Polar ↔ Rectangular Equations

20 Practice Problems · Self-Study Worksheet

From Basics to Exam Level

Name: ______________________________ **Date:** ____________________

Score: _____ / 20 **Time:** _____________

Essential Conversion Formulas

Polar → Rectangular: $x = r\cdot\cos(\theta)$ $y = r\cdot\sin(\theta)$ $r^2 = x^2 + y^2$ $\tan(\theta) = y/x$

Rectangular → Polar: $r = \sqrt{(x^2+y^2)}$ $x = r\cdot\cos\theta$ $y = r\cdot\sin\theta$ Double angle: $\sin(2\theta)=2\sin\theta\cos\theta$, $\cos(2\theta)=\cos^2\theta-\sin^2\theta$

Key Shapes: $r=a$ (circle at origin) | $r=a\cos\theta$ (circle, center $(a/2,0)$) | $r=a\sin\theta$ (circle, center $(0,a/2)$) | $r=a/(1\pm e\cos\theta)$ (conic, e=eccentricity)

Part 1: Polar → Rectangular (Basics)

Q1

Basic Conversion Formulas

■ Concept: Bridge formula: $x = r\cdot\cos\theta$, $y = r\cdot\sin\theta$, $r^2 = x^2 + y^2$, $\tan(\theta) = y/x$

Question: Which are the correct conversion formulas? Convert r = 5 to rectangular form.

Step 1: Step 1: x in terms of r and θ

What is x equal to in polar-to-rectangular conversion?

A) $x = r\cdot\sin(\theta)$

B) $x = r\cdot\cos(\theta)$

C) $x = r/\cos(\theta)$

D) $x = \cos(\theta)/r$

Answer: (B) B) $x = r\cdot\cos(\theta)$

Why: $x = r\cdot\cos\theta$ — x-component of the radius vector.

Step 2: Step 2: Relationship between r, x, y

Which equation correctly relates r, x, and y?

A) $r = x + y$

B) $r^2 = x^2 + y^2$

C) $r = x^2 + y^2$

D) $r^2 = x + y$

Answer: (B) B) $r^2 = x^2 + y^2$

Why: $r^2 = x^2 + y^2$ — Pythagorean theorem applied to the right triangle.

Step 3: Step 3: Convert r = 5 to rectangular

The rectangular form of r = 5 is:

A) $x + y = 5$

B) $x^2 + y^2 = 5$

C) $x^2 + y^2 = 25$

D) $x^2 - y^2 = 25$

Answer: (C) C) $x^2 + y^2 = 25$

Why: $r = 5 \to r^2 = 25 \to x^2 + y^2 = 25$ (circle, radius 5, centered at origin).

Work Space (use extra paper if needed):

Q2

Converting θ = constant (Lines)

■ Concept: $\theta = c$ is a line through the origin. Use $\tan(\theta) = y/x \to y = x\cdot\tan(\theta)$

Question: Convert $\theta = \pi/4$ to rectangular form.

Step 1: Step 1: Which trig function relates θ, x, y?

What identity connects θ, x, and y?

A) $\sin(\theta)=x/r$

B) $\cos(\theta)=y/x$

C) $\tan(\theta)=y/x$

D) $\cot(\theta)=y/x$

Answer: (C) C) $\tan(\theta)=y/x$

Why: $\tan(\theta) = y/x$ from the definitions $x=r\cos\theta$, $y=r\sin\theta$.

Step 2: Step 2: Find tan(π/4)

What is the value of $\tan(\pi/4)$?

A) 0

B) $\sqrt{3}$

C) 1

D) $1/\sqrt{2}$

Answer: (C) C) 1

Why: $\tan(\pi/4) = \sin(\pi/4)/\cos(\pi/4) = (\sqrt{2}/2)/(\sqrt{2}/2) = 1$.

Step 3: Step 3: Write the rectangular equation

The rectangular form of $\theta = \pi/4$ is:

A) $x = 1$

B) $y = 1$

C) $y = x$

D) $x + y = 0$

Answer: (C) C) $y = x$

Why: $1 = y/x \to y = x$ (line through origin, slope 1).

Work Space (use extra paper if needed):

Q3

$r = 2\cos(\theta)$ — Circle

■ Concept: Multiply by r: $r^2 = 2r\cdot\cos\theta \to x^2 + y^2 = 2x$. Complete the square!

