

# MATH MASTER WORKSHEET

Algebra 1 & Geometry — 20 Key Problems

Designed for Korean Students — Common Challenging Topics

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Class: \_\_\_\_\_

Score: \_\_\_\_\_ / 20

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## INSTRUCTIONS:

- \* Each problem includes a Key Concept, Example, and 3 Practice Questions.
- \* Circle the correct answer for each question (A, B, C, or D).
- \* All 3 questions must be correct for full credit on that problem.
- \* Show your work in the space provided.
- \* Answers and explanations are at the bottom of each problem.

## SECTION A: ALGEBRA 1

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### Problem 1 | Solving Linear Equations [Linear Equations]

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**KEY CONCEPT:**

A linear equation has the variable to the first power. Solve by using inverse operations to isolate the variable.

$$ax + b = c \Rightarrow x = (c - b) / a$$

**EXAMPLE PROBLEM & SOLUTION:**

Solve:  $3x + 5 = 20$

Step 1: Subtract 5 from both sides  $\Rightarrow 3x = 15$

Step 2: Divide both sides by 3  $\Rightarrow x = 5$

**Answer:**  $x = 5$

**PRACTICE QUESTIONS:**

**Q1. Solve for x:  $2x + 7 = 15$**

- A.  $x = 1$
- B.  $x = 4$
- C.  $x = 11$
- D.  $x = 7$

Work / Notes:

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**Q2. Solve for x:  $5x - 3 = 22$**

- A.  $x = 4$
- B.  $x = 5$
- C.  $x = 19$
- D.  $x = 3$

Work / Notes:

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**Q3. Solve for x:  $x/3 + 4 = 9$**

- A.  $x = 15$
- B.  $x = 5$
- C.  $x = 39$
- D.  $x = 10$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Subtract 7:  $2x = 8$ . Divide by 2:  $x = 4$ .

Q2: Answer B — Add 3:  $5x = 25$ . Divide by 5:  $x = 5$ .

Q3: Answer A — Subtract 4:  $x/3 = 5$ . Multiply by 3:  $x = 15$ .

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## Problem 2 | Solving Inequalities [Inequalities]

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### KEY CONCEPT:

Inequalities work like equations EXCEPT: when you multiply or divide both sides by a NEGATIVE number, you must FLIP the inequality sign.

**If  $-ax > b$ , then  $x < -b/a$  (sign flips!)**

### EXAMPLE PROBLEM & SOLUTION:

Solve:  $-2x + 3 > 9$

Step 1: Subtract 3  $\Rightarrow -2x > 6$

Step 2: Divide by -2 (FLIP sign!)  $\Rightarrow x < -3$

**Answer:  $x < -3$**

### PRACTICE QUESTIONS:

**Q1. Solve:  $-3x + 6 > 15$**

- A.  $x > -3$
- B.  $x < -3$
- C.  $x > 3$
- D.  $x < 3$

Work / Notes:

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**Q2. Which value of  $x$  satisfies  $2x - 5 \leq 7$  ?**

- A.  $x = 7$
- B.  $x = 6$
- C.  $x = 5$
- D.  $x = 8$

Work / Notes:

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**Q3. Solve:  $-4x \geq 20$**

- A.  $x \geq -5$
- B.  $x \leq -5$
- C.  $x \geq 5$
- D.  $x \leq 5$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Subtract 6:  $-3x > 9$ . Divide by  $-3$  (flip!):  $x < -3$ .

Q2: Answer B —  $2x \leq 12$ , so  $x \leq 6$ . Among choices,  $x = 6$  satisfies  $x \leq 6$ .

Q3: Answer B — Divide by  $-4$  and flip sign:  $x \leq -5$ .

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**Problem 3 | Slope and Rate of Change [Linear Functions]**

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**KEY CONCEPT:**

The slope ( $m$ ) measures steepness of a line: rise over run.

