

Polar Equations of Conics

Worksheet — 20 Problems with Step-by-Step Practice

Name: _____ Date: _____ Score: _____ / 20

$e < 1 \rightarrow$ Ellipse	$e = 1 \rightarrow$ Parabola	$e > 1 \rightarrow$ Hyperbola
$r = ed / (1 \pm e \cos \theta)$ [vertical directrix]	$r = ed / (1 \pm e \sin \theta)$ [horizontal directrix]	$a = ed / (1 - e^2)$ Latus rectum = $2ed$

Q1. Standard Form [★ Basic]

Concept:

All conics with a focus at the origin: $r = ed / (1 \pm e \cos \theta)$ or $r = ed / (1 \pm e \sin \theta)$.

$e < 1$: Ellipse | $e = 1$: Parabola | $e > 1$: Hyperbola

Use \cos when directrix is vertical; \sin when horizontal.

Problem:

Identify the type of conic: $r = 12 / (3 + 4 \sin \theta)$

Step 1 — What is the eccentricity e ?

- (A) $e = 3$ (B) $e = 4/3$
(C) $e = 1/3$ (D) $e = 4$

Work / Notes:

Step 2 — Since $e = 4/3$, what type of conic is this?

- (A) Parabola (B) Ellipse
(C) Hyperbola (D) Circle

Work / Notes:

Step 3 — The directrix is:

- (A) Vertical $x = d$ (B) Horizontal $y = d$
(C) Vertical $x = -d$ (D) No directrix

Work / Notes:

Q2. Finding ed and d [★ Basic]

Concept:

In standard form $r = ed / (1 \pm e \cdot \cos\theta)$, numerator = ed.

Divide denominator to get leading 1, then $d = (\text{numerator}) / e$.

Problem:

For $r = 9 / (3 - 6\cos\theta)$, find d (distance from focus to directrix).

Step 1 — Divide by 3 — standard form is:

(A) $r = 3/(1 - 2\cos\theta)$

(B) $r = 9/(1 - 6\cos\theta)$

(C) $r = 3/(1 + 2\cos\theta)$

(D) $r = 9/(3 - \cos\theta)$

Work / Notes:

Step 2 — What is e?

(A) $e = 3$

(B) $e = 1/2$

(C) $e = 6$

(D) $e = 2$

Work / Notes:

Step 3 — What is d?

(A) $d = 6$

(B) $d = 3$

(C) $d = 3/2$

(D) $d = 9$

Work / Notes:

Q3. Parabola (e=1) [★★ Medium]

Concept:

Parabola in polar form has $e = 1$.

Vertex at $r = d/2$. Opens direction depends on sign/trig:

$1 - \cos\theta$: opens left | $1 + \cos\theta$: opens right

$1 - \sin\theta$: opens down | $1 + \sin\theta$: opens up

Problem:

Find the vertex and direction for: $r = 8 / (2 - 2\sin\theta)$

Step 1 — Standard form (divide by 2):

(A) $r = 4/(1 - \sin\theta)$

(B) $r = 8/(1 - \sin\theta)$

(C) $r = 4/(1 + \sin\theta)$

(D) $r = 8/(2 - \sin\theta)$

Work / Notes:

Step 2 — Which direction does it open?

(A) Upward

(B) Downward

(C) Left

(D) Right

Work / Notes:

Step 3 — Vertex (r-value at vertex angle):

(A) $r=2$ at $\theta=\pi/2$

(B) $r=8$ at $\theta=-\pi/2$

(C) $r=4$ at $\theta=\pi/2$

(D) $r=2$ at $\theta=3\pi/2$

Work / Notes:

Q4. Ellipse Properties [★★ Medium]

Concept:

For ellipse $r = ed/(1-e\cos\theta)$: $a = ed/(1-e^2)$

Perihelion $r_1 = a(1-e)$, Aphelion $r_2 = a(1+e)$, $r_1+r_2 = 2a$

Problem:

For the ellipse $r = 6 / (1 - (1/2)\cos\theta)$, find $r_1 + r_2$.

Step 1 — Values of e and ed:

(A) $e=2$, $ed=6$

(B) $e=1/2$, $ed=6$

(C) $e=6$, $ed=1/2$

(D) $e=1/2$, $ed=3$

Work / Notes:

Step 2 — Semi-major axis $a = ed/(1-e^2)$:

(A) $a=6$

(B) $a=12$

(C) $a=8$

(D) $a=4$

Work / Notes:

Step 3 — $r_1 + r_2 = 2a =$

(A) 8

(B) 12

(C) 16

(D) 24

Work / Notes:

Q5. Hyperbola Vertices [★★ Medium]

Concept:

Hyperbola ($e > 1$): vertices at $\theta = 0$ and $\theta = \pi$.

