

# Hyperbolic Functions

sinh · cosh · tanh

Master Worksheet — 20 Problems

Basics · Identities · Derivatives · Integrals · Inverse · Applications · Exam Problems

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 100

## [ FORMULA REFERENCE SHEET ]

### DEFINITIONS

$$\sinh x = (e^x - e^{-x}) / 2$$

$$\cosh x = (e^x + e^{-x}) / 2$$

$$\tanh x = \sinh x / \cosh x = (e^x - e^{-x}) / (e^x + e^{-x})$$

$$\operatorname{sech} x = 1/\cosh x, \operatorname{csch} x = 1/\sinh x, \operatorname{coth} x = \cosh x/\sinh x$$

### KEY IDENTITIES

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh(2x) = 2 \sinh x \cosh x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1, \operatorname{coth}^2 x - \operatorname{csch}^2 x = 1$$

### DERIVATIVES

$$d/dx [\sinh x] = \cosh x$$

$$d/dx [\cosh x] = \sinh x \text{ (NOTE: POSITIVE, not negative!)}$$

$$d/dx [\tanh x] = \operatorname{sech}^2 x$$

$$d/dx [\sinh^{-1} x] = 1/\sqrt{x^2+1}$$

$$d/dx [\cosh^{-1} x] = 1/\sqrt{x^2-1} \text{ (} x > 1 \text{)}$$

$$d/dx [\tanh^{-1} x] = 1/(1-x^2) \text{ (} |x| < 1 \text{)}$$

### INTEGRALS

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\int \tanh x \, dx = \ln(\cosh x) + C$$

$$\int dx/\sqrt{x^2+a^2} = \sinh^{-1}(x/a) + C = \ln(x + \sqrt{x^2+a^2}) + C$$

### INVERSE HYPERBOLIC (LOG FORMS)

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}), \text{ domain: all real}$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2-1}), \text{ domain: } x \geq 1$$

$$\tanh^{-1} x = (1/2) \ln[(1+x)/(1-x)], \text{ domain: } |x| < 1$$

## Q1 Definition of sinh and cosh

★ Easy | Basics

(A) The value of  $\cosh(0)$  is: (A) 0 (B) 1 (C) -1 (D) e

(B)  $\sinh x$  is a(n) \_\_\_ function (satisfies  $f(-x) = -f(x)$ ): (A) Even (B) Odd (C) Neither (D) Periodic

(C)  $\cosh^2 x - \sinh^2 x = ?$  (A) 0 (B) 2 (C) 1 (D) -1

Answer  
s:

(A): [\_\_\_]

(B): [\_\_\_]

(C): [\_\_\_]

## Q2 Domain and Range of tanh

★ Easy | Basics

(A) The range of  $\tanh x$  is: (A) All reals (B)  $(-1, 1)$  (C)  $[-1, 1]$  (D)  $(0, \infty)$

(B)  $\lim_{x \rightarrow -\infty} \tanh x = ?$  (A) 1 (B) 0 (C) -1 (D)  $\infty$

(C) The minimum value of  $\cosh x$  is: (A) 0 (B) 1 (C) -1 (D) No minimum

Answer  
s:

(A): [\_\_\_]

(B): [\_\_\_]

(C): [\_\_\_]

### Q3 Hyperbolic Identities

★★ Medium | Identities

(A) If  $\sinh x = 3/4$ , then  $\cosh x = ?$  (A)  $5/4$  (B)  $7/4$  (C)  $\sqrt{7}/4$  (D)  $\sqrt{25/16+1}$

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(B)  $\sinh(2x) = ?$  (A)  $2\sinh^2 x$  (B)  $2 \sinh x \cosh x$  (C)  $\cosh^2 x - \sinh^2 x$  (D)  $\sinh^2 x + \cosh^2 x$

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(C)  $\cosh(2x)$  expressed with only  $\cosh x$ : (A)  $2\cosh x - 1$  (B)  $2\cosh^2 x - 1$  (C)  $1 - 2\cosh^2 x$  (D)  $2\cosh^2 x + 1$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