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Slope-intercept form:  $y = mx + b$  ( $m = \text{slope}$ ,  $b = \text{y-intercept}$ )

**EXAMPLE PROBLEM & SOLUTION:**

Find slope through  $(1, 3)$  and  $(4, 9)$ :

$$m = (9 - 3) / (4 - 1) = 6 / 3 = 2$$

Answer: slope = 2

**PRACTICE QUESTIONS:**

**Q1. What is the slope of the line through  $(2, 5)$  and  $(6, 13)$ ?**

- A.  $m = 1$
- B.  $m = 2$
- C.  $m = 3$
- D.  $m = 4$

Work / Notes:

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**Q2. In  $y = -3x + 7$ , what is the slope?**

- A. 7
- B. 3
- C.  $-3$
- D.  $-7$

Work / Notes:

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**Q3. What is the y-intercept of the line  $y = 4x - 5$ ?**

- A. 4
- B.  $-5$
- C. 5
- D.  $-4$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $m = (13-5)/(6-2) = 8/4 = 2$ .

Q2: Answer C — In  $y = mx + b$ ,  $m$  is the slope. Here  $m = -3$ .

Q3: Answer B — In  $y = mx + b$ ,  $b$  is the y-intercept. Here  $b = -5$ .

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**Problem 4 | Systems of Equations [Systems of Equations]**

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**KEY CONCEPT:**

A system of equations has 2+ equations with the same variables.

Methods: Substitution (solve one for a variable, plug in) or Elimination (add/subtract to remove one variable).

**EXAMPLE PROBLEM & SOLUTION:**

Solve:  $x + y = 5$  and  $2x - y = 4$

Add equations:  $3x = 9 \Rightarrow x = 3$

Substitute:  $3 + y = 5 \Rightarrow y = 2$

**Answer:  $(x, y) = (3, 2)$**

**PRACTICE QUESTIONS:**

**Q1. Solve the system:  $x + y = 7$  and  $x - y = 3$**

A. (3, 4)

B. (5, 2)

C. (4, 3)

D. (2, 5)

Work / Notes:

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**Q2. Solve:  $2x + y = 9$  and  $x - y = 3$**

A. (3, 3)

B. (4, 1)

C. (2, 5)

D. (5, 0)

Work / Notes:

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**Q3. If  $y = 2x + 1$  and  $y = -x + 7$ , find  $x$ .**

A.  $x = 1$

B.  $x = 2$

C.  $x = 3$

D.  $x = 4$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Add:  $2x = 10 \Rightarrow x = 5$ . Then  $y = 7 - 5 = 2$ . Answer: (5, 2).

Q2: Answer B — Add:  $3x = 12 \Rightarrow x = 4$ . Then  $y = 9 - 8 = 1$ . Answer: (4, 1).

Q3: Answer B — Set equal:  $2x + 1 = -x + 7 \Rightarrow 3x = 6 \Rightarrow x = 2$ .

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**Problem 5 | Factoring Trinomials [Polynomials & Factoring]**

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**KEY CONCEPT:**

To factor  $x^2 + bx + c$ , find two numbers that multiply to  $c$  and add to  $b$ .

$$x^2 + bx + c = (x + p)(x + q) \text{ where } p \cdot q = c \text{ and } p + q = b$$

**EXAMPLE PROBLEM & SOLUTION:**

Factor:  $x^2 + 5x + 6$

Find numbers that multiply to 6 and add to 5: 2 and 3

**Answer:  $(x + 2)(x + 3)$**

**PRACTICE QUESTIONS:**

**Q1. Factor:  $x^2 + 7x + 12$**

- A.  $(x+3)(x+4)$
- B.  $(x+2)(x+6)$
- C.  $(x+1)(x+12)$
- D.  $(x+4)(x+4)$

Work / Notes:

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**Q2. Factor:  $x^2 - 5x + 6$**

- A.  $(x-1)(x-6)$
- B.  $(x-2)(x-3)$
- C.  $(x+2)(x-3)$
- D.  $(x-2)(x+3)$

Work / Notes:

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**Q3. What are the solutions of  $x^2 - x - 6 = 0$  ?**

- A.  $x = 2, x = 3$

- B.  $x = -2, x = 3$
- C.  $x = 2, x = -3$
- D.  $x = -2, x = -3$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer A —  $3 \times 4 = 12$  and  $3 + 4 = 7$ . Answer:  $(x+3)(x+4)$ .

