IB Mathematics
Core 20 Problems Worksheet

Topics: Algebra · Functions · Calculus · Statistics · Vectors · Probability

Designed for Korean IB Students · Print-Ready · No Color

Name: _____ Date: _____ Score: _____ / 20

Time Started: _____ Time Finished: _____ Total Time: _____

Instructions:

- Each problem has a concept review, a worked example, and 3 multiple choice sub-questions (a), (b), (c).
- Circle your answer for each sub-question. All 3 must be correct to earn full credit.
- Show all working in the space provided. Answers without working may receive no credit.
- Calculators are permitted. State any distribution (e.g. X~B(n,p)) before calculating.

ANSWER SHEET

Circle your answers below. Transfer from your working.

Q1 (Algebra): (a) A B C D (b) A B C D (c) A B C D | **Q2 (Function):** (a) A B C D (b) A B C D (c) A B C D

Q3 (Exponent): (a) A B C D (b) A B C D (c) A B C D | **Q4 (Trigonom):** (a) A B C D (b) A B C D (c) A B C D

Q5 (Differen): (a) A B C D (b) A B C D (c) A B C D | **Q6 (Integrat):** (a) A B C D (b) A B C D (c) A B C D

Q7 (Normal D): (a) A B C D (b) A B C D (c) A B C D | **Q8 (Binomial):** (a) A B C D (b) A B C D (c) A B C D

Q9 (Vectors): (a) A B C D (b) A B C D (c) A B C D | **Q10 (Conditio):** (a) A B C D (b) A B C D (c) A B C D

Q11 (Matrices): (a) A B C D (b) A B C D (c) A B C D | **Q12 (Complex):** (a) A B C D (b) A B C D (c) A B C D

Q13 (Differen): (a) A B C D (b) A B C D (c) A B C D | **Q14 (Series):** (a) A B C D (b) A B C D (c) A B C D

Q15 (Graph Th): (a) A B C D (b) A B C D (c) A B C D | **Q16 (3D Vecto):** (a) A B C D (b) A B C D (c) A B C D

Q17 (Mathemat): (a) A B C D (b) A B C D (c) A B C D | **Q18 (Optimiza):** (a) A B C D (b) A B C D (c) A B C D

Q19 (Limits): (a) A B C D (b) A B C D (c) A B C D | **Q20 (Permutat):** (a) A B C D (b) A B C D (c) A B C D

Problem 1 of 20 | Topic: Algebra | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Arithmetic Sequences: a_n = a_1 + (n-1)d, Sum S_n = n/2 * (2a_1 + (n-1)d)

Geometric Sequences: a_n = a_1 * r^(n-1), Sum S_n = a_1(1-r^n)/(1-r)

Key: Identify constant difference (arithmetic) or constant ratio (geometric) first!

[WORKED EXAMPLE]

Example: a_1=3, d=4, find S_10.

Solution: S_10 = 10/2 x (2x3 + 9x4) = 5 x 42 = 210

Question: An arithmetic sequence has first term 5 and common difference 3. Find the sum of the first 20 terms.

(a) What is the 20th term (a_20)?

A) 57

B) 62

C) 65

D) 60

Working / Scratch space:

(b) What is the sum of the first 10 terms (S_10)?

A) 175

B) 185

C) 195

D) 200

Working / Scratch space:

(c) What is the sum of the first 20 terms (S_20)?

A) 670

B) 650

C) 680

D) 660

Working / Scratch space:

Problem 2 of 20 | Topic: Functions | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Composite: (f o g)(x) = f(g(x)) - apply g first, then f

Inverse: f^(-1)(x) exists if f is one-to-one. To find: swap x and y, solve for y.

Key: f(f^(-1)(x)) = x always!

[WORKED EXAMPLE]

Example: f(x)=2x+1, g(x)=x^2. Find (f o g)(3).

Solution: g(3)=9, then f(9)=2(9)+1 = 19

Question: Let f(x) = 3x - 2 and g(x) = x^2 + 1.

(a) Find (f o g)(2)

A) 11

B) 13

C) 15

D) 17

Working / Scratch space:

(b) Find (g o f)(2)

A) 13

B) 15

C) 16

D) 17

Working / Scratch space:

(c) Find f^(-1)(x)

A) (x+2)/3

B) (x-2)/3

C) 3x+2

D) 3/(x-2)

Working / Scratch space:

Problem 3 of 20 | Topic: Exponential & Logarithm | Difficulty: Hard

[CONCEPT & KEY FORMULA]

log(ab) = log a + log b

log(a/b) = log a - log b

log(a^n) = n*log a

Change of base: log_a(b) = ln(b)/ln(a)

[WORKED EXAMPLE]

Example: Simplify log_2(8) + log_2(4).