### Q4 Derivative of $\sinh x$

★★ Medium | Differentiation

(A)  $d/dx [\sinh(x^2)] = ?$  (A)  $\cosh(x^2)$  (B)  $2x \cosh(x^2)$  (C)  $2x \sinh(x^2)$  (D)  $x \cosh(x^2)$

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(B)  $d/dx [\cosh^2 x] = ?$  (A)  $2\cosh x$  (B)  $\sinh(2x)$  (C)  $2\sinh x$  (D)  $2\cosh x \sinh x$

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(C)  $d/dx [x \sinh x] = ?$  (A)  $\sinh x + x \cosh x$  (B)  $x \cosh x$  (C)  $\cosh x$  (D)  $\sinh x - x \cosh x$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

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## Q5 Derivative of tanh and sech

★★ Medium | Differentiation

(A)  $\frac{d}{dx} [\tanh^3 x] = ?$  (A)  $3\tanh^2 x$  (B)  $3\operatorname{sech}^2 x$  (C)  $3\tanh^2 x \cdot \operatorname{sech}^2 x$  (D)  $\tanh^2 x \cdot \operatorname{sech} x$

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(B)  $\frac{d}{dx} [\ln(\cosh x)] = ?$  (A)  $1/\cosh x$  (B)  $\tanh x$  (C)  $\coth x$  (D)  $\operatorname{sech} x$

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(C) If  $y = e^{\tanh x}$ , then  $y' = ?$  (A)  $e^{\tanh x}$  (B)  $\operatorname{sech}^2 x \cdot e^{\tanh x}$  (C)  $\tanh x \cdot e^{\tanh x}$  (D)  $e^{\operatorname{sech}^2 x}$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q6 Integration of sinh and cosh

★★ Medium | Integration

(A)  $\int \sinh(3x) dx = ?$  (A)  $3\cosh(3x)+C$  (B)  $\cosh(3x)/3+C$  (C)  $-\cosh(3x)/3+C$  (D)  $3\sinh(3x)+C$

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(B)  $\int \cosh^2 x dx = ?$  (Hint: use identity) (A)  $(x+\sinh(2x))/2$  (B)  $\sinh^2 x+C$  (C)  $(x + \sinh(2x))/2 + C$  (D)  $\sinh(2x)/4+C$

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(C)  $\int \tanh x dx = ?$  (A)  $\operatorname{sech}^2 x+C$  (B)  $-\ln(\cosh x)+C$  (C)  $\ln(\cosh x)+C$  (D)  $\ln|\sinh x|+C$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

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## Q7 Integration by Substitution

★★★ Hard | Integration

(A)  $\int \sinh x \cosh^2 x \, dx = ?$  (A)  $\cosh^3 x/3+C$  (B)  $\sinh^3 x/3+C$  (C)  $\cosh^2 x/2+C$  (D)  $-\cosh^3 x/3+C$

(B)  $\int_0^{\ln 2} \sinh x \, dx = ?$  (A)  $3/4$  (B)  $5/4$  (C)  $3/8$  (D)  $1/2$

(C)  $\int \cosh x / \sinh^2 x \, dx = ?$  (A)  $-1/\sinh x+C$  (B)  $\ln|\sinh x|+C$  (C)  $-\cosh x/\sinh x+C$  (D)  $1/\cosh x+C$

Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q8 Inverse Hyperbolic: $\sinh^{-1}$

★★★ Hard | Inverse Functions

(A) The domain of  $\cosh^{-1} x$  is: (A) All reals (B)  $x>0$  (C)  $x \geq 1$  (D)  $-1 \leq x \leq 1$

(B)  $d/dx [\sinh^{-1} x] = ?$  (A)  $1/\sqrt{x^2-1}$  (B)  $1/\sqrt{x^2+1}$  (C)  $1/(1-x^2)$  (D)  $\sqrt{x^2+1}$

(C)  $\tanh^{-1}(1/2) = ?$  (A)  $\ln(3)/2$  (B)  $\ln(2)$  (C)  $\ln(3/2)$  (D)  $\ln(\sqrt{3})$

Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q9 Derivatives of Inverse Hyperbolic

