Algebra 2 & Geometry

Self-Study Problem Set — Core Topics

ALGEBRA 2

A-01 QUADRATIC FUNCTIONS

Vertex Form Conversion

- *VERTEX = complete the square -> y = a(x-h)^2 + k*

EXAMPLE

Example: y = x^2 - 6x + 5 -> y = (x-3)^2 - 4, Vertex: (3, -4)

Q: Convert y = x^2 - 4x + 7 to vertex form. What is the value of k?

Hint: Half of -4 is -2; (-2)^2 = 4. Add and subtract 4.

Answer: _____

A-02 COMPLEX NUMBERS

Powers of i

- *i CYCLE every 4: i^1=i, i^2=-1, i^3=-i, i^4=1 -> divide exponent by 4, use remainder*

EXAMPLE

Example: i^23 = i^(4x5+3) -> remainder 3 -> i^3 = -i

Q: Simplify i^47. Enter: i, -1, -i, or 1.

Hint: 47 / 4 = 11 remainder 3.

Answer: _____

A-03 POLYNOMIAL FUNCTIONS

Remainder Theorem

- *REMAINDER THEOREM: f(a) = remainder when f(x) / (x - a)*

EXAMPLE

Example: f(x) = x^3 - 2x + 1 divided by (x-2). f(2) = 8-4+1 = 5

Q: Find the remainder when f(x) = x^3 - 3x^2 + 2x - 5 is divided by (x - 2).

Hint: Just plug x = 2 into f(x). No long division!

Answer: _____

A-04 EXPONENTIAL & LOGARITHMS

Log to Exponential Switch

■ *LOG SWITCH: log_b(a) = c <-> b^c = a ("base stays base")*

EXAMPLE

Example: log_2(8) = 3 <-> 2^3 = 8

Q: Solve: log_3(x) = 4. What is x?

Hint: Switch to exponential: 3^4 = x.

Answer: _____

A-05 RATIONAL EXPRESSIONS

Excluded Values

■ *EXCLUDED VALUES: set denominator = 0, solve for x*

EXAMPLE

Example: f(x) = (x+1)/((x-2)(x+3)) -> excluded: x=2 and x=-3

Q: For f(x) = (x^2 - 1) / (x^2 - x - 6), find the SUM of all excluded values.

Hint: Factor: x^2 - x - 6 = (x-3)(x+2)

Answer: _____

A-06 SEQUENCES & SERIES

Geometric Series Sum

■ *GEOMETRIC SUM: S_n = a(1 - r^n) / (1 - r)*

EXAMPLE

Example: 2+6+18+54: a=2, r=3, n=4 -> S_4 = 2(1-81)/(1-3) = 80

Q: Find the sum of the first 5 terms: 3, 6, 12, 24, 48, ...

Hint: a=3, r=2, n=5. Or just add them up!

Answer: _____

A-07 MATRICES

Matrix Multiplication

■ *MATRIX MULT: (m x n)(n x p) = (m x p) — inner dimensions MUST match*

EXAMPLE

Example: [2 1; 3 0] x [1; 4] = [2+4; 3+0] = [6; 3]

Q: A = [[2,3],[1,4]], B = [[1],[2]]. Find the TOP entry of AB.

Hint: Top row of A times column B: (2)(1) + (3)(2)

Answer: _____

A-08 CONIC SECTIONS

Circle Equation

- *CIRCLE: (x-h)^2 + (y-k)^2 = r^2 -> center (h,k), radius r (SIGNS FLIP!)*

EXAMPLE

Example: (x+2)^2 + (y-3)^2 = 25 -> center (-2, 3), r = 5

Q: Find the radius of: (x-5)^2 + (y+1)^2 = 49.

Hint: r^2 = 49, so r = ?

Answer: _____

A-09 TRIGONOMETRY

Unit Circle Exact Values

- *sin = y-coord, cos = x-coord, tan = sin/cos | Q2: sin(+), cos(-)*

EXAMPLE

Example: sin(150deg): Q2, ref angle=30deg, sin is + -> sin(150deg) = 1/2

Q: What is the exact value of sin(150 degrees)? Enter as fraction.

Hint: Reference angle = 30 degrees. Q2 so sin is positive.

Answer: _____

A-10 PROBABILITY

Binomial Probability

- *BINOMIAL: P = C(n,k) * p^k * (1-p)^(n-k) — don't forget C(n,k)!*

EXAMPLE

Example: 3 coins, P(2 heads) = C(3,2)(1/2)^2(1/2)^1 = 3/8

Q: Fair coin flipped 4 times. P(exactly 3 heads)? Enter as fraction.

*Hint: C(4,3) = 4. P = 4 * (1/2)^3 * (1/2)^1*

Answer: _____

GEOMETRY

G-01 SIMILAR TRIANGLES

Proportional Sides

- *SIMILAR: corresponding sides proportional -> set up ratios, CROSS MULTIPLY*

EXAMPLE

Example: AB/DE = BC/EF -> 4/6 = 6/EF -> EF = 9

Q: Triangle ABC ~ Triangle DEF. AB=6, BC=9, DE=8. Find EF.