Solution: log_2(32) = log_2(2^5) = 5

Question: Solve the following logarithm problems.

(a) Evaluate: log_3(81)

A) 2

B) 3

C) 4

D) 5

Working / Scratch space:

(b) Simplify: log_2(32) - log_2(8)

A) 1

B) 2

C) 3

D) 4

Working / Scratch space:

(c) Solve 2^x = 50 (3 sig. fig.)

A) 5.58

B) 5.62

C) 5.64

D) 5.72

Working / Scratch space:

Problem 4 of 20 | Topic: Trigonometry | Difficulty: Hard

[CONCEPT & KEY FORMULA]

sin^2(x) + cos^2(x) = 1

sin(2x) = 2sin(x)cos(x)

cos(2x) = cos^2(x) - sin^2(x) = 1 - 2sin^2(x)

Key: ALWAYS check domain for multiple solutions!

[WORKED EXAMPLE]

Example: Solve sin(x) = 0.5 for 0 le x le 2pi.

Solution: x = pi/6 AND x = 5pi/6 (two solutions!)

Question: Work with trigonometric identities and equations.

(a) If sin(x) = 3/5 and x is in Q1, find cos(x)

A) 3/4

B) 3/5

C) 4/5

D) 5/4

Working / Scratch space:

(b) Find sin(2x) if sin(x)=3/5, cos(x)=4/5

A) 7/25

B) 12/25

C) 18/25

D) 24/25

Working / Scratch space:

(c) How many solutions does cos(2x)=0.5 have for 0<=x<=2pi?

A) 1

B) 2

C) 3

D) 4

Working / Scratch space:

Problem 5 of 20 | Topic: Differentiation | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Power rule: d/dx[x^n] = n*x^(n-1)

Chain rule: d/dx[f(g(x))] = f'(g(x))*g'(x)

Product rule: d/dx[uv] = u'v + uv'

Key: Chain rule is #1 mistake — identify inner/outer function!

[WORKED EXAMPLE]

Example: Differentiate y = (3x^2+1)^4.

*Solution: dy/dx = 4(3x^2+1)^3 * 6x = 24x(3x^2+1)^3*

Question: Differentiate the following functions.

(a) Find dy/dx if y = 5x^3 - 4x^2 + 7x - 2

A) 15x^2-8x+7

B) 15x^2+8x-7

C) 5x^2-4x+7

D) 15x^3-8x+7

Working / Scratch space:

(b) Differentiate y = sin(3x^2)

A) cos(3x^2)

B) 3cos(3x^2)

C) 2x*cos(3x^2)

D) 6x*cos(3x^2)

Working / Scratch space:

(c) If f(x) = x^2 * e^x, find f'(x)

A) 2x*e^x

B) e^x(x^2+2x)

C) 2x*e^(2x)

D) x^2*e^x + e^x

Working / Scratch space:

Problem 6 of 20 | Topic: Integration | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Power rule: integral(x^n)dx = x^(n+1)/(n+1) + C

integral(e^x)dx = e^x + C

Definite integral = area (can be negative if f(x) < 0!)

Key: Don't forget +C for indefinite integrals!

[WORKED EXAMPLE]

Example: Find integral from 0 to 2 of (2x+3)dx.

Solution: [x^2+3x] from 0 to 2 = (4+6) - 0 = 10

Question: Evaluate the following integrals.

(a) Find integral(6x^2 - 4x + 1)dx

A) 12x-4+C

B) 6x^3-4x^2+x+C

C) 2x^3-2x^2+x+C

D) 3x^2-2x+C

Working / Scratch space:

(b) Evaluate integral from 1 to 3 of x^3 dx

A) 16

B) 18

C) 19

D) 20

Working / Scratch space:

(c) Find area bounded by y = x(4-x) and x-axis

A) 8

B) 32/3

C) 12

D) 16/3

Working / Scratch space:

Problem 7 of 20 | Topic: Normal Distribution | Difficulty: Medium

[CONCEPT & KEY FORMULA]

X ~ N(mu, sigma^2): bell-shaped, symmetric around mu

Standardize: Z = (X - mu)/sigma

Use GDC for probabilities

Key: Always write X ~ N(mu, sigma^2) notation first in exam!

[WORKED EXAMPLE]

Example: X ~ N(50, 16). Find P(X > 54).