★★★ Hard | Differentiation

(A)  $\frac{d}{dx} [\tanh^{-1}(x^2)] = ?$  (A)  $\frac{2x}{(1-x^2)}$  (B)  $\frac{2x}{(1-x^4)}$  (C)  $\frac{1}{(1-x^4)}$  (D)  $\frac{2}{(1-x^2)}$

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(B)  $\int \frac{dx}{\sqrt{x^2+4}} = ?$  (A)  $\sinh^{-1}(x/2)+C$  (B)  $\sinh^{-1}(x)+C$  (C)  $(1/2)\sinh^{-1}(2x)+C$  (D)  $\ln(x+\sqrt{x^2+4})+C$

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(C)  $\frac{d}{dx} [\cosh^{-1}(\sec x)] = ?$  (A)  $\csc x$  (B)  $-\csc x$  (C)  $\sec x \tan x$  (D)  $\frac{1}{\sqrt{\sec^2 x - 1}}$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q10 Hyperbolic Limits

★★ Medium | Limits

(A)  $\lim_{x \rightarrow 0} \frac{\tanh(2x)}{\sin(3x)} = ?$  (A)  $\frac{2}{3}$  (B)  $\frac{3}{2}$  (C) 1 (D) 0

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(B)  $\lim_{x \rightarrow 0} (\cosh x - 1)/x^2 = ?$  (A) 0 (B) 1 (C)  $\frac{1}{2}$  (D) inf

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(C)  $\lim_{x \rightarrow \infty} (\cosh x - \sinh x) = ?$  (A) inf (B) 1 (C) 0 (D)  $\frac{1}{2}$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

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## Q1 Catenary Curve Application

1 ★★★ Hard | Applications

(A) Arc length of  $y = a \cosh(x/a)$  from  $x=-a$  to  $x=a$ : (A)  $2a \sinh(1)$  (B)  $a \sinh(a)$  (C)  $2\sinh(a)$  (D)  $2a \cosh(1)$

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(B) Surface area formula for rotating  $y=\cosh x$ ,  $0 \leq x \leq 1$  about x-axis: (A)  $2\pi \int \cosh x \, dx$  (B)  $2\pi \int \cosh^2 x \, dx$  (C)  $2\pi \int \cosh x \sqrt{1+\sinh^2 x} \, dx$  (D)  $\pi \int \cosh^2 x \, dx$

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(C) For  $y = \cosh x$ , the y-coordinate of the lowest point is: (A) 0 (B) 1 (C) e (D)  $1/e$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q1 Taylor Series of sinh and cosh

2 ★★★ Hard | Series

(A) Coefficient of  $x^5$  in the Taylor series of  $\sinh x$ : (A)  $1/120$  (B)  $-1/120$  (C)  $1/24$  (D)  $1/5$

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(B) How does  $\cosh x$  series differ from  $\cos x$  series? (A) Same (B) Signs of odd-power terms differ (C) All terms positive (no  $(-1)^n$ ) (D)  $\cosh$  has odd powers too

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(C)  $\lim_{x \rightarrow 0} [\cosh x - 1 - x^2/2] / x^4 = ?$  (A) 0 (B)  $1/24$  (C)  $1/4$  (D)  $1/12$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

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### Q1 Proving Hyperbolic Identities

3 ★★★ Hard | Proof / Exam

(A)  $\cosh(x+y) = ?$  (A)  $\cosh x \cdot \cosh y - \sinh x \cdot \sinh y$  (B)  $\cosh x \cdot \cosh y + \sinh x \cdot \sinh y$  (C)  $\sinh x \cdot \cosh y + \cosh x \cdot \sinh y$  (D)  $\cosh x \cdot \sinh y + \sinh x \cdot \cosh y$

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(B)  $\cosh(2x)$  when  $y=x$  in  $\cosh(x+y)$ : (A)  $2\cosh^2 x$  (B)  $\sinh^2 x + \cosh^2 x$  (C)  $2\cosh x - 1$  (D)  $1 + \sinh x$