Question: Convert $r = 2\cos(\theta)$ to rectangular and identify the shape.

Step 1: Step 1: Multiply both sides by r

After multiplying $r = 2\cos\theta$ by r, you get:

A) $r = 2r\cdot\cos\theta$

B) $r^2 = 2\cos\theta$

C) $r^2 = 2r\cdot\cos\theta$

D) $r\cdot\cos\theta=2$

Answer: (C) C) $r^2 = 2r\cdot\cos\theta$

Why: $r\cdot r = r^2$ and $r\cdot 2\cos\theta = 2r\cdot\cos\theta$.

Step 2: Step 2: Substitute $r^2=x^2+y^2$, $r\cos\theta=x$

Rectangular form after substitution:

A) $x^2+y^2=2x$

B) $x^2+y^2=2y$

C) $x^2+y^2=4$

D) $x^2-y^2=2x$

Answer: (A) A) $x^2+y^2=2x$

Why: $r^2 = x^2+y^2$ and $r\cdot\cos\theta = x \rightarrow x^2+y^2 = 2x$.

Step 3: Step 3: Complete the square

Standard form after completing the square:

A) $(x-1)^2+y^2=1$

B) $(x+1)^2+y^2=1$

C) $(x-1)^2+y^2=4$

D) $x^2+(y-1)^2=1$

Answer: (A) A) $(x-1)^2+y^2=1$

Why: $x^2-2x+y^2=0 \rightarrow (x-1)^2-1+y^2=0 \rightarrow (x-1)^2+y^2=1$. Circle: center (1,0), radius 1.

Work Space (use extra paper if needed):

Q4

$y = x^2 \rightarrow$ Polar

■ Concept: Substitute $x=r\cos\theta$, $y=r\sin\theta$. Factor and solve for r.

Question: Convert $y = x^2$ to polar form.

Step 1: Step 1: Substitute x and y

After substituting into $y = x^2$:

A) $r\cdot\sin\theta=r\cdot\cos^2\theta$

B) $r\cdot\sin\theta=r^2\cdot\cos^2\theta$

C) $\sin\theta=r\cdot\cos^2\theta$

D) $r=\sin\theta/\cos^2\theta$

Answer: (B) B) $r\cdot\sin\theta=r^2\cdot\cos^2\theta$

Why: $y=r\sin\theta$, $x^2=r^2\cos^2\theta \rightarrow r\sin\theta = r^2\cos^2\theta$.

Step 2: Step 2: Divide by r ($r \neq 0$)

After dividing both sides by r:

A) $\sin\theta=r^2\cdot\cos^2\theta$

B) $\sin\theta=r\cdot\cos^2\theta$

C) $r=\sin\theta\cdot\cos^2\theta$

D) $r\cdot\sin\theta=\cos^2\theta$

Answer: (B) B) $\sin\theta=r\cdot\cos^2\theta$

Why: Divide by r: $\sin\theta = r\cdot\cos^2\theta$.

Step 3: Step 3: Solve for r

r = ? (final polar form):

A) $r=\sin\theta\cdot\cos^2\theta$

B) $r=\sin\theta/\cos^2\theta$

C) $r=\tan\theta\cdot\sec\theta$

D) Both B and C

Answer: (D) D) Both B and C

Why: $r = \sin\theta/\cos^2\theta = (\sin\theta/\cos\theta)\cdot(1/\cos\theta) = \tan\theta\cdot\sec\theta$. B and C are equivalent!

Work Space (use extra paper if needed):

Q5

r = 6sin(θ) — Circle

■ Concept: r=2a·sinθ gives circle at (0,a) with radius a. Multiply by r first!

Question: Convert r = 6sin(θ) to rectangular. Find center and radius.

Step 1: Step 1: Multiply by r and substitute

Rectangular equation after substitution:

A) $x^2+y^2=6y$

B) $x^2+y^2=6x$

C) $x^2+y^2=36$

D) $y=6x$

Answer: (A) A) $x^2+y^2=6y$

Why: $r\cdot\sin\theta = y \rightarrow r^2=6r\cdot\sin\theta \rightarrow x^2+y^2=6y$.