Negative r means point is on the opposite branch.

Problem:

Find the vertices of: $r = 5 / (1 - 3\cos\theta)$

Step 1 — Eccentricity:

(A) $e=5$

(B) $e=1/3$

(C) $e=3$

(D) $e=1/5$

Work / Notes:

Step 2 — r at $\theta = 0$:

(A) $r=5/4$

(B) $r=5/2$

(C) $r=-5/2$

(D) $r=5$

Work / Notes:

Step 3 — r at $\theta = \pi$:

(A) $r=5/4$

(B) $r=-5/4$

(C) $r=5/2$

(D) $r=5$

Work / Notes:

Q6. Polar → Rectangular [★★ Medium]

Concept:

Key conversions: $x=r\cos\theta$, $y=r\sin\theta$, $r^2=x^2+y^2$.

Multiply both sides by denominator, substitute $r\cos\theta=x$ or $r\sin\theta=y$, then square.

Problem:

Convert to rectangular form: $r = 3 / (1 + \sin\theta)$

Step 1 — Multiply both sides by $(1+\sin\theta)$:

(A) $r + r\sin\theta = 3$

(B) $r + \sin\theta = 3$

(C) $r\sin\theta = 3$

(D) $r - r\sin\theta = 3$

Work / Notes:

Step 2 — Substitute $r \cdot \sin\theta = y$, solve for r :

(A) $r = 3+y$

(B) $r = 3-y$

(C) $r = y-3$

(D) $r = 3y$

Work / Notes:

Step 3 — Square both sides ($r^2=x^2+y^2$):

(A) $x^2+y^2=9-y^2$

(B) $x^2=9-6y$

(C) $x^2=-6y+9$

(D) $x^2+2y^2=9$

Work / Notes:

Q7. Finding Directrix [★★ Medium]

Concept:

$+\cos\theta: x=d$ | $-\cos\theta: x=-d$ | $+\sin\theta: y=d$ | $-\sin\theta: y=-d$

$d = ed/e$ (numerator of standard form divided by e)

Problem:

Find the directrix equation for: $r = 10 / (5 + 2\sin\theta)$

Step 1 — Divide by 5 — standard form:

(A) $r=2/(1+0.4\sin\theta)$

(B) $r=10/(1+0.4\sin\theta)$

(C) $r=2/(5+2\sin\theta)$

(D) $r=0.4/(1+2\sin\theta)$

Work / Notes:

Step 2 — e and ed :

(A) $e=5, ed=2$

(B) $e=2/5, ed=2$

(C) $e=2, ed=10$

(D) $e=0.4, ed=10$

Work / Notes:

Step 3 — Directrix equation:

(A) $x=5$

(B) $y=-5$

(C) $y=5$

(D) $x=2$

Work / Notes:

Q8. Rotated Conics [★★★ Hard]

Concept:

Replace θ with $(\theta - \alpha)$ to rotate conic by angle α counterclockwise:

$$r = ed / (1 - e \cdot \cos(\theta - \alpha))$$

Problem:

For $r = 6 / (1 - 2\cos(\theta - \pi/6))$, find the vertex closest to the focus.

Step 1 — Conic type and e:

(A) Parabola $e=1$

(B) Ellipse $e=2$

(C) Hyperbola $e=2$

(D) Ellipse $e=1/2$

Work / Notes:

Step 2 — θ that maximizes denominator $1-2\cos(\theta-\pi/6)$:

(A) $\theta=\pi/6$

(B) $\theta=7\pi/6$

(C) $\theta=\pi/2$

(D) $\theta=\pi/3$

Work / Notes:

Step 3 — r-value at closest vertex:

(A) $r=2$

(B) $r=3$

(C) $r=6$

(D) $r=1$

Work / Notes:

Q9. Latus Rectum [★★ Medium]

Concept:

Latus rectum = chord through focus perpendicular to major axis.

Length = $2 \cdot ed$. Found by setting $\theta = \pi/2$ or $3\pi/2$.