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(C) Which identity is WRONG? (A)  $\sinh(-x) = -\sinh x$  (B)  $\cosh(-x) = \cosh x$  (C)  $\tanh(x+y) = (\tanh x + \tanh y) / (1 + \tanh x \cdot \tanh y)$  (D)  $\cosh^2 x - \sinh^2 x = -1$

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Answers:

(A):

(B):

(C):

### Q1 Integration with Substitution

4 ★★★ Hard | Integration / Exam

(A)  $\int dx/(1-x^2)$  for  $|x| < 1$ : (A)  $\tanh^{-1} x + C$  (B)  $(1/2)\ln|(1+x)/(1-x)| + C$  (C) Both A and B are equivalent (D)  $\coth^{-1} x + C$

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(B)  $\int_0^{3/5} dx/(1-x^2) = ?$  (A)  $\tanh^{-1}(3/5)$  (B)  $(1/2)\ln(4)$  (C)  $(1/2)\ln(2)$  (D) 1

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(C)  $\int x dx/\sqrt{x^4+1} = ?$  (A)  $(1/2)\sinh^{-1}(x^2) + C$  (B)  $\sinh^{-1}(x^2) + C$  (C)  $(1/2)\ln(x^2 + \sqrt{x^4+1}) + C$  (D) Both A and C equivalent

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Answers:

(A):

(B):

(C):

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**Q1** Differential Equations with Hyperbolic  
**5** ★★★ Hard | Diff. Equations

(A) General solution of  $y'' - y = 0$ : (A)  $A \cos x + B \sin x$  (B)  $A \cosh x + B \sinh x$  (C)  $A e^x + B$  (D)  $A \sin x + B \cos x$

(B) Solve  $y'' - y = 0$ ,  $y(0) = 3$ ,  $y'(0) = 1$ . Then  $y = ?$  (A)  $3 \cosh x + \sinh x$  (B)  $3 \sinh x + \cosh x$  (C)  $2e^x + e^{-x}$  (D)  $\cosh x + 3 \sinh x$

(C) For which ODE is  $y = \tanh x$  NOT a solution? (A)  $y' = 1 - y^2$  (B)  $y' = \operatorname{sech}^2 x$  (C)  $y'' = 2y \cdot y'$  (D)  $y'' = y$

Answer s: (A): [ ] (B): [ ] (C): [ ]

**Q1** Hyperbolic vs Trig — Sign Traps  
**6** ★★ Medium | Concept / Exam

(A) Which derivative is CORRECT? (A)  $\frac{d}{dx}[\cosh x] = -\sinh x$  (B)  $\frac{d}{dx}[\sinh x] = -\cosh x$  (C)  $\frac{d}{dx}[\cosh x] = \sinh x$  (D)  $\frac{d}{dx}[\tanh x] = -\operatorname{sech}^2 x$

(B) Which identity is CORRECT? (A)  $\cosh^2 x + \sinh^2 x = 1$  (B)  $\cosh^2 x - \sinh^2 x = -1$  (C)  $\sinh^2 x - \cosh^2 x = 1$  (D)  $\cosh^2 x - \sinh^2 x = 1$

(C)  $\int \operatorname{sech}^2 x \, dx = ?$  (A)  $-\tanh x + C$  (B)  $\tanh x + C$  (C)  $\coth x + C$  (D)  $-\coth x + C$

Answer s: (A): [ ] (B): [ ] (C): [ ]

**Q1 sinh<sup>-1</sup> as Logarithm**

7 ★★★ Hard | Inverse / Exam

(A)  $\sinh^{-1}(1) = ?$  (A)  $\ln(1+\sqrt{2})$  (B)  $\ln(2)$  (C)  $\pi/4$  (D) 1

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(B)  $\tanh^{-1}(1/3)$  as logarithm: (A)  $(1/2)\ln(2)$  (B)  $(1/3)\ln(2)$  (C)  $\ln(3/2)$  (D)  $(1/2)\ln(3)$

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(C)  $\int_0^1 dx/\sqrt{x^2+1} = ?$  (A)  $\ln(1+\sqrt{2})$  (B)  $\sqrt{2}$  (C)  $\ln(2)$  (D)  $\sinh^{-1}(2)$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