Step 2: Step 2: Complete the square on y

After completing the square on y:

A) $x^2+(y+6)^2=36$

B) $x^2+(y-6)^2=36$

C) $x^2+(y-3)^2=9$

D) $x^2+(y+3)^2=9$

Answer: (C) C) $x^2+(y-3)^2=9$

Why: $(y^2-6y)+x^2=0 \rightarrow (y-3)^2-9+x^2=0 \rightarrow x^2+(y-3)^2=9$.

Step 3: Step 3: Identify center and radius

Center and radius of the circle:

A) Center(3,0),r=9

B) Center(0,3),r=3

C) Center(0,3),r=9

D) Center(0,-3),r=3

Answer: (B) B) Center(0,3),r=3

Why: Center (0,3), radius 3.

Work Space (use extra paper if needed):

Part 2: Rectangular → Polar

Q6

$x^2+y^2=4y$ → Polar

■ Concept: Replace x^2+y^2 with r^2, y with $r\cdot\sin\theta$, then divide by r.

Question: Convert $x^2 + y^2 = 4y$ to polar form.

Step 1: Step 1: Substitute

After substituting polar forms:

A) $r^2=4r\cdot\cos\theta$

B) $r^2=4r\cdot\sin\theta$

C) $r=4\sin\theta$

D) $r^2=4\sin\theta$

Answer: (B) B) $r^2=4r\cdot\sin\theta$

Why: $x^2+y^2=r^2$, $4y=4r\cdot\sin\theta \rightarrow r^2=4r\cdot\sin\theta$.

Step 2: Step 2: Divide by r

Result of dividing by r:

A) $r=4\cos\theta$

B) $r=4\sin\theta$

C) $r=\sin\theta/4$

D) $r^2=4\sin\theta$

Answer: (B) B) $r=4\sin\theta$

Why: Divide by r: $r=4\sin\theta$.

Step 3: Step 3: Identify shape

$r=4\sin\theta$ represents:

A) A line

B) Circle at (0,2) r=2

C) A parabola

D) Circle at (2,0) r=4

Answer: (B) B) Circle at (0,2) r=2

Why: $r=2a\cdot\sin\theta$, a=2 → center (0,2), radius 2.

Work Space (use extra paper if needed):

Q7

r = 4/(2cosθ–sinθ) — Line

■ Concept: Multiply by denominator, distribute r, then use $r\cos\theta=x$ and $r\sin\theta=y$.

Question: Convert $r = 4/(2\cos\theta - \sin\theta)$ to rectangular form.

Step 1: Step 1: Multiply by denominator

After multiplying:

A) $r=4(2\cos\theta-\sin\theta)$

B) $r(2\cos\theta-\sin\theta)=4$

C) $2\cos\theta-\sin\theta=4/r$

D) $r\cdot\cos\theta=4$

Answer: (B) B) $r(2\cos\theta-\sin\theta)=4$

Why: $r(2\cos\theta-\sin\theta)=4$.

Step 2: Step 2: Distribute r

After distributing r:

A) $2r\cdot\cos\theta-r\cdot\sin\theta=4$

B) $2\cos\theta-\sin\theta=4r$

C) $r^2\cdot\cos\theta=4$

D) $2x+y=4$

Answer: (A) A) $2r\cdot\cos\theta-r\cdot\sin\theta=4$

Why: $2r\cdot\cos\theta - r\cdot\sin\theta = 4$.

Step 3: Step 3: Substitute and simplify

Final rectangular form:

A) $2x+y=4$

B) $2x-y=4$

C) $x-2y=4$

D) $x+2y=4$

Answer: (B) B) $2x-y=4$

Why: $2x - y = 4$ (a straight line).

Work Space (use extra paper if needed):

Q8

$x^2–y^2=1 \rightarrow$ Polar (Double Angle)

■ Concept: Use identity: $\cos^2\theta - \sin^2\theta = \cos(2\theta)$. Key exam identity!

Question: Convert $x^2 – y^2 = 1$ to polar using the double-angle identity.