Z = (54-50)/4 = 1. P(Z>1) = 0.159

Question: A test score X follows N(70, 64). [mean=70, variance=64, sd=8]

(a) Find P(X < 78)

A) 0.682

B) 0.750

C) 0.841

D) 0.950

Working / Scratch space:

(b) Find P(62 < X < 78)

A) 0.683

B) 0.750

C) 0.841

D) 0.955

Working / Scratch space:

(c) Find k such that P(X < k) = 0.9

A) 78.0

B) 79.2

C) 80.1

D) 80.3

Working / Scratch space:

Problem 8 of 20 | Topic: Binomial Distribution | Difficulty: Medium

[CONCEPT & KEY FORMULA]

X ~ B(n, p): n trials, P(success)=p, independent

P(X=r) = C(n,r)*p^r*(1-p)^(n-r)

Mean: E(X) = np, Variance: np(1-p)

Key: Check all 4 conditions before applying B!

[WORKED EXAMPLE]

*Example: X~B(5,0.4). P(X=2) = C(5,2)*0.4^2*0.6^3 = 0.346*

Question: A fair coin is tossed 10 times. X = number of heads.

(a) State the distribution of X

A) X~N(5,2.5)

B) X~B(10,0.5)

C) X~P(5)

D) X~U(0,10)

Working / Scratch space:

(b) Find P(X = 6)

A) 0.172

B) 0.193

C) 0.205

D) 0.220

Working / Scratch space:

(c) Find P(X >= 8)

A) 0.0547

B) 0.0645

C) 0.0723

D) 0.0898

Working / Scratch space:

Problem 9 of 20 | Topic: Vectors | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Dot product: a.b = a1*b1+a2*b2+a3*b3 = |a||b|cos(theta)

Perpendicular iff a.b = 0

Magnitude: |a| = sqrt(a1^2+a2^2+a3^2)

Key: Dot product gives angle; cross product gives area!

[WORKED EXAMPLE]

Example: a=(2,1,-1), b=(1,3,2). a.b = 2+3-2 = 3

Question: Let a = (3, -1, 2) and b = (1, 4, -2).

(a) Find a . b

A) -2

B) -5

C) -9

D) -8

Working / Scratch space:

(b) Find |a| (magnitude of a)

A) sqrt(12)

B) sqrt(14)

C) sqrt(16)

D) sqrt(18)

Working / Scratch space:

(c) Find angle between a and b (nearest degree)

A) 95

B) 100

C) 105

D) 107

Working / Scratch space:

Problem 10 of 20 | Topic: Conditional Probability | Difficulty: Hard

[CONCEPT & KEY FORMULA]

P(A|B) = P(A intersect B) / P(B)

Bayes: P(A|B) = P(B|A)*P(A) / P(B)

Independent if P(A|B) = P(A)

Key: Always draw a tree diagram first!

[WORKED EXAMPLE]

Example: P(A)=0.6, P(B|A)=0.3, P(B|A')=0.5.

*P(B)=0.6*0.3+0.4*0.5=0.38. P(A|B)=0.18/0.38≈0.474*

Question: 60% of students study math (M). Of M students, 70% pass. Of non-M students, 30% pass.

(a) Find P(passes)

A) 0.54

B) 0.60

C) 0.63

D) 0.66

Working / Scratch space:

(b) Find P(studies math | passes exam)

A) 0.667

B) 0.722

C) 0.778

D) 0.800

Working / Scratch space:

(c) Are M and 'passes' independent?

A) Yes, P(M)=0.6

B) No, P(pass|M)≠P(pass)

C) Yes, always

D) Cannot determine

Working / Scratch space:

Problem 11 of 20 | Topic: Matrices | Difficulty: Hard

[CONCEPT & KEY FORMULA]

det(A) = ad-bc for 2x2 matrix [[a,b],[c,d]]

Inverse: A^(-1) = (1/det)*[[d,-b],[-c,a]]

System Ax=b -- x=A^(-1)*b

Key: Check det ≠ 0 before inverting!

[WORKED EXAMPLE]

*A=[[2,1],[3,4]]. det(A)=2*4-1*3=5*

Question: A = [[3, 2], [1, 4]] and B = [[1, -2], [3, 0]].

(a) Find det(A)

A) 8

B) 9

C) 10

D) 12

Working / Scratch space:

(b) Find A^(-1)

A) (1/10)[[4,-2],[-1,3]]

B) (1/10)[[4,2],[1,3]]

C) (1/5)[[4,-2],[-1,3]]

D) [[4,-2],[-1,3]]

Working / Scratch space:

(c) Find AB

A) [[9,6],[13,2]]

B) [[5,-4],[11,-2]]

C) [[3,-6],[13,-2]]

D) [[9,-6],[13,-2]]

Working / Scratch space:

Problem 12 of 20 | Topic: Complex Numbers | Difficulty: Hard

[CONCEPT & KEY FORMULA]

z = a+bi, i^2 = -1

Modulus: |z| = sqrt(a^2+b^2)

Argument: arg(z) = arctan(b/a) — watch quadrant!