**Q1 sech, csch, coth Functions**

8 ★★ Medium | All Functions

(A)  $d/dx [\operatorname{csch} x] = ?$  (A)  $-\operatorname{csch} x \cdot \operatorname{coth} x$  (B)  $\operatorname{csch} x \cdot \operatorname{coth} x$  (C)  $-\operatorname{coth}^2 x$  (D)  $\operatorname{sech} x \cdot \tanh x$

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(B)  $\int \operatorname{sech} x \tanh x \, dx = ?$  (A)  $\tanh x + C$  (B)  $-\operatorname{sech} x + C$  (C)  $\operatorname{sech} x + C$  (D)  $\ln(\operatorname{sech} x) + C$

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(C)  $\operatorname{coth}^2 x - \operatorname{csch}^2 x = ?$  (A) 0 (B) -1 (C) 1 (D)  $\sinh^2 x$

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Answer  
s:

(A): [ ]

(B): [ ]

(C): [ ]

## Q1 Definite Integral Evaluation

9 ★★★ Hard | Integration / Exam

(A) First step for  $\int \cosh^2 x \, dx$ , rewrite  $\cosh^2 x$  as: (A)  $(1 - \cosh 2x)/2$  (B)  $(1 + \cosh 2x)/2$  (C)  $(\cosh 2x - 1)/2$  (D)  $(\sinh 2x + 1)/2$

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(B)  $\int_0^{\ln 2} \cosh^2 x \, dx = ?$  (given  $\sinh(2\ln 2) = 15/8$ ) (A)  $\ln(2)/2 + 3/4$  (B)  $\ln(2)/2 + 15/32$  (C)  $3/4 + \ln 2$  (D)  $15/8 + \ln 2$

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(C)  $\int \sinh^2 x \, dx = ?$  (A)  $\sinh(2x)/4 - x/2 + C$  (B)  $\cosh(2x)/4 + C$  (C)  $\sinh^2 x/2 + C$  (D)  $x/2 + \sinh(2x)/4 + C$

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Answer  
s:

(A):

(B):

(C):

## Q2 Ultimate Exam Challenge

0 ★★★★★ Expert | Mixed / Exam

(A)  $\int \coth x \, dx = ?$  (A)  $\ln|\cosh x| + C$  (B)  $\ln|\sinh x| + C$  (C)  $-\operatorname{csch}^2 x + C$  (D)  $\tanh x + C$

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(B) For  $y = \ln(\cosh x)$ ,  $y'' = \operatorname{sech}^2 x$ . Then  $y''(0) = ?$  (A) 0 (B) 1 (C)  $1/2$  (D)  $\operatorname{sech}(0) = 1$

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(C) Using  $x = \sinh t$ ,  $\int_0^1 \sqrt{x^2 + 1} \, dx = ?$  (A)  $(\sinh^{-1} 1 + \sqrt{2})/2$  (B)  $(\sqrt{2} + \ln(1 + \sqrt{2}))/2$  (C)  $\sinh(1)\cosh(1)$  (D)  $1/2$

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Answer  
s:

(A):

(B):

(C):

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## [ ANSWER KEY ]

<b>Q1</b> A: B B: B C: C	<b>Q2</b> A: B B: C C: B	<b>Q3</b> A: A B: B C: B	<b>Q4</b> A: B B: D C: A
<b>Q5</b> A: C B: B C: B	<b>Q6</b> A: B B: C C: C	<b>Q7</b> A: A B: A C: A	<b>Q8</b> A: C B: B C: A
<b>Q9</b> A: B B: A C: A	<b>Q10</b> A: A B: C C: C	<b>Q11</b> A: A B: C C: B	<b>Q12</b> A: A B: C C: B
<b>Q13</b> A: B B: B C: D	<b>Q14</b> A: C B: A C: D	<b>Q15</b> A: B B: A C: D	<b>Q16</b> A: C B: D C: B
<b>Q17</b> A: A B: A C: A	<b>Q18</b> A: A B: B C: C	<b>Q19</b> A: B B: B C: A	<b>Q20</b> A: B B: D C: B

All 3 parts of each question must be correct to earn full credit (5 pts each). Total: 100 pts.