Step 1: Step 1: Substitute

x^2-y^2 in polar is:

A) $r^2(\cos\theta-\sin\theta)$

B) $r(\cos^2\theta-\sin^2\theta)$

C) $r^2(\cos^2\theta-\sin^2\theta)$

D) $r^2\cos^2\theta+r^2\sin^2\theta$

Answer: (C) C) $r^2(\cos^2\theta-\sin^2\theta)$

Why: $x^2=r^2\cos^2\theta$, $y^2=r^2\sin^2\theta \rightarrow x^2-y^2=r^2(\cos^2\theta-\sin^2\theta)$.

Step 2: Step 2: Apply cos(2θ) identity

Using $\cos(2\theta)=\cos^2\theta-\sin^2\theta$:

A) $r^2\cdot\sin(2\theta)$

B) $r^2\cdot\cos(2\theta)$

C) $2r^2\cdot\cos\theta\sin\theta$

D) $r\cdot\cos(2\theta)$

Answer: (B) B) $r^2\cdot\cos(2\theta)$

Why: $r^2\cdot\cos(2\theta)$ by the double-angle identity.

Step 3: Step 3: Final polar form

Polar form of $x^2-y^2=1$:

A) $r^2=\sin(2\theta)$

B) $r=\cos(2\theta)$

C) $r^2=\cos(2\theta)$

D) $r^2=1$

Answer: (C) C) $r^2=\cos(2\theta)$

Why: $r^2\cdot\cos(2\theta)=1 \rightarrow r^2=\sec(2\theta)$ or written as $r^2=\cos(2\theta)$ when RHS=1.

Work Space (use extra paper if needed):

Q9

r = 3 — Circle at Origin

■ Concept: $r=c \to r^2=c^2 \to x^2+y^2=c^2$. Circle centered at origin, radius c.

Question: Convert r = 3 to rectangular. Find its shape and area.

Step 1: Step 1: Convert

Rectangular form of r=3:

A) $x+y=3$

B) $x^2+y^2=3$

C) $x^2+y^2=9$

D) $\sqrt{(x^2+y^2)}=9$

Answer: (C) C) $x^2+y^2=9$

Why: $r^2=9 \to x^2+y^2=9$.

Step 2: Step 2: Identify shape

$x^2+y^2=9$ represents:

A) A line

B) A parabola

C) Circle at origin r=3

D) Circle at (3,3)

Answer: (C) C) Circle at origin r=3

Why: $x^2+y^2=r^2$ is a circle centered at origin with radius 3.

Step 3: Step 3: Calculate area

Area of the circle:

A) 3π

B) 6π

C) 9π

D) 12π

Answer: (C) C) 9π

Why: Area = $\pi r^2 = \pi \cdot 9 = 9\pi$.

Work Space (use extra paper if needed):

Q10

$2x+3y=6 \to$ Polar (Line)

■ Concept: $ax+by=c$: substitute $x=r\cos\theta$, $y=r\sin\theta$, factor r, then solve for r.

Question: Convert the line 2x + 3y = 6 to polar form.

Step 1: Step 1: Substitute

After substituting:

A) $2r\cdot\cos\theta+3r\cdot\sin\theta=6$

B) $r(2+3)\cos\theta=6$

C) $2\cos\theta+3\sin\theta=6r$

D) $r+3r=6$

Answer: (A) A) $2r\cdot\cos\theta+3r\cdot\sin\theta=6$

Why: $2r\cdot\cos\theta + 3r\cdot\sin\theta = 6$.

Step 2: Step 2: Factor r

After factoring r:

A) $r(2+3)(\cos\theta+\sin\theta)=6$

B) $r(2\cos\theta+3\sin\theta)=6$

C) $r^2(2\cos\theta+3\sin\theta)=6$

D) $r=6(...)$

Answer: (B) B) $r(2\cos\theta+3\sin\theta)=6$

Why: $r(2\cos\theta + 3\sin\theta) = 6$.

Step 3: Step 3: Solve for r

Final polar form:

A) $r=6\cdot(2\cos\theta+3\sin\theta)$

B) $r=2\cos\theta+3\sin\theta$

C) $r=6/(2\cos\theta+3\sin\theta)$

D) $r=1/(2\cos\theta+3\sin\theta)$

Answer: (C) C) $r=6/(2\cos\theta+3\sin\theta)$

Why: $r = 6/(2\cos\theta + 3\sin\theta)$.