Q2: Answer B —  $(-2)(-3) = 6$  and  $-2 + (-3) = -5$ . Answer:  $(x-2)(x-3)$ .

Q3: Answer B — Factor:  $(x-3)(x+2) = 0$ . So  $x = 3$  or  $x = -2$ .

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**Problem 6 | Quadratic Formula [Quadratic Equations]**

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**KEY CONCEPT:**

For  $ax^2 + bx + c = 0$ , use the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant ( $b^2 - 4ac$ ):  $> 0$  means 2 solutions,  $= 0$  means 1 solution,  $< 0$  means no real solutions.

**EXAMPLE PROBLEM & SOLUTION:**

Solve:  $x^2 - 5x + 6 = 0$  ( $a=1, b=-5, c=6$ )

$$x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2}$$

**Answer:  $x = 3$  or  $x = 2$**

**PRACTICE QUESTIONS:**

**Q1. For  $x^2 - 4x + 4 = 0$ , what does the discriminant equal?**

- A. 0
- B. 4
- C. 8
- D. 16

Work / Notes:

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**Q2. Solve  $x^2 - 6x + 8 = 0$  using the quadratic formula.**

- A.  $x = 2, x = 4$
- B.  $x = -2, x = -4$
- C.  $x = 1, x = 8$
- D.  $x = 3, x = 3$

Work / Notes:

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**Q3. How many real solutions does  $x^2 + 2x + 5 = 0$  have?**

- A. 2 real solutions
- B. 1 real solution
- C. 0 real solutions
- D. 3 real solutions

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer A — Discriminant =  $(-4)^2 - 4(1)(4) = 16 - 16 = 0$ .

Q2: Answer A —  $x = (6 \pm \sqrt{36-32}) / 2 = (6 \pm 2) / 2$ . So  $x = 4$  or  $x = 2$ .

Q3: Answer C — Discriminant =  $4 - 20 = -16 < 0$ . Negative discriminant => no real solutions.

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**Problem 7 | Functions & Domain/Range [Functions]**

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**KEY CONCEPT:**

A function maps each input (domain) to exactly one output (range).

Vertical Line Test: if any vertical line crosses a graph more than once, it is NOT a function.

**f(x) notation: f(3) means plug in x = 3**

**EXAMPLE PROBLEM & SOLUTION:**

Given  $f(x) = 2x - 1$ , find  $f(4)$ :

$$f(4) = 2(4) - 1 = 8 - 1 = 7$$

**Answer:  $f(4) = 7$**

**PRACTICE QUESTIONS:**

**Q1. If  $f(x) = 3x + 2$ , what is  $f(5)$ ?**

- A. 13
- B. 15
- C. 17
- D. 10

Work / Notes:

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**Q2. What is the domain of  $f(x) = \sqrt{x - 4}$ ?**

- A.  $x > 4$
- B.  $x < 4$
- C.  $x \geq 4$
- D. All real numbers

Work / Notes:

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**Q3. If  $g(x) = x^2 - 3$ , what is  $g(-2)$ ?**

- A. 7
- B. 1
- C. -7
- D. -1

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer C —  $f(5) = 3(5) + 2 = 15 + 2 = 17$ .

Q2: Answer C — The expression under a square root must be  $\geq 0$ . So  $x - 4 \geq 0 \Rightarrow x \geq 4$ .

Q3: Answer B —  $g(-2) = (-2)^2 - 3 = 4 - 3 = 1$ .