De Moivre: z^n = r^n * (cos(n*theta) + i*sin(n*theta))

[WORKED EXAMPLE]

*z=1+i*sqrt(3): |z|=sqrt(1+3)=2, arg(z)=pi/3*

Question: Let z = sqrt(3) + i.

(a) Find |z|

A) 1

B) 2

C) sqrt(3)

D) 3

Working / Scratch space:

(b) Find arg(z) in radians

A) pi/6

B) pi/4

C) pi/3

D) pi/2

Working / Scratch space:

(c) Find z^3 using De Moivre

A) 4+4i

B) -8i

C) 8i

D) -8+8i

Working / Scratch space:

Problem 13 of 20 | Topic: Differential Equations | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Separable ODE: separate variables then integrate both sides

f(y)dy = g(x)dx -- integrate both sides

Apply initial condition to find constant C

Key: Always write +C until you use initial condition!

[WORKED EXAMPLE]

dy/dx=2xy, y(0)=1: dy/y=2x dx -- ln|y|=x^2+C -- y=e^(x^2)

Question: Solve dy/dx = 3x^2 * y, given y = 2 when x = 0.

(a) First step to solve this ODE?

A) Multiply both sides by y

B) Integrate directly

C) Separate: dy/y = 3x^2 dx

D) Quadratic formula

Working / Scratch space:

(b) After integrating both sides:

A) ln|y|=x^3+C

B) ln|y|=3x^3+C

C) y^2=x^3+C

D) y=3x^3+C

Working / Scratch space:

(c) Particular solution y = ?

A) y=e^(x^3)

B) y=2e^(x^3)

C) y=e^(3x^3)

D) y=2e^(3x^2)

Working / Scratch space:

Problem 14 of 20 | Topic: Series | Difficulty: Medium

[CONCEPT & KEY FORMULA]

Geometric series sum to infinity: S_inf = a/(1-r), only if |r| < 1

If |r| >= 1: series DIVERGES (no sum to infinity)

Finite sum: S_n = a(1-r^n)/(1-r)

Key: Check |r|<1 FIRST!

[WORKED EXAMPLE]

Sum of 3(0.5)^(n-1) for n=1 to inf: S=3/(1-0.5)=6

Question: A geometric series has first term 12 and common ratio 1/3.

(a) Does this series converge? Why?

A) Yes, |r|=1/3 less than 1

B) No, terms don't decrease

C) Yes, because a=12>0

D) Cannot determine

Working / Scratch space:

(b) Find the sum to infinity

A) 16

B) 17

C) 18

D) 20

Working / Scratch space:

(c) Sum of first 5 terms (3 sig.fig.)

A) 17.2

B) 17.6

C) 17.7

D) 17.9

Working / Scratch space:

Problem 15 of 20 | Topic: Graph Theory | Difficulty: Medium

[CONCEPT & KEY FORMULA]

Euler's formula (planar): V - E + F = 2

Eulerian circuit: ALL vertices must have EVEN degree

Eulerian path: exactly 0 or 2 vertices with ODD degree

Key: Count vertex degrees carefully!

[WORKED EXAMPLE]

V=4, E=6: F = 2-4+6 = 4 faces. Planar if E le 3V-6: 6 le 6 ok

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Question: A connected planar graph has 8 vertices and 14 edges.

(a) How many faces does this graph have?

A) 6

B) 7

C) 8

D) 9

Working / Scratch space:

(b) If all 8 vertices have degree 4, how many edges?

A) 14

B) 16

C) 18

D) 20

Working / Scratch space:

(c) Degrees: 2,3,3,4,4. Does Eulerian circuit exist?

A) Yes, connected

B) Yes always

C) No, never

D) No, odd degree vertices exist

Working / Scratch space:

Problem 16 of 20 | Topic: 3D Vectors/Lines | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Line: r = a + t*d (position + t*direction)

Plane: r.n = d (n = normal vector)

Dist. point to plane: |ax0+by0+cz0-d|/sqrt(a^2+b^2+c^2)

Key: Substitute line into plane to find intersection!

[WORKED EXAMPLE]

Line: r=(1,2,3)+t(1,-1,2). Plane: 2x+y-z=5.

Sub: 2(1+t)+(2-t)-(3+2t)=1≠5 -- parallel

Question: Line L: r = (2,1,-1) + t(1,2,3). Plane P: x + y - z = 4.