Work Space (use extra paper if needed):

Part 3: Mixed Conversions

Q11

$r\cdot\sin\theta = 4$ — Horizontal Line

■ Concept: $r\cdot\sin\theta = y$ directly! $r\cdot\sin\theta=k$ means $y=k$ (horizontal line).

Question: Convert $r\cdot\sin(\theta) = 4$ to rectangular form.

Step 1: Step 1: Recall substitution

$r\cdot\sin\theta$ equals:

A) x

B) y

C) r^2

D) $\tan\theta$

Answer: (B) B) y

Why: $r\cdot\sin\theta = y$ by definition.

Step 2: Step 2: Write equation

Rectangular form:

A) $x=4$

B) $y=4$

C) $x^2+y^2=4$

D) $y=4x$

Answer: (B) B) $y=4$

Why: $r\cdot\sin\theta=4 \rightarrow y=4$.

Step 3: Step 3: Identify shape

$y = 4$ represents:

A) A circle

B) Vertical line $x=4$

C) Horizontal line $y=4$

D) Diagonal line

Answer: (C) C) Horizontal line $y=4$

Why: $y=4$ is a horizontal line parallel to x-axis.

Work Space (use extra paper if needed):

Q12

r = sec(θ) — Vertical Line

■ Concept: sec(θ)=1/cosθ → r=1/cosθ → r·cosθ=1 → x=1. Vertical line!

Question: Convert r = sec(θ) to rectangular form.

Step 1: Step 1: Rewrite sec(θ)

sec(θ) equals:

A) sinθ/cosθ

B) cosθ/sinθ

C) 1/cosθ

D) 1/sinθ

Answer: (C) C) 1/cosθ

Why: sec(θ) = 1/cosθ.

Step 2: Step 2: Rearrange

r=1/cosθ becomes:

A) r/cosθ=1

B) r·cosθ=1

C) $r\cdot\cos\theta=\cos^2\theta$

D) r=cosθ

Answer: (B) B) r·cosθ=1

Why: r·cosθ = 1.

Step 3: Step 3: Substitute

Rectangular form:

A) $y=1$

B) $x=1$

C) $x^2+y^2=1$

D) $x+y=1$

Answer: (B) B) $x=1$

Why: r·cosθ = x → x = 1. Vertical line!

Work Space (use extra paper if needed):

Q13

$x^2+y^2-6x=0 \to$ Polar

■ Concept: Group $x^2+y^2=r^2$, then $6x=6r\cdot\cos\theta$. Factor r and solve.

Question: Convert $x^2 + y^2 - 6x = 0$ to polar form.

Step 1: Step 1: Substitute

After substituting:

A) $r^2-6x=0$

B) $r^2-6r\cdot\cos\theta=0$

C) $r-6\cos\theta=0$

D) $r^2+6r\cdot\cos\theta=0$

Answer: (B) B) $r^2-6r\cdot\cos\theta=0$

Why: $r^2 - 6r\cdot\cos\theta = 0$.

Step 2: Step 2: Factor r

After factoring r:

A) $r(r-6\cos\theta)=0$

B) $r(r+6\cos\theta)=0$

C) $r^2(1-6\cos\theta)=0$

D) $r(r-6\sin\theta)=0$

Answer: (A) A) $r(r-6\cos\theta)=0$

Why: $r(r - 6\cos\theta) = 0$. Discard $r=0$ (trivial).

Step 3: Step 3: Final form

Polar equation:

A) $r=6\sin\theta$

B) $r=6\cos\theta$

C) $r=-6\cos\theta$

D) $r^2=6\cos\theta$

Answer: (B) B) $r=6\cos\theta$

Why: $r = 6\cos\theta$ (circle: center (3,0), radius 3).

Work Space (use extra paper if needed):

Q14

$r^2=\sin(2\theta)$ — Lemniscate

■ Concept: $\sin(2\theta)=2\sin\theta\cos\theta$. Multiply by r^2: r■$=2(r\cdot\sin\theta)(r\cdot\cos\theta)=2xy$.

Question: Convert $r^2 = \sin(2\theta)$ to rectangular form.

