

MATH MASTERY

Algebra 2 & Geometry 2 — Self-Study Practice Sheet

20 Key Questions · High-Frequency Mistakes · Memory Points

SECTION A — ALGEBRA 2

A-01 · QUADRATICS — VERTEX

If $f(x) = x^2 - 6x + 5$, what is the minimum value of $f(x)$?

■ **MEMORY: KEY:** vertex $x = -b/2a$, then plug back in. Min value = $f(\text{vertex } x)$.

Example: e.g. $g(x) = x^2 - 4x + 7 \rightarrow x = 2 \rightarrow g(2) = 3$ (minimum)

■ *Common error: answering $x = 3$ (the x -coord) instead of $f(3) = -4$.*

Answer: _____

A-02 · LOGARITHMS — LOG LAWS

Solve: $\log \blacksquare x = 1 + \log \blacksquare 3$

■ **MEMORY: KEY:** $\log A + \log B = \log AB$. Rewrite $1 = \log \blacksquare 2$, then combine.

Example: e.g. $\log \blacksquare x = 1 + \log \blacksquare 2 \rightarrow \log \blacksquare (x/2) = 1 \rightarrow x = 6$

■ *Rewrite the constant "1" as a log first before combining.*

Answer: _____

A-03 · ARITHMETIC SEQUENCE — nth TERM

The 3rd term = 11, 7th term = 27. Find the common difference d .

■ **MEMORY: KEY:** $d = (a_n - a_m) \div (n - m)$. Count the GAPS, not the terms.

Example: e.g. $a_3=5, a_7=21 \rightarrow d = (21-5)/4 = 4$

■ *From term 3 to term 7 = 4 gaps (not 5). Easy counting mistake!*

Answer: _____

A-04 · RATIONAL EXPRESSIONS — SIMPLIFY

Simplify: $(x^2 - 9) / (x^2 - x - 6)$. What is the constant in the numerator?

■ **MEMORY: KEY:** Factor BOTH top and bottom, then cancel. $a^2-b^2 = (a+b)(a-b)$.

Example: e.g. $(x^2-4)/(x^2-x-2) = (x+2)(x-2)/(x-2)(x+1) = (x+2)/(x+1)$

■ *Always factor completely before canceling — never cancel individual terms.*

Answer: _____

A-05 · EXPONENTIALS — SAME BASE

Solve: $2^{(x-1)} = 16$

■ **MEMORY: KEY:** Make SAME BASE \rightarrow set exponents equal. $16 = 2^4$.

Example: e.g. $3^{(x+1)} = 27 \rightarrow 3^{(x+1)} = 3^3 \rightarrow x+1=3 \rightarrow x=2$

■ *After setting $x-1 = 4$, don't forget to add 1: $x = 5$.*

Answer: _____

A-06 · FUNCTIONS — COMPOSITION

$f(x) = 2x+1, g(x) = x-3$. Find $f(g(4))$.

■ **MEMORY: KEY:** INSIDE OUT — evaluate $g(4)$ first, then plug into f .

Example: e.g. $f(x)=3x, g(x)=x+2, f(g(1))$: $g(1)=3 \rightarrow f(3)=9$

■ *$f(g(4)) \neq g(f(4))$. Composition is NOT commutative!*

Answer: _____

A-07 · POLYNOMIALS — REMAINDER THEOREM

Find the remainder when $p(x) = x^3 - 2x^2 + x - 3$ is divided by $(x - 2)$.

■ **MEMORY: KEY:** Dividing by $(x - a)$? Just compute $p(a)$. No long division needed!

Example: e.g. $q(x)=x^2+3x-1 \div (x-1)$: $q(1)=1+3-1=3$ (remainder)

■ *Dividing by $(x-2)$ means plug in $x = +2$, NOT -2 .*

Answer: _____

A-08 · GEOMETRIC SERIES — INFINITE SUM

Find the sum: $8 + 4 + 2 + 1 + \dots$

■ **MEMORY: KEY:** $S = a \div (1 - r)$, only valid when $|r| < 1$. Here $r = 1/2$.

Example: e.g. $6 + 2 + 2/3 + \dots$: $a=6$, $r=1/3 \rightarrow S = 6/(1-1/3) = 9$

■ *Check r first! Formula invalid if $|r| \geq 1$.*

Answer: _____

A-09 · COMPLEX NUMBERS — MULTIPLY

Simplify $(1 + i)^2$. What is the coefficient of i in the result?

■ **MEMORY: KEY:** Expand normally, then replace $i^2 = -1$. That's the only rule.

Example: e.g. $(2+i)(2-i) = 4 - i^2 = 4+1 = 5$ (real number)

■ *Don't forget to replace i^2 with -1 after expanding.*

Answer: _____

A-10 · MATRICES — DETERMINANT 2x2

Find det $\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$.

■ **MEMORY: KEY:** $\det = ad - bc$. Multiply down-right diagonal, subtract down-left.

Example: e.g. $\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$: $\det = 3 \times 5 - 1 \times 2 = 15 - 2 = 13$

■ *Order matters: it's $ad - bc$, NOT $bc - ad$.*

Answer: _____

SECTION G — GEOMETRY 2

G-01 · CIRCLES — INSCRIBED ANGLE

An inscribed angle intercepts an arc of 80° . What is the angle?