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**Problem 8 | Exponent Rules [Exponents]**

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**KEY CONCEPT:**

**Product rule:**  $a^m \cdot a^n = a^{(m+n)}$

**Quotient rule:**  $a^m / a^n = a^{(m-n)}$

**Power rule:**  $(a^m)^n = a^{(m \cdot n)}$

**Zero:**  $a^0 = 1$  (a not equal 0)

**Negative:**  $a^{(-n)} = 1 / a^n$

**EXAMPLE PROBLEM & SOLUTION:**

Simplify:  $(x^3)^2 \cdot x^{(-1)}$

$$= x^6 \cdot x^{(-1)} = x^{(6-1)} = x^5$$

**Answer:**  $x^5$

**PRACTICE QUESTIONS:**

**Q1. Simplify:  $x^3 \cdot x^5$**

- A.  $x^{15}$
- B.  $x^8$
- C.  $x^2$
- D.  $2x^8$

Work / Notes:

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**Q2. Simplify:  $y^7 / y^3$**

- A.  $y^4$
- B.  $y^{10}$

C.  $y^{21}$

D.  $y^{-4}$

Work / Notes:

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**Q3. What is the value of  $(2^3)^2$  ?**

A. 12

B. 32

C. 64

D. 48

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Product rule:  $x^3 * x^5 = x^{(3+5)} = x^8$ .

Q2: Answer A — Quotient rule:  $y^7 / y^3 = y^{(7-3)} = y^4$ .

Q3: Answer C — Power rule:  $(2^3)^2 = 2^{(3*2)} = 2^6 = 64$ .

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**Problem 9 | Absolute Value Equations [Absolute Value]**

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**KEY CONCEPT:**

$|x|$  = the distance from  $x$  to 0 on the number line. Always non-negative.

To solve  $|ax + b| = c$  ( $c \geq 0$ ), split into TWO cases:

**$ax + b = c$  OR  $ax + b = -c$**

**EXAMPLE PROBLEM & SOLUTION:**

Solve:  $|2x - 3| = 7$

Case 1:  $2x - 3 = 7 \Rightarrow x = 5$

Case 2:  $2x - 3 = -7 \Rightarrow x = -2$

**Answer:  $x = 5$  or  $x = -2$**

**PRACTICE QUESTIONS:**

**Q1. Solve:  $|x + 3| = 8$**

A.  $x = 5$  or  $x = 11$

B.  $x = 5$  or  $x = -11$

C.  $x = -5$  or  $x = 11$

D.  $x = -5$  or  $x = -11$

Work / Notes:

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**Q2. How many solutions does  $|2x - 1| = -5$  have?**

- A. Two solutions
- B. One solution
- C. No solution
- D. Infinitely many

Work / Notes:

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**Q3. Solve:  $|3x| = 12$**

- A.  $x = 4$  only
- B.  $x = -4$  only
- C.  $x = 4$  or  $x = -4$
- D.  $x = 36$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $x+3 = 8 \Rightarrow x = 5$ . Or  $x+3 = -8 \Rightarrow x = -11$ .

Q2: Answer C — Absolute value is always  $\geq 0$ . It can never equal  $-5$ . So no solution.

Q3: Answer C —  $3x = 12 \Rightarrow x = 4$ . Or  $3x = -12 \Rightarrow x = -4$ .

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**Problem 10 | Word Problems with Linear Models [Applications]**

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**KEY CONCEPT:**

When reading word problems, identify: What does  $x$  represent? Translate words into math.

Key words: 'more than' (+), 'less than' (-), 'times' ( $\times$ ), 'per' (divided by)

**EXAMPLE PROBLEM & SOLUTION:**

A taxi charges \$3 base fee + \$2 per mile. Total bill = \$15. How many miles?

Equation:  $3 + 2m = 15 \Rightarrow 2m = 12 \Rightarrow m = 6$

**Answer: 6 miles**

**PRACTICE QUESTIONS:**

**Q1. A plumber charges \$50 for the visit plus \$30 per hour. The total bill is \$170. How many hours did the plumber work?**

- A. 3 hours
- B. 4 hours
- C. 5 hours
- D. 6 hours

Work / Notes:

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**Q2. Sam has 3 times as many marbles as Tom. Together they have 48. How many does Tom have?**

- A. 8
- B. 12
- C. 16
- D. 24

Work / Notes:

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**Q3. The perimeter of a rectangle is 36. The length is 6 more than the width. What is the width?**

- A. 6
- B. 8
- C. 12
- D. 15

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $50 + 30h = 170 \Rightarrow 30h = 120 \Rightarrow h = 4$  hours.