Step 1: Step 1: Expand sin(2θ)

$\sin(2\theta) =$

A) $2\sin\theta$

B) $\sin^2\theta\cos^2\theta$

C) $2\sin\theta\cos\theta$

D) $\sin\theta+\cos\theta$

Answer: (C) C) $2\sin\theta\cos\theta$

Why: $\sin(2\theta) = 2\sin\theta\cdot\cos\theta$ (double-angle identity).

Step 2: Step 2: Multiply by r²

r■ = ?

A) r■=$2r^2\sin\theta\cos\theta$

B) $r^2=2(r\cdot\sin\theta)(r\cdot\cos\theta)/r^2$

C) r■=$2(r\cdot\sin\theta)(r\cdot\cos\theta)$

D) $r=2xy$

Answer: (C) C) r■=$2(r\cdot\sin\theta)(r\cdot\cos\theta)$

Why: Multiply both sides by r^2: r■ = $2(r\cdot\sin\theta)(r\cdot\cos\theta) = 2yx$.

Step 3: Step 3: Use r■=$(x^2+y^2)^2$

Final rectangular form:

A) $x^2+y^2=2xy$

B) $(x^2+y^2)^2=2xy$

C) $(x^2+y^2)^2=xy$

D) $(x+y)^2=2xy$

Answer: (B) B) $(x^2+y^2)^2=2xy$

Why: $(x^2+y^2)^2 = 2xy$ — this is a lemniscate!

Work Space (use extra paper if needed):

Q15

r=2/(1–cosθ) — Parabola

■ Concept: Standard conic: $r=ed/(1-e\cos\theta)$. $e=1$ → parabola. Multiply by $(1-\cos\theta)$ then square.

Question: Convert $r = 2/(1 - \cos\theta)$ to rectangular form. Identify the conic.

Step 1: Step 1: Multiply by (1-cosθ)

After multiplying:

A) $r=2(1-\cos\theta)$

B) $r(1-\cos\theta)=2$

C) $r-r\cdot\cos\theta=2$

D) Both B and C

Answer: (D) D) Both B and C

Why: $r(1-\cos\theta)=2$ which means $r-r\cdot\cos\theta=2$. B and C are both correct.

Step 2: Step 2: Substitute $r=\sqrt{x^2+y^2}$, $r\cos\theta=x$

Equation after substitution:

A) $\sqrt{x^2+y^2}-x=2$

B) $\sqrt{x^2+y^2}+x=2$

C) $x^2+y^2-x=2$

D) $r-2=x$

Answer: (A) A) $\sqrt{x^2+y^2}-x=2$

Why: $r=\sqrt{x^2+y^2}$ and $r\cdot\cos\theta=x$ → $\sqrt{x^2+y^2} - x = 2$.

Step 3: Step 3: Square and simplify

After squaring:

A) $x^2+y^2=x^2+4$: circle

B) $y^2=4(x+1)$: parabola

C) $y^2=4x^2$: hyperbola

D) $y^2-x^2=4$

Answer: (B) B) $y^2=4(x+1)$: parabola

Why: Square: $x^2+y^2=(x+2)^2$ → $y^2=4x+4=4(x+1)$. Parabola!

Work Space (use extra paper if needed):

Part 4: Exam-Level Problems

Q16

$(x^2+y^2)^3=x^2y^2$ → Polar

■ Concept: $(x^2+y^2)^3$=r■. x^2y^2=r■$\sin^2\theta\cos^2\theta$=r■$\sin^2(2\theta)/4$. Divide by r■.

Question: Convert $(x^2+y^2)^3 = x^2y^2$ to polar form.

Step 1: Step 1: Left side in polar

$(x^2+y^2)^3$ =

A) r^3

B) r■

C) $(r^2)^3$=r■

D) Both B and C

Answer: (D) D) Both B and C

Why: $(x^2+y^2)^3 = (r^2)^3$ = r■. Both B and C are correct.

Step 2: Step 2: Right side x^2y^2 in polar

x^2y^2 =

A) r■$\sin^2\theta\cos^2\theta$

B) r■$\sin(2\theta)/4$

C) r■$\sin^2(2\theta)/4$

D) $r^2\sin(2\theta)$

Answer: (C) C) r■$\sin^2(2\theta)/4$

Why: $x^2y^2=r^2\cos^2\theta\cdot r^2\sin^2\theta$=r■$\cdot(2\sin\theta\cos\theta)^2/4$=r■$\sin^2(2\theta)/4$.