(a) Direction vector of line L?

A) (2,1,-1)

B) (1,2,3)

C) (1,1,-1)

D) (4,4,-4)

Working / Scratch space:

(b) Does line L lie in plane P?

A) No, they are parallel

B) No, they intersect at one point

C) Yes, every point of L is on P

D) Cannot determine

Working / Scratch space:

(c) Angle between L and plane P?

A) 0 degrees (lies in plane)

B) 30 degrees

C) 45 degrees

D) 60 degrees

Working / Scratch space:

Problem 17 of 20 | Topic: Mathematical Induction | Difficulty: Medium

[CONCEPT & KEY FORMULA]

Step 1: Base case (prove for n=1)

Step 2: Inductive hypothesis: Assume true for n=k

Step 3: Prove true for n=k+1

Key: BOTH steps must be clearly labeled for full marks!

[WORKED EXAMPLE]

Prove sum(i)=n(n+1)/2:

Base n=1: LHS=1=RHS=1 ok

Assume: sum k terms = k(k+1)/2

Add (k+1): k(k+1)/2+(k+1)=(k+1)(k+2)/2 ok

Question: Prove by induction: 1+3+5+...+(2n-1) = n^2.

(a) Base case n=1:

A) LHS=1, RHS=1^2=1 ok

B) LHS=1, RHS=2

C) LHS=2, RHS=1

D) Not needed

Working / Scratch space:

(b) We assume 1+3+...+(2k-1)=k^2. We must prove:

A) k^2=(k+1)^2

B) LHS+(2k)=k^2+k

C) 1+3+...+(2k+1)=(k+1)^2

D) LHS=(k-1)^2

Working / Scratch space:

(c) k^2 + (2k+1) = ?

A) k^2+2k

B) (k+1)^2

C) k^2+1

D) 2k^2+1

Working / Scratch space:

Problem 18 of 20 | Topic: Optimization | Difficulty: Hard

[CONCEPT & KEY FORMULA]

Find critical points: f'(x)=0

Second derivative test: f''(x)>0 -- min; f''(x)<0 -- max

For word problems: define variables, write constraint, substitute

Key: Always verify max/min with second derivative!

[WORKED EXAMPLE]

f(x)=x^3-3x: f'=3x^2-3=0 -- x=+-1.

f''(1)=6>0 (min), f''(-1)=-6<0 (max)

Question: A rectangular box with square base has total surface area 600 cm^2. Maximize volume.

(a) If base side=x, height=h. Express h using SA=600:

A) h=(600-2x^2)/(4x)

B) h=(600-x^2)/(4x)

C) h=(300-x^2)/(2x)

D) h=600/(4x)

Working / Scratch space:

(b) Express Volume V in terms of x only:

A) V=300x-x^3/2

B) V=150x-x^3/3

C) V=x(300-x^2)/2

D) V=150x^2-x^4

Working / Scratch space:

(c) Value of x that maximizes V:

A) x=5*sqrt(2)

B) x=8

C) x=9

D) x=10

Working / Scratch space:

Problem 19 of 20 | Topic: Limits | Difficulty: Medium

[CONCEPT & KEY FORMULA]

L'Hopital's Rule: if lim gives 0/0 or inf/inf form,

lim f/g = lim f'/g'

For polynomials/exponentials as x->inf: exponential wins

Key: Check indeterminate form FIRST before applying L'Hopital!

[WORKED EXAMPLE]

lim(x->0) sin(x)/x: form 0/0 -- L'Hopital: cos(x)/1 = 1

Question: Evaluate the following limits.

(a) lim(x->2) of (x^2-4)/(x-2)

A) 4

B) 0

C) infinity

D) 2

Working / Scratch space:

(b) lim(x->0) of (1-cos x)/x^2

A) 0

B) 1

C) 1/2

D) infinity

Working / Scratch space:

(c) lim(x->inf) of (3x^2+2x)/(5x^2-1)

A) 0

B) 3/5

C) 1

D) infinity

Working / Scratch space:

Problem 20 of 20 | Topic: Permutations & Combinations | Difficulty: Medium

[CONCEPT & KEY FORMULA]

Permutation (order MATTERS): P(n,r)=n!/(n-r)!

Combination (order doesn't matter): C(n,r)=n!/[r!(n-r)!]

With restrictions: fix restricted items first, then arrange rest

Key: Ask 'does ORDER matter?' Yes->P, No->C

[WORKED EXAMPLE]

Choose 3 from 8: C(8,3)=56 (order doesn't matter)

Arrange 3 from 8: P(8,3)=336 (order matters)

Question: A committee of 4 is chosen from 6 men and 5 women.