Problem:

Find the latus rectum length for: $r = 6 / (2 + 3\cos\theta)$

Step 1 — Standard form (divide by 2):

(A) $r=3/(1+1.5\cos\theta)$

(B) $r=6/(1+1.5\cos\theta)$

(C) $r=3/(2+3\cos\theta)$

(D) $r=6/(1+\cos\theta)$

Work / Notes:

Step 2 — ed (numerator in standard form):

(A) $ed=6$

(B) $ed=2$

(C) $ed=3$

(D) $ed=9$

Work / Notes:

Step 3 — Full latus rectum length:

(A) 3

(B) 6

(C) 12

(D) 9

Work / Notes:

Q10. Semi-major Axis [★★★ Hard]

Concept:

$r_{\min} = ed/(1+e)$, $r_{\max} = ed/(1-e)$, $2a = r_{\min} + r_{\max}$

Center is at distance $c = ae$ from focus.

Problem:

For $r = 8 / (4 - 2\cos\theta)$, find a and c .

Step 1 — Standard form (divide by 4):

(A) $r=2/(1-0.5\cos\theta)$

(B) $r=8/(1-0.5\cos\theta)$

(C) $r=2/(4-\cos\theta)$

(D) $r=4/(1-2\cos\theta)$

Work / Notes:

Step 2 — $a = ed/(1-e^2)$:

- (A) $a=4$ (B) $a=8/3$
(C) $a=2$ (D) $a=3$

Work / Notes:

Step 3 — $c = ae$:

- (A) $c=4/3$ (B) $c=8/3$
(C) $c=2$ (D) $c=1$

Work / Notes:

Q11. Kepler Orbit [★★★ Hard]

Concept:

Kepler's First Law: planets orbit in ellipses with Sun at one focus.

$$r = a(1-e^2)/(1-e\cos\theta), \text{ perihelion}=a(1-e), \text{ aphelion}=a(1+e)$$

Problem:

Comet: perihelion=2 AU, aphelion=18 AU. Find e and polar orbit equation.

Step 1 — Semi-major axis $a = (\text{perihelion} + \text{aphelion})/2$:

- (A) $a=8$ AU (B) $a=10$ AU
(C) $a=16$ AU (D) $a=20$ AU

Work / Notes:

Step 2 — Eccentricity e from perihelion = $a(1-e)$:

- (A) $e=0.2$ (B) $e=0.8$
(C) $e=0.5$ (D) $e=0.9$

Work / Notes:

Step 3 — $a(1-e^2)$ in the orbit equation equals:

(A) 3.6 AU

(B) 18 AU

(C) 36 AU

(D) 1.8 AU

Work / Notes:

Q12. Polar → Std Form [★★ Medium]

Concept:

Convert: multiply by denominator → substitute $r \cdot \cos\theta = x$ or $r \cdot \sin\theta = y$ → square ($r^2 = x^2 + y^2$) → complete the square.

Problem:

Convert $r = 2/(1 - \cos\theta)$ to rectangular form and identify the conic.

Step 1 — Multiply by $(1 - \cos\theta)$ and expand:

(A) $r - x = 2$

(B) $r + x = 2$

(C) $r \cdot \cos\theta = 2$

(D) $r - r \cdot \cos\theta = 2$

Work / Notes:

Step 2 — Substitute $r \cdot \cos\theta = x$, square $r = 2 + x$:

(A) $x^2 + y^2 = (x + 2)^2$

(B) $x^2 + y^2 = (x - 2)^2$

(C) $x^2 = y^2 + 4$

(D) $y = x^2 - 4$

Work / Notes:

Step 3 — Simplify and identify conic:

(A) $y^2 = 4x - 4$ (ellipse)

(B) $y^2 = 4x + 4$ (parabola)

(C) $y^2 = 4x + 4$ (ellipse)

(D) $y^2 = 4(x + 1)$ (parabola opening right)

Work / Notes:

Q13. Hyperbola Asymptotes [★★★ Hard]

Concept:

Asymptotes where denominator=0: $1 - e \cdot \cos\theta = 0 \rightarrow \cos\theta = 1/e \rightarrow \theta = \pm \arccos(1/e)$

Problem:

Find asymptote angles for: $r = 4 / (1 - \sqrt{2} \cdot \cos\theta)$

Step 1 — $\cos\theta$ value that makes denominator 0:

(A) $\cos\theta = \sqrt{2}$

(B) $\cos\theta = 1/\sqrt{2}$

(C) $\cos\theta = -1/\sqrt{2}$

(D) $\cos\theta = 4$

Work / Notes:

Step 2 — Confirm $e > 1$ (hyperbola):

(A) $e=4$, hyperbola

(B) $e=1/\sqrt{2} < 1$, ellipse

(C) $e=\sqrt{2} > 1$, hyperbola

(D) $e=1$, parabola

Work / Notes:

Step 3 — Asymptote angles:

(A) $\theta = \pm\pi/3$

(B) $\theta = \pm\pi/6$

(C) $\theta = \pm\pi/4$

(D) $\theta = \pm\pi/2$

Work / Notes:

Q14. Rect→Polar [★★★ Hard]

Concept:

From focus-directrix definition $PF = e \cdot PD$.