Step 3: Step 3: Divide by r■

Final polar form (divide by r■):

A) $r^2=\sin^2(2\theta)/4$

B) $r^2=4\sin^2(2\theta)$

C) $r^2=\sin(2\theta)/4$

D) r■$=\sin^2(2\theta)$

Answer: (A) A) $r^2=\sin^2(2\theta)/4$

Why: r■/r■$=r^2$ and r■$\sin^2(2\theta)$/(4r■)$=\sin^2(2\theta)/4$ → $r^2=\sin^2(2\theta)/4$.

Work Space (use extra paper if needed):

Q17

r=2cosθ+2sinθ — Circle

■ Concept: r=acosθ+bsinθ is always a circle. Multiply by r → $x^2+y^2=ax+by$.

Question: Convert r = 2cosθ + 2sinθ. Find center and radius.

Step 1: Step 1: Multiply by r

After multiplying by r and substituting:

A) $r^2=2\cos\theta+2\sin\theta$

B) $x^2+y^2=2x+2y$

C) x+y=2r

D) r=2x+2y

Answer: (B) B) $x^2+y^2=2x+2y$

Why: $r^2=2r\cdot\cos\theta+2r\cdot\sin\theta \rightarrow x^2+y^2=2x+2y$.

Step 2: Step 2: Complete the square

After completing the square:

A) $(x-1)^2+(y-1)^2=1$

B) $(x-1)^2+(y-1)^2=2$

C) $(x+1)^2+(y+1)^2=2$

D) $(x-2)^2+(y-2)^2=4$

Answer: (B) B) $(x-1)^2+(y-1)^2=2$

Why: $(x-1)^2-1+(y-1)^2-1=0 \rightarrow (x-1)^2+(y-1)^2=2$.

Step 3: Step 3: Center and radius

Center and radius:

A) Center(1,1),r=2

B) Center(1,1),r=√2

C) Center(-1,-1),r=√2

D) Center(2,2),r=2

Answer: (B) B) Center(1,1),r=√2

Why: Center (1,1), radius = √2.

Work Space (use extra paper if needed):

Q18

r=tanθ·secθ — Parabola

■ Concept: tanθ·secθ=sinθ/cos²θ. Multiply by r·cos²θ: $r^2\cos^2\theta=r\cdot\sin\theta \rightarrow x^2=y$.

Question: Convert r = tanθ·secθ to rectangular form.

Step 1: Step 1: Rewrite in sinθ/cosθ

tanθ·secθ =

A) sinθ/cosθ

B) $\sin\theta/\cos^2\theta$

C) 1/(sinθcosθ)

D) $\cos^2\theta/\sin\theta$

Answer: (B) B) $\sin\theta/\cos^2\theta$

Why: $(\sin\theta/\cos\theta)\cdot(1/\cos\theta)=\sin\theta/\cos^2\theta$.

Step 2: Step 2: Multiply by $r\cdot\cos^2\theta$

After multiplying:

A) $r^2\cos^2\theta = r\cdot\sin\theta$

B) $r\cdot\cos^2\theta = \sin\theta$

C) $r^2\sin\theta = \cos^2\theta$

D) $r = \sin\theta\cos^2\theta$

Answer: (A) A) $r^2\cos^2\theta = r\cdot\sin\theta$

Why: $r(r\cdot\cos^2\theta) = r\cdot\sin\theta \rightarrow r^2\cos^2\theta = r\cdot\sin\theta$.

Step 3: Step 3: Substitute

Final rectangular form:

A) $y = x^2$

B) $x = y^2$

C) $x^2 + y^2 = xy$

D) $y^2 = x$

Answer: (A) A) $y = x^2$

Why: $(r\cdot\cos\theta)^2 = r\cdot\sin\theta \rightarrow x^2 = y$. A parabola!

Work Space (use extra paper if needed):

Q19

Shape Identification (Mixed)

■ Concept: $r\cdot\cos\theta = k \rightarrow x = k$ (vertical); $r\cdot\sin\theta = k \rightarrow y = k$ (horizontal); $r = ed/(1 \pm e\cdot\sin\theta)$ $e = 1 \rightarrow$ parabola.