■ **MEMORY: KEY:** Inscribed angle = HALF the intercepted arc. Central angle = arc.

Example: e.g. arc = $120^\circ \rightarrow$ inscribed angle = 60°

■ Many confuse inscribed ($\div 2$) and central (= arc). Identify type first!

Answer: _____

G-02 · SIMILARITY — SIDE RATIOS

Similar triangles: sides 4, 6, 8 and ?, 9, 12. Find the missing side.

■ **MEMORY: KEY:** Find SCALE FACTOR first: $9/6 = 1.5$. Apply to all sides.

Example: e.g. sides 3,5,7 and ?,10,14: scale=2 \rightarrow missing=6

■ All three ratios must be equal. Verify: $9/6 = 12/8 = 1.5 \checkmark$

Answer: _____

G-03 · TRIGONOMETRY — SOH-CAH-TOA

$\sin \theta = 0.6$ and hypotenuse = 50. Find the opposite side.

■ **MEMORY: KEY:** SOH: opposite = $\sin \times$ hypotenuse. CAH: adjacent = $\cos \times$ hyp.

Example: e.g. $\cos \theta = 0.8$, hyp=20 \rightarrow adjacent = $0.8 \times 20 = 16$

■ sin uses OPPOSITE; cos uses ADJACENT. Don't swap them!

Answer: _____

G-04 · CIRCLES — EXTERNAL ANGLE

Tangent-secant from external point: far arc= 80° , near arc= 30° . External angle?

■ **MEMORY: KEY:** External angle = (far arc - near arc) $\div 2$. Always bigger minus smaller.

Example: e.g. far= 100° , near= $40^\circ \rightarrow$ angle = $(100-40)/2 = 30^\circ$

■ It's a DIFFERENCE divided by 2, not a sum!

Answer: _____

G-05 · SIMILARITY — AREA RATIO

Similar triangles with side ratio 1:3. Smaller area = 4. Larger area = ?

■ **MEMORY: KEY:** Area ratio = (side ratio) 2 . If sides 1:3, areas 1:9.

Example: e.g. side ratio 1:2, small area=5 \rightarrow large area = $5 \times 4 = 20$

■ #1 mistake: using side ratio directly instead of squaring it!

Answer: _____

G-06 · COORDINATE GEOMETRY — DISTANCE

Find the distance between (1, 2) and (4, 6).

■ **MEMORY: KEY:** distance = $\sqrt{(\Delta x)^2 + (\Delta y)^2}$. It's Pythagorean theorem!

Example: e.g. (0,0) to (3,4): $\sqrt{(9+16)} = \sqrt{25} = 5$ (3-4-5 triple)

■ Watch for Pythagorean triples: 3-4-5, 5-12-13, 8-15-17.

Answer: _____

G-07 · POLYGONS — INTERIOR ANGLES

What is each interior angle of a regular hexagon?

■ **MEMORY: KEY:** Each angle = $(n-2) \times 180 \div n$. Total sum = $(n-2) \times 180$.

Example: e.g. pentagon $n=5$: sum= $540^\circ \rightarrow$ each = 108°

■ Hexagon has $n=6$. Do NOT use 6×180 — that's exterior + interior!

Answer: _____

G-08 · CIRCLES — INTERSECTING CHORDS

Two chords cross: segments 3 & 4 on one; 6 & ? on the other. Find ?

■ **MEMORY: KEY:** part \times part = part \times part. Intersecting chords: $a \cdot b = c \cdot d$.

Example: e.g. $(2)(6) = (3)(x) \rightarrow x = 4$

■ *Only works for chords intersecting INSIDE the circle.*

Answer: _____

G-09 · 3D GEOMETRY — CONE VOLUME

Cone with radius=3, height=4. Volume = ? π (give integer coefficient only)

■ **MEMORY: KEY:** $V = (1/3)\pi r^2 h$. "One-third of the cylinder."

Example: e.g. $r=3, h=7: V = (1/3)\pi(9)(7) = 21\pi$

■ *Don't forget the 1/3 factor. Cone \neq cylinder!*

Answer: _____

G-10 · TRANSFORMATIONS — ROTATION

A(4,0) maps to A'(2 $\sqrt{2}$, 2 $\sqrt{2}$) by CCW rotation. Angle of rotation = ? $^\circ$

■ **MEMORY: KEY:** $\cos \theta = 2\sqrt{2}/4 = \sqrt{2}/2$. When $x=y$ after rotation $\rightarrow 45^\circ$.

Example: $45^\circ \rightarrow (\sqrt{2}/2 \cdot r, \sqrt{2}/2 \cdot r)$ $60^\circ \rightarrow (1/2 \cdot r, \sqrt{3}/2 \cdot r)$ $90^\circ \rightarrow (0, r)$

■ *Both coordinates equal? \rightarrow that's always a 45° rotation signature!*

Answer: _____

ANSWER KEY

ALGEBRA 2

Q	Answer	Q	Answer	Q	Answer
A-01	-4	A-02	6	A-03	4
A-04	3	A-05	5	A-06	3
A-07	-1	A-08	16	A-09	2
A-10	2				

GEOMETRY 2

Q	Answer	Q	Answer	Q	Answer
G-01	40°	G-02	6	G-03	30
G-04	25°	G-05	36	G-06	5
G-07	120°	G-08	2	G-09	12
G-10	45°				