Focus at origin, directrix $x=d \rightarrow r = ed / (1 + e \cdot \cos\theta)$

Directrix $x=-d \rightarrow r = ed / (1 - e \cdot \cos\theta)$

Problem:

Write polar equation: ellipse, focus at origin, $e=1/2$, directrix $x=-6$.

Step 1 — d = distance to directrix:

(A) $d=6$

(B) $d=-6$

(C) $d=3$

(D) $d=12$

Work / Notes:

Step 2 — Compute ed:

(A) $ed=12$

(B) $ed=3$

(C) $ed=6$

(D) $ed=1$

Work / Notes:

Step 3 — Directrix $x=-d$ (left side) gives equation:

(A) $r=3/(1+\cos\theta)$

(B) $r=3/(1-\cos\theta)$

(C) $r=6/(1-\cos\theta)$

(D) $r=3/(1+\sin\theta)$

Work / Notes:

Q15. Focus & Vertex [★★ Medium]

Concept:

In polar form, focus is always at the pole (0,0).

Vertex = angle where r is minimum (ellipse) or characteristic.

Closest vertex = where denominator is largest.

Problem:

Find the vertex closest to the focus for: $r = 10/(1 + 5\cos\theta)$

Step 1 — e and conic type:

(A) $e=10$, hyperbola

(B) $e=5$, hyperbola

(C) $e=1/5$, ellipse

(D) $e=1$, parabola

Work / Notes:

Step 2 — θ that maximizes $1+5\cos\theta$ (closest vertex):

(A) $\theta=\pi$

(B) $\theta=\pi/2$

(C) $\theta=0$

(D) $\theta=3\pi/2$

Work / Notes:

Step 3 — r-value at closest vertex:

(A) $r=5/3$

(B) $r=5$

(C) $r=5/3$

(D) $r=10$

Work / Notes:

Q16. Graph Matching [★★ Medium]

Concept:

Quick ID: $e<1$ →Ellipse, $e=1$ →Parabola, $e>1$ →Hyperbola

$\cos\theta$ →horizontal axis, $\sin\theta$ →vertical axis

$+\cos\theta$ →opens left, $-\cos\theta$ →opens right

Problem:

Which of these is an ellipse with major axis along the y-axis?

Step 1 — Which equation could be an ellipse ($e<1$)?

(A) $r=5/(1-3\sin\theta)$

(B) $r=4/(1-\sin\theta)$

(C) $r=6/(2-\sin\theta)$

(D) $r=3/(1-4\cos\theta)$

Work / Notes:

Step 2 — For $r=3/(1-0.5\sin\theta)$, major axis direction:

(A) x-axis (horizontal)

(B) y-axis (vertical)

(C) z-axis

(D) diagonal

Work / Notes:

Step 3 — The farthest vertex is at:

(A) $\theta=0$ (right)

(B) $\theta=\pi$ (left)

(C) $\theta=3\pi/2$ (below)

(D) $\theta=\pi/2$ (above)

Work / Notes:

Q17. Perihelion/Aphelion [★★ Medium]

Concept:

For $r = ed/(1 - e \cdot \cos\theta)$: r_{\min} at largest denominator, r_{\max} at smallest.

With $-\cos\theta$: largest denom when $\cos\theta = -1$ ($\theta = \pi$) $\rightarrow r_{\min}$

smallest denom when $\cos\theta = 1$ ($\theta = 0$) $\rightarrow r_{\max}$

Problem:

For $r = 3/(1 - 0.75\cos\theta)$, find closest and farthest distances.

Step 1 — r at $\theta = 0$ (check if max or min):

(A) $r = 3/0.25 = 12$ (farthest)

(B) $r = 3/1.75 = 12/7$ (closest)

(C) $r = 3/1.25 = 2.4$

(D) $r = 3/0.75 = 4$

Work / Notes:

Step 2 — Which θ gives the CLOSEST distance?