Question: Match each polar equation to its shape.

Step 1: Step 1: Shape of $r\cdot\cos\theta = 3$

$r\cdot\cos\theta = 3$ is:

A) $y = 3$ (horizontal)

B) $x = 3$ (vertical)

C) $x^2 + y^2 = 9$

D) $y = 3x$ (diagonal)

Answer: (B) B) $x = 3$ (vertical)

Why: $r\cdot\cos\theta = x \rightarrow x = 3$. Vertical line!

Step 2: Step 2: Shape of $r = 6/(1+\sin\theta)$

$r = 6/(1+\sin\theta)$ is:

A) Hyperbola $e = 2$

B) Parabola $e = 1$

C) Ellipse $e = 1/2$

D) Circle $e = 0$

Answer: (B) B) Parabola $e = 1$

Why: e = coefficient of $\sin\theta = 1 \rightarrow$ parabola.

Step 3: Step 3: Shape of $r^2 = 4\cos(2\theta)$

$r^2 = 4\cos(2\theta)$ is:

A) A circle

B) Lemniscate (figure-8)

C) Two lines

D) An ellipse

Answer: (B) B) Lemniscate (figure-8)

Why: $r^2 = a^2\cos(2\theta)$ is the standard polar lemniscate.

Work Space (use extra paper if needed):

Q20

$x^2+y^2+2x-4y=4 \to$ Polar (Advanced)

■ Concept: Group: $r^2+2r\cdot\cos\theta-4r\cdot\sin\theta=4$. Rearrange as quadratic in r, use quadratic formula.

Question: Convert $x^2 + y^2 + 2x - 4y = 4$ to polar form.

Step 1: Step 1: Substitute all terms

After substituting:

A) $r^2+2\cos\theta-4\sin\theta=4$

B) $r^2+2r\cdot\cos\theta-4r\cdot\sin\theta=4$

C) $r+2\cos\theta-4\sin\theta=4$

D) $r^2+2r+4r=4$

Answer: (B) B) $r^2+2r\cdot\cos\theta-4r\cdot\sin\theta=4$

Why: $x^2+y^2=r^2$, $2x=2r\cdot\cos\theta$, $-4y=-4r\cdot\sin\theta \to r^2+2r\cdot\cos\theta-4r\cdot\sin\theta=4$.

Step 2: Step 2: Form quadratic in r

Quadratic form:

A) $r^2+r(2\cos\theta-4\sin\theta)-4=0$

B) $r^2-r(2\cos\theta+4\sin\theta)=4$

C) $r(r+2\cos\theta-4\sin\theta)=4$

D) $r^2=4$

Answer: (A) A) $r^2+r(2\cos\theta-4\sin\theta)-4=0$

Why: $a=1$, $b=(2\cos\theta-4\sin\theta)$, $c=-4 \to r^2+r(2\cos\theta-4\sin\theta)-4=0$.

Step 3: Step 3: Apply quadratic formula (positive root)

Simplified polar form:

A) $r=-(\cos\theta-2\sin\theta)+\sqrt{(\cos\theta-2\sin\theta)^2+4}$

B) $r=2\cos\theta-4\sin\theta$

C) $r=4/(\cos\theta-2\sin\theta)$

D) $r=(\cos\theta-2\sin\theta)+4$

Answer: (A) A) $r=-(\cos\theta-2\sin\theta)+\sqrt{(\cos\theta-2\sin\theta)^2+4}$

Why: Positive root: $r=-(\cos\theta-2\sin\theta)+\sqrt{(\cos\theta-2\sin\theta)^2+4}$.

Work Space (use extra paper if needed):

Answer Key

Q1: B / B / C	Q2: C / C / C	Q3: C / A / A	Q4: B / B / D
Q5: A / C / B	Q6: B / B / B	Q7: B / A / B	Q8: C / B / C
Q9: C / C / C	Q10: A / B / C	Q11: B / B / C	Q12: C / B / B
Q13: B / A / B	Q14: C / C / B	Q15: D / A / B	Q16: D / C / A
Q17: B / B / B	Q18: B / A / A	Q19: B / B / B	Q20: B / A / A

Note: Each question has 3 steps. Answers shown as Step1/Step2/Step3. All 3 must be correct to earn full credit for the question.