Q2: Answer B — Tom =  $t$ , Sam =  $3t$ . So  $4t = 48 \Rightarrow t = 12$ .

Q3: Answer A —  $2(w + w+6) = 36 \Rightarrow 4w + 12 = 36 \Rightarrow w = 6$ .

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## SECTION B: GEOMETRY

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### Problem 11 | Triangle Angle Sum [Triangle Properties]

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**KEY CONCEPT:**

The sum of all interior angles in any triangle is always 180 degrees.

$$\text{angle A} + \text{angle B} + \text{angle C} = 180 \text{ degrees}$$

Exterior Angle Theorem: An exterior angle = sum of the two non-adjacent interior angles.

**EXAMPLE PROBLEM & SOLUTION:**

A triangle has angles 55 degrees and 70 degrees. Find the third angle.

$$\text{Third angle} = 180 - 55 - 70 = 55 \text{ degrees}$$

**Answer: 55 degrees**

**PRACTICE QUESTIONS:**

**Q1. A triangle has angles of 45 degrees and 85 degrees. What is the third angle?**

- A. 40 degrees
- B. 50 degrees
- C. 60 degrees
- D. 70 degrees

Work / Notes:

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**Q2. An exterior angle of a triangle is 110 degrees. The two non-adjacent interior angles are equal. What is each?**

- A. 40 degrees
- B. 55 degrees
- C. 70 degrees
- D. 80 degrees

Work / Notes:

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**Q3. In a right triangle, one acute angle is 37 degrees. What is the other acute angle?**

- A. 43 degrees
- B. 53 degrees
- C. 63 degrees
- D. 73 degrees

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $180 - 45 - 85 = 50$  degrees.

Q2: Answer B — Each =  $110 / 2 = 55$  degrees (exterior angle theorem).

Q3: Answer B —  $90 + 37 + x = 180 \Rightarrow x = 53$  degrees.

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## Problem 12 | Pythagorean Theorem [Right Triangles]

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### KEY CONCEPT:

In a right triangle, the square of the hypotenuse ( $c$ ) = sum of squares of the legs ( $a$  and  $b$ ).

$$a^2 + b^2 = c^2$$

Common triples: (3,4,5), (5,12,13), (8,15,17)

### EXAMPLE PROBLEM & SOLUTION:

Find the hypotenuse of a right triangle with legs 6 and 8:

$$c^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$c = \sqrt{100} = 10$$

**Answer: Hypotenuse = 10**

### PRACTICE QUESTIONS:

**Q1. A right triangle has legs of 5 and 12. What is the hypotenuse?**

- A. 13
- B. 15
- C. 17
- D. 10

Work / Notes:

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**Q2. The hypotenuse of a right triangle is 10 and one leg is 6. What is the other leg?**

- A. 4
- B. 6
- C. 8
- D. 16

Work / Notes:

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**Q3. Is a triangle with sides 7, 24, and 25 a right triangle?**

- A. Yes
- B. No
- C. Cannot be determined
- D. Only if the angle is 90 degrees

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer A —  $c^2 = 5^2 + 12^2 = 25 + 144 = 169$ .  $c = \sqrt{169} = 13$ .

Q2: Answer C —  $b^2 = 10^2 - 6^2 = 100 - 36 = 64$ .  $b = \sqrt{64} = 8$ .

Q3: Answer A —  $7^2 + 24^2 = 49 + 576 = 625 = 25^2$ . Yes, it is a right triangle!