(a) Ways to choose 4 from 11 (no restriction)?

A) 280

B) 300

C) 330

D) 360

Working / Scratch space:

(b) Ways if committee must have at least 2 women?

A) 210

B) 230

C) 250

D) 260

Working / Scratch space:

(c) 4 members sit in a row (no restriction) - arrangements?

A) 330

B) 660

C) 1320

D) 7920

Working / Scratch space:

ANSWER KEY & EXPLANATIONS

For teacher/self-check use. Review each explanation carefully.

Problem 1: Algebra (Hard)

An arithmetic sequence has first term 5 and common difference 3. Find the sum of the first 20 terms.

(a) (a) What is the 20th term (a_20)?

Correct Answer: B) 62

Explanation: a_20 = 5 + (20-1)x3 = 5 + 57 = 62

(b) (b) What is the sum of the first 10 terms (S_10)?

Correct Answer: B) 185

Explanation: S_10 = 10/2 x (2x5 + 9x3) = 5 x 37 = 185

(c) (c) What is the sum of the first 20 terms (S_20)?

Correct Answer: A) 670

Explanation: S_20 = 20/2 x (2x5 + 19x3) = 10 x 67 = 670

Problem 2: Functions (Hard)

Let f(x) = 3x - 2 and g(x) = x^2 + 1.

(a) (a) Find (f o g)(2)

Correct Answer: B) 13

Explanation: g(2)=5, f(5)=3(5)-2=13

(b) (b) Find (g o f)(2)

Correct Answer: D) 17

Explanation: f(2)=4, g(4)=16+1=17

(c) (c) Find f^(-1)(x)

Correct Answer: A) (x+2)/3

Explanation: Swap: x=3y-2, solve: y=(x+2)/3

Problem 3: Exponential & Logarithm (Hard)

Solve the following logarithm problems.

(a) (a) Evaluate: log_3(81)

Correct Answer: C) 4

Explanation: 3^4=81, so log_3(81)=4

(b) (b) Simplify: log_2(32) - log_2(8)

Correct Answer: B) 2

Explanation: log_2(32/8) = log_2(4) = 2

(c) (c) Solve 2^x = 50 (3 sig. fig.)

Correct Answer: C) 5.64

Explanation: x = log(50)/log(2) = ln50/ln2 ≈ 5.64

Problem 4: Trigonometry (Hard)

Work with trigonometric identities and equations.

(a) (a) If sin(x) = 3/5 and x is in Q1, find cos(x)

Correct Answer: C) 4/5

Explanation: cos^2 = 1 - 9/25 = 16/25, so cos(x) = 4/5

(b) (b) Find sin(2x) if sin(x)=3/5, cos(x)=4/5

Correct Answer: D) 24/25

Explanation: sin(2x) = 2(3/5)(4/5) = 24/25

(c) (c) How many solutions does cos(2x)=0.5 have for 0<=x<=2pi?

Correct Answer: D) 4

Explanation: Let u=2x, 0 le u le 4pi: u=pi/3,5pi/3,7pi/3,11pi/3 -- 4 solutions

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Problem 5: Differentiation (Hard)

Differentiate the following functions.

(a) (a) Find dy/dx if y = 5x^3 - 4x^2 + 7x - 2

Correct Answer: A) 15x^2-8x+7

Explanation: Power rule on each term: 15x^2 - 8x + 7

(b) (b) Differentiate y = sin(3x^2)

Correct Answer: D) 6x*cos(3x^2)

Explanation: Chain rule: cos(3x^2) * 6x = 6x*cos(3x^2)

(c) (c) If f(x) = x^2 * e^x, find f'(x)

Correct Answer: B) e^x(x^2+2x)

Explanation: Product rule: 2x*e^x + x^2*e^x = e^x(x^2+2x)

Problem 6: Integration (Hard)

Evaluate the following integrals.