Hint: 6/8 = 9/EF. Cross multiply.

Answer: _____

G-02 PYTHAGOREAN THEOREM

Pythagorean Triples

- *TRIPLES: 3-4-5, 5-12-13, 8-15-17, 7-24-25 (and multiples!)*

EXAMPLE

Example: Legs 6, 8 -> 3-4-5 x2 -> hypotenuse = 10

Q: Right triangle with legs 5 and 12. Find the hypotenuse.

Hint: Classic triple! 5-12-?

Answer: _____

G-03 CIRCLE THEOREMS

Inscribed Angle Theorem

- *INSCRIBED ANGLE = (1/2) x intercepted arc*

EXAMPLE

Example: Central angle = 80 deg -> Inscribed angle = 40 deg

Q: Inscribed angle intercepts arc of 110 degrees. Find the inscribed angle.

Hint: Inscribed angle = (1/2) x 110

Answer: _____

G-04 VOLUME & SURFACE AREA

Cylinder vs Cone

- *CYLINDER: V = pi*r^2*h | CONE: V = (1/3)*pi*r^2*h*

EXAMPLE

Example: Cone r=3, h=4 -> V = (1/3)*pi*9*4 = 12*pi

Q: Cone with radius 3 and height 4. Find the volume. (Enter as: 12pi)

*Hint: V = (1/3)*pi*r^2*h*

Answer: _____

G-05 COORDINATE GEOMETRY

Distance Formula

■ *DISTANCE: d = sqrt((x2-x1)^2 + (y2-y1)^2) | MIDPOINT: ((x1+x2)/2, (y1+y2)/2)*

EXAMPLE

Example: A(2,4) B(8,0) -> d = sqrt(36+16) = sqrt(52) = 2*sqrt(13)

Q: Find the distance between A(1,2) and B(4,6).

Hint: d = sqrt((4-1)^2 + (6-2)^2) = sqrt(9+16)

Answer: _____

G-06 TRIANGLE CONGRUENCE

SSA is NOT Congruence!

■ *CONGRUENCE: SSS, SAS, ASA, AAS, HL (right tri only) | SSA and AAA = NOT congruent*

EXAMPLE

Example: AAA = SIMILAR only, not congruent (different sizes possible)

Q: Two triangles have three pairs of equal angles (AAA). Are they congruent? YES or NO.

Hint: Does AAA guarantee same SIZE?

Answer: _____

G-07 TRANSFORMATIONS

Reflection Rules

■ *over x-axis: (x,y) -> (x,-y) | over y-axis: (x,y) -> (-x,y) | over y=x: (x,y) -> (y,x)*

EXAMPLE

Example: (3,-5) over y-axis -> (-3,-5)

Q: P(4,-3) reflected over the x-axis. What is the new y-coordinate?

Hint: x-axis reflection flips the sign of y.

Answer: _____

G-08 ANGLES & PARALLEL LINES

Transversal Angle Pairs

■ *Alternate interior = EQUAL | Co-interior (same-side) = SUPPLEMENTARY (sum 180)*

EXAMPLE

Example: One angle = 65 deg -> co-interior = 180-65 = 115 deg

Q: Parallel lines cut by transversal. One co-interior angle = 72 deg. Find the other.

Hint: Co-interior angles add to 180.

Answer: _____

G-09 TRIGONOMETRY IN TRIANGLES

SOHCAHTOA

■ *SOH: sin = Opp/Hyp | CAH: cos = Adj/Hyp | TOA: tan = Opp/Adj*

EXAMPLE

Example: angle=30, hyp=10 -> opp = 10*sin(30) = 5

Q: Right triangle, angle A = 45 deg, hypotenuse = 10. Find the side opposite angle A.

Hint: sin(45) = opp/10. sin(45) = sqrt(2)/2

Answer: _____

G-10 PROOFS & LOGIC

Contrapositive

■ *CONTRAPOSITIVE: "If P then Q" <-> "If NOT Q then NOT P" (always equivalent!)*

EXAMPLE

Example: "If rains, ground is wet" Contrapositive: "If not wet, did not rain"

Q: Contrapositive of "If equilateral, then all angles equal"?
A) If angles equal, equilateral
B) If not equilateral, not all angles equal
C) If not all angles equal, not equilateral
D) If equal angles, equilateral

Hint: Flip AND negate BOTH parts.

Answer: _____

ANSWER KEY

A-01 3	**A-02** -i	**A-03** -5	**A-04** 81	**A-05** 1
A-06 93	**A-07** 8	**A-08** 7	**A-09** 1/2	**A-10** 1/4
G-01 12	**G-02** 13	**G-03** 55	**G-04** 12pi	**G-05** 5
G-06 NO	**G-07** 3	**G-08** 108	**G-09** 5*sqrt(2)	**G-10** C

Score: _____ / 20

Algebra 2: _____ / 10 Geometry: _____ / 10