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**Problem 13 | Area of Triangles and Quadrilaterals [Area & Perimeter]**

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**KEY CONCEPT:**

**Triangle:**  $A = (1/2) \times \text{base} \times \text{height}$

**Rectangle:**  $A = \text{length} \times \text{width}$

**Parallelogram:**  $A = \text{base} \times \text{height}$

**Trapezoid:**  $A = (1/2)(b_1 + b_2) \times h$

**EXAMPLE PROBLEM & SOLUTION:**

Area of trapezoid with bases 5 and 9, height 4:

$$A = (1/2)(5 + 9) \times 4 = (1/2)(14)(4) = 28$$

**Answer: 28 sq units**

**PRACTICE QUESTIONS:**

**Q1. What is the area of a triangle with base 10 and height 6?**

- A. 60
- B. 30
- C. 16
- D. 20

Work / Notes:

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**Q2. What is the area of a trapezoid with bases 4 and 10, and height 5?**

- A. 35
- B. 200
- C. 70
- D. 50

Work / Notes:

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**Q3. A parallelogram has base 8 cm and height 6 cm. What is its area?**

- A. 28 cm<sup>2</sup>
- B. 48 cm<sup>2</sup>
- C. 24 cm<sup>2</sup>
- D. 96 cm<sup>2</sup>

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $A = (1/2) \times 10 \times 6 = 30$  square units.

Q2: Answer A —  $A = (1/2)(4+10) \times 5 = (1/2)(14)(5) = 35$  square units.

Q3: Answer B —  $A = \text{base} \times \text{height} = 8 \times 6 = 48 \text{ cm}^2$ .

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**Problem 14 | Circle Properties [Circles]**

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**KEY CONCEPT:**

For a circle with radius  $r$ :

**Circumference:  $C = 2 * \pi * r = \pi * d$**

**Area:  $A = \pi * r^2$**

**EXAMPLE PROBLEM & SOLUTION:**

Circle with radius 7:

$C = 2 * \pi * (7) = 14 * \pi$  approximately 43.98

$A = \pi * (7^2) = 49 * \pi$  approximately 153.94

**Answer:  $C = 14 * \pi$ ,  $A = 49 * \pi$**

**PRACTICE QUESTIONS:**

**Q1. What is the area of a circle with radius 5? (Leave in terms of  $\pi$ )**

- A.  $10 * \pi$
- B.  $25 * \pi$
- C.  $20 * \pi$
- D.  $50 * \pi$

Work / Notes:

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**Q2. The circumference of a circle is  $18 * \pi$ . What is the radius?**

- A. 6
- B. 9
- C. 18
- D. 36

Work / Notes:

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**Q3. A circle has diameter 10. What is its area? (Use  $\pi$  approximately 3.14)**

- A. 31.4

- B. 78.5
- C. 62.8
- D. 314

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $A = \pi * r^2 = \pi * (5^2) = 25\pi$ .

Q2: Answer B —  $C = 2\pi r = 18\pi \Rightarrow r = 9$ .

Q3: Answer B — Radius = 5.  $A = 3.14 * 5^2 = 3.14 * 25 = 78.5$ .

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**Problem 15 | Similar Triangles & Proportions [Similarity]**

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**KEY CONCEPT:**

Two triangles are similar (~) if corresponding angles are equal and sides are proportional.

**If triangle ABC ~ triangle DEF, then  $AB/DE = BC/EF = AC/DF$**

Criteria: AA, SAS, SSS

**EXAMPLE PROBLEM & SOLUTION:**

If triangle ABC ~ triangle DEF, AB = 4, DE = 6, BC = 5, find EF:

$$4/6 = 5/EF \Rightarrow EF = 5 * 6 / 4 = 7.5$$

**Answer: EF = 7.5**

**PRACTICE QUESTIONS:**

**Q1. Triangle ABC ~ Triangle DEF. If AB = 3, DE = 9, and BC = 5, what is EF?**

- A. 10
- B. 15
- C. 25
- D. 45

Work / Notes:

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**Q2. Two similar triangles have sides in ratio 2:3. If the smaller triangle's area is 16, what is the larger area?**

- A. 24
- B. 32
- C. 36
- D. 48

Work / Notes:

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**Q3. A 6-ft person casts a 4-ft shadow. A tree casts a 20-ft shadow at the same time. How tall is the tree?**

- A. 25 ft
- B. 28 ft
- C. 30 ft
- D. 32 ft

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Ratio =  $\frac{9}{3} = 3$ . EF =  $5 \times 3 = 15$ .