(a) (a) Find integral(6x^2 - 4x + 1)dx

Correct Answer: C) 2x^3-2x^2+x+C

Explanation: 6x^3/3 - 4x^2/2 + x = 2x^3 - 2x^2 + x + C

(b) (b) Evaluate integral from 1 to 3 of x^3 dx

Correct Answer: D) 20

Explanation: [x^4/4] from 1 to 3 = 81/4 - 1/4 = 80/4 = 20

(c) (c) Find area bounded by y = x(4-x) and x-axis

Correct Answer: B) 32/3

Explanation: Roots at x=0,4. Integral(0 to 4)(4x-x^2)dx = 32 - 64/3 = 32/3

Problem 7: Normal Distribution (Medium)

A test score X follows N(70, 64). [mean=70, variance=64, sd=8]

(a) (a) Find P(X < 78)

Correct Answer: C) 0.841

Explanation: Z=(78-70)/8=1. P(Z less than 1) = 0.841

(b) (b) Find P(62 < X < 78)

Correct Answer: A) 0.683

Explanation: P(-1 less than Z less than 1) ≈ 0.683 (68% rule)

(c) (c) Find k such that P(X < k) = 0.9

Correct Answer: D) 80.3

Explanation: z_0.9 ≈ 1.282. k = 70 + 1.282x8 ≈ 80.3

Problem 8: Binomial Distribution (Medium)

A fair coin is tossed 10 times. X = number of heads.

(a) (a) State the distribution of X

Correct Answer: B) X~B(10,0.5)

Explanation: n=10, P(head)=0.5, independent -- X~B(10,0.5)

(b) (b) Find P(X = 6)

Correct Answer: C) 0.205

Explanation: C(10,6)*0.5^10 = 210/1024 ≈ 0.205

(c) (c) Find P(X >= 8)

Correct Answer: A) 0.0547

Explanation: (45+10+1)/1024 = 56/1024 ≈ 0.0547

Problem 9: Vectors (Hard)

Let a = (3, -1, 2) and b = (1, 4, -2).

(a) (a) Find a . b

Correct Answer: B) -5

Explanation: a.b = 3(1)+(-1)(4)+2(-2) = 3-4-4 = -5

(b) (b) Find |a| (magnitude of a)

Correct Answer: B) sqrt(14)

Explanation: |a| = sqrt(9+1+4) = sqrt(14)

(c) (c) Find angle between a and b (nearest degree)

Correct Answer: D) 107

Explanation: cos(theta)=-5/(sqrt(14)*sqrt(21))=-0.292, theta≈107 degrees

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Problem 10: Conditional Probability (Hard)

60% of students study math (M). Of M students, 70% pass. Of non-M students, 30% pass.

(a) (a) Find P(passes)

Correct Answer: A) 0.54

Explanation: P(pass)=0.6*0.7+0.4*0.3=0.42+0.12=0.54

(b) (b) Find P(studies math | passes exam)

Correct Answer: C) 0.778

Explanation: P(M|pass)=0.42/0.54=7/9≈0.778

(c) (c) Are M and 'passes' independent?

Correct Answer: B) No, P(pass|M)≠P(pass)

Explanation: P(pass|M)=0.7 ≠ P(pass)=0.54, NOT independent

Problem 11: Matrices (Hard)

A = [[3, 2], [1, 4]] and B = [[1, -2], [3, 0]].

(a) (a) Find det(A)

Correct Answer: C) 10

Explanation: det(A)=3*4-2*1=10

(b) (b) Find A^(-1)

Correct Answer: A) (1/10)[[4,-2],[-1,3]]

Explanation: A^(-1) = (1/10)*[[4,-2],[-1,3]]

(c) (c) Find AB

Correct Answer: D) [[9,-6],[13,-2]]

Explanation: Row1: [3+6,-6+0]=[9,-6]. Row2: [1+12,-2+0]=[13,-2]

Problem 12: Complex Numbers (Hard)

Let z = sqrt(3) + i.

(a) (a) Find |z|

Correct Answer: B) 2

Explanation: |z|=sqrt(3+1)=sqrt(4)=2

(b) (b) Find arg(z) in radians

Correct Answer: A) pi/6

Explanation: arg=arctan(1/sqrt(3))=pi/6

(c) (c) Find z^3 using De Moivre

Correct Answer: C) 8i

Explanation: z=2*e^(i*pi/6), z^3=8*e^(i*pi/2)=8i

Problem 13: Differential Equations (Hard)

Solve dy/dx = 3x^2 * y, given y = 2 when x = 0.

(a) (a) First step to solve this ODE?

Correct Answer: C) Separate: dy/y = 3x^2 dx

Explanation: Divide by y, move dx -- dy/y = 3x^2 dx

(b) (b) After integrating both sides:

Correct Answer: A) ln|y|=x^3+C

Explanation: Integral(dy/y)=ln|y|, Integral(3x^2)dx=x^3

(c) (c) Particular solution y = ?

Correct Answer: B) y=2e^(x^3)

Explanation: y(0)=2: C=ln2. y=2e^(x^3)

Problem 14: Series (Medium)

A geometric series has first term 12 and common ratio 1/3.

(a) (a) Does this series converge? Why?