(A) $\theta = 0$ ($r = 12$)

(B) $\theta = \pi/2$ ($r = 3$)

(C) $\theta = \pi$ ($r = 12/7 \approx 1.71$)

(D) $\theta = \pi/4$

Work / Notes:

Step 3 — Ratio r_{\max} / r_{\min} :

(A) $4/7$

(B) 7

(C) $7/4$

(D) 2

Work / Notes:

Q18. Complex Denominator [★★★ Hard]

Concept:

Always divide through to make constant term = 1 first.

$$r = A/(B + C \cdot \cos\theta) \rightarrow r = (A/B)/(1 + (C/B) \cdot \cos\theta)$$

Then $e = |C/B|$ and $ed = A/B$.

Problem:

Find e , conic type, and ed for: $r = 15/(5 + 3\cos\theta)$

Step 1 — Divide by 5 — standard form:

(A) $r = 3/(1 + 0.6\cos\theta)$

(B) $r = 15/(1 + 3\cos\theta)$

(C) $r=3/(5+\cos\theta)$

(D) $r=5/(1+\cos\theta)$

Work / Notes:

Step 2 — e and conic type:

(A) $e=5$, hyperbola

(B) $e=0.6$, ellipse

(C) $e=3$, hyperbola

(D) $e=1$, parabola

Work / Notes:

Step 3 — ed and d:

(A) $ed=3$, $d=5$

(B) $ed=15$, $d=25$

(C) $ed=5$, $d=3$

(D) $ed=3$, $d=0.6$

Work / Notes:

Q19. Polar Area [★★★ Hard]

Concept:

Area: $A = (1/2)\int r^2 d\theta$

For full ellipse: $A = \pi \cdot a \cdot b$ where $b = a \cdot \sqrt{1-e^2}$

Problem:

Find the area enclosed by: $r = 6/(3 - \cos\theta)$

Step 1 — Standard form (divide by 3):

(A) $r=2/(1-(1/3)\cos\theta)$

(B) $r=6/(1-(1/3)\cos\theta)$

(C) $r=2/(3-\cos\theta)$

(D) $r=3/(1-\cos\theta)$

Work / Notes:

Step 2 — $a = ed/(1-e^2)$ with $e=1/3$, $ed=2$:

(A) $a=9/4$

(B) $a=2$

(C) $a=9/8$

(D) $a=3$

Work / Notes:

Step 3 — Area $A = \pi \cdot a \cdot b$ where $b = a \cdot \sqrt{1 - e^2}$:

(A) $A = 9\pi\sqrt{2}/4$

(B) $A = 9\pi\sqrt{8}/8$

(C) $A = 27\pi\sqrt{2}/8$

(D) $A = 9\pi/(4\sqrt{8})$

Work / Notes:

Q20. Comprehensive [★★★ Hard]

Concept:

Complete analysis: 1) Divide for leading 1. 2) Read e & type. 3) Compute ed, d, a, b, c. 4) Latus rectum = 2ed. 5) Directrix from sign/trig type.

Problem:

Full analysis of $r = 12/(4 + 3\sin\theta)$. Find type, e, semi-major axis a, latus rectum.

Step 1 — Divide by 4 — e and conic type:

(A) $e = 3/4$, ellipse

(B) $e = 4/3$, hyperbola

(C) $e = 3$, hyperbola

(D) $e = 1/4$, ellipse

Work / Notes:

Step 2 — $a = ed/(1 - e^2)$ with $ed = 3$, $e = 3/4$:

(A) $a = 48/7$

(B) $a = 3$

(C) $a = 4$

(D) $a = 12/7$

Work / Notes:

Step 3 — Latus rectum length = 2·ed:

(A) 2

(B) 12

(C) 6

(D) 3

Work / Notes:

Answer Key

Q#	Part 1	Part 2	Part 3
Q1 (Standard Form)	B	C	B
Q2 (Finding e and d)	A	D	C
Q3 (Parabola ($e=1$))	A	B	D
Q4 (Ellipse Properties)	B	C	C
Q5 (Hyperbola Vertices)	C	C	A
Q6 (Polar→Rectangular)	A	B	B
Q7 (Finding Directrix)	A	B	C
Q8 (Rotated Conics)	C	B	A
Q9 (Latus Rectum)	A	C	B
Q10 (Semi-major Axis)	A	B	A
Q11 (Kepler Orbit)	B	B	A
Q12 (Polar→Std Form)	D	A	D
Q13 (Hyperbola Asymptotes)	B	C	C
Q14 (Rect→Polar)	A	B	B
Q15 (Focus & Vertex)	B	C	C
Q16 (Graph Matching)	C	B	D
Q17 (Perihelion/Aphelion)	A	C	B
Q18 (Complex Denominator)	A	B	A
Q19 (Polar Area)	A	A	C
Q20 (Comprehensive)	A	A	C