ . ( $x$  = short leg)

Example: 30-60-90 with hypotenuse 10: short leg = 5, long leg =  $5\sqrt{3}$

**In a 45-45-90 triangle, the hypotenuse is  $8\sqrt{2}$ . Find each leg.**

- (A) 8 (C) 16  
(B)  $4\sqrt{2}$  (D) 4

**G-09 Polygon Interior Angles — Sum of Interior Angles**

★ MEMORY KEY: Sum =  $(n-2) \times 180^\circ$ . Each angle of regular  $n$ -gon =  $(n-2) \times 180^\circ / n$

Example: Hexagon ( $n=6$ ):  $(6-2) \times 180 = 720^\circ$ . Each angle =  $720/6 = 120^\circ$

**What is each interior angle of a regular nonagon (9 sides)?**

- (A)  $120^\circ$  (C)  $160^\circ$   
(B)  $140^\circ$  (D)  $135^\circ$

**G-10 Transformations — Rotations —  $90^\circ, 180^\circ, 270^\circ$**

★ MEMORY KEY:  $90^\circ$  CCW:  $(x,y) \rightarrow (-y,x)$ .  $180^\circ$ :  $(x,y) \rightarrow (-x,-y)$ .  $270^\circ$  CCW =  $90^\circ$  CW:  $(x,y) \rightarrow (y,-x)$

Example: Point (3,2) rotated  $90^\circ$  CCW:  $\rightarrow (-2, 3)$

**Point P(4, -3) is rotated  $270^\circ$  counterclockwise about the origin. New coordinates?**

(A) (-3, -4)

(C) (-4, 3)

(B) (3, 4)

(D) (3, -4)

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## ANSWER KEY

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Q	ANS	Q	ANS	Q	ANS	Q	ANS
A-01	B	A-02	C	A-03	B	A-04	A
A-05	A	A-06	B	A-07	B	A-08	A
A-09	A	A-10	C	G-01	B	G-02	A
G-03	A	G-04	C	G-05	B	G-06	B
G-07	A	G-08	A	G-09	B	G-10	A

Score: \_\_\_\_ / 20 Date: \_\_\_\_\_ Name: \_\_\_\_\_