Correct Answer: A) Yes, |r|=1/3 less than 1

Explanation: |r|=1/3 less than 1, so the series converges

(b) (b) Find the sum to infinity

Correct Answer: C) 18

Explanation: S_inf = 12/(1-1/3) = 12/(2/3) = 18

(c) (c) Sum of first 5 terms (3 sig.fig.)

Correct Answer: D) 17.9

Explanation: S_5 = 12*(1-(1/3)^5)/(2/3) ≈ 17.93

Problem 15: Graph Theory (Medium)

A connected planar graph has 8 vertices and 14 edges.

(a) (a) How many faces does this graph have?

Correct Answer: C) 8

Explanation: V-E+F=2: 8-14+F=2, F=8

(b) (b) If all 8 vertices have degree 4, how many edges?

Correct Answer: B) 16

Explanation: Sum of degrees=2E: 8*4=2E, E=16

(c) (c) Degrees: 2,3,3,4,4. Does Eulerian circuit exist?

Correct Answer: D) No, odd degree vertices exist

Explanation: Eulerian circuit needs ALL even degrees. Degrees 3,3 are odd -- No circuit

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Problem 16: 3D Vectors/Lines (Hard)

Line L: r = (2,1,-1) + t(1,2,3). Plane P: x + y - z = 4.

(a) (a) Direction vector of line L?

Correct Answer: B) (1,2,3)

Explanation: In r=a+td, d=(1,2,3) is the direction vector

(b) (b) Does line L lie in plane P?

Correct Answer: C) Yes, every point of L is on P

Explanation: Sub: (2+t)+(1+2t)-(-1+3t)=4 -- 4=4 always true

(c) (c) Angle between L and plane P?

Correct Answer: A) 0 degrees (lies in plane)

Explanation: Since L lies in P, the angle between them is 0

Problem 17: Mathematical Induction (Medium)

Prove by induction: 1+3+5+...+(2n-1) = n^2.

(a) (a) Base case n=1:

Correct Answer: A) LHS=1, RHS=1^2=1 ok

Explanation: n=1: LHS=2(1)-1=1. RHS=1^2=1. LHS=RHS ok

(b) (b) We assume 1+3+...+(2k-1)=k^2. We must prove:

Correct Answer: C) 1+3+...+(2k+1)=(k+1)^2

Explanation: For n=k+1: prove sum to (2(k+1)-1)=(k+1)^2

(c) (c) k^2 + (2k+1) = ?

Correct Answer: B) (k+1)^2

Explanation: k^2+2k+1=(k+1)^2. Induction complete!

Problem 18: Optimization (Hard)

A rectangular box with square base has total surface area 600 cm^2. Maximize volume.

(a) (a) If base side=x, height=h. Express h using SA=600:

Correct Answer: A) h=(600-2x^2)/(4x)

Explanation: SA=2x^2+4xh=600 -- h=(600-2x^2)/(4x)=(300-x^2)/(2x)

(b) (b) Express Volume V in terms of x only:

Correct Answer: C) V=x(300-x^2)/2

Explanation: V=x^2*h=x^2*(300-x^2)/(2x)=x(300-x^2)/2

(c) (c) Value of x that maximizes V:

Correct Answer: D) x=10

Explanation: V'=150-3x^2/2=0 -- x^2=100 -- x=10

Problem 19: Limits (Medium)

Evaluate the following limits.

(a) (a) lim(x->2) of (x^2-4)/(x-2)

Correct Answer: A) 4

Explanation: Factor: (x-2)(x+2)/(x-2)=x+2. At x=2: 4

(b) (b) lim(x->0) of (1-cos x)/x^2

Correct Answer: C) 1/2

Explanation: L'Hopital twice: sin x/(2x) -- cos x/2 = 1/2

(c) (c) lim(x->inf) of (3x^2+2x)/(5x^2-1)

Correct Answer: B) 3/5

Explanation: Divide by x^2: (3+2/x)/(5-1/x^2) -- 3/5

Problem 20: Permutations & Combinations (Medium)

A committee of 4 is chosen from 6 men and 5 women.

(a) (a) Ways to choose 4 from 11 (no restriction)?

Correct Answer: C) 330

Explanation: C(11,4)=11!/(4!7!)=330

(b) (b) Ways if committee must have at least 2 women?

Correct Answer: A) 210

Explanation: 2W2M: C(5,2)*C(6,2)=150; 3W1M: C(5,3)*C(6,1)=60; 4W: C(5,4)=5. Total=210

(c) (c) 4 members sit in a row (no restriction) - arrangements?

Correct Answer: D) 7920

Explanation: C(11,4)*4! = 330*24 = 7920