Q2: Answer C — Area ratio =  $(2:3)^2 = 4:9$ . Larger area =  $16 \times (\frac{9}{4}) = 36$ .

Q3: Answer C —  $\frac{6}{4} = \frac{h}{20} \Rightarrow h = \frac{6 \times 20}{4} = 30$  feet.

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**Problem 16 | Volume of 3D Solids [3D Geometry]**

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**KEY CONCEPT:**

**Rectangular prism:**  $V = l \times w \times h$

**Cylinder:**  $V = \pi \times r^2 \times h$

**Cone:**  $V = (\frac{1}{3}) \times \pi \times r^2 \times h$

**Sphere:**  $V = (\frac{4}{3}) \times \pi \times r^3$

**EXAMPLE PROBLEM & SOLUTION:**

Volume of cylinder with  $r = 3$ ,  $h = 10$ :

$V = \pi \times (3^2) \times 10 = 90\pi$  approximately 282.7

**Answer:**  $V = 90\pi$

**PRACTICE QUESTIONS:**

**Q1. What is the volume of a rectangular prism with  $l=5$ ,  $w=4$ ,  $h=3$ ?**

- A. 24
- B. 47
- C. 60
- D. 94

Work / Notes:

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**Q2. A cone has radius 3 and height 7. What is its volume in terms of  $\pi$ ?**

- A.  $21\pi$
- B.  $63\pi$
- C.  $9\pi$
- D.  $7\pi$

Work / Notes:

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**Q3. A sphere has radius 6. What is its volume in terms of pi?**

- A.  $144\pi$
- B.  $216\pi$
- C.  $288\pi$
- D.  $72\pi$

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer C —  $V = 5 \times 4 \times 3 = 60$  cubic units.

Q2: Answer A —  $V = \frac{1}{3}\pi(3^2)(7) = \frac{1}{3}\pi 9 \cdot 7 = 21\pi$ .

Q3: Answer C —  $V = \frac{4}{3}\pi(6^3) = \frac{4}{3}\pi 216 = 288\pi$ .

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**Problem 17 | Parallel Lines & Transversals [Angle Relationships]**

**KEY CONCEPT:**

When a transversal crosses parallel lines:

Corresponding angles are EQUAL (same position).

Alternate interior angles are EQUAL (Z-shape).

Co-interior (same-side interior) angles are SUPPLEMENTARY (add to 180 degrees).

Vertical angles are always EQUAL.

**EXAMPLE PROBLEM & SOLUTION:**

Parallel lines cut by transversal, one angle = 65 degrees:

Alternate interior angle = 65 degrees

Co-interior angle =  $180 - 65 = 115$  degrees

**PRACTICE QUESTIONS:**

**Q1. Two parallel lines are cut by a transversal. One angle measures 72 degrees. What is the alternate interior angle?**

- A. 18 degrees
- B. 72 degrees
- C. 108 degrees
- D. 118 degrees

Work / Notes:

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**Q2. Two parallel lines are cut by a transversal. One co-interior angle is 65 degrees. What is the other co-interior angle?**

- A. 65 degrees
- B. 90 degrees
- C. 115 degrees
- D. 125 degrees

Work / Notes:

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**Q3. Two lines intersect. One angle is 48 degrees. What is the vertically opposite angle?**

- A. 42 degrees
- B. 48 degrees
- C. 132 degrees
- D. 138 degrees

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Alternate interior angles are equal when lines are parallel. Answer: 72 degrees.

Q2: Answer C — Co-interior angles are supplementary:  $180 - 65 = 115$  degrees.

Q3: Answer B — Vertical angles are always equal. Answer: 48 degrees.

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**Problem 18 | Coordinate Geometry [Coordinate Geometry]**

**KEY CONCEPT:**

**Distance:**  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Midpoint:**  $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

**EXAMPLE PROBLEM & SOLUTION:**

Midpoint of (2, 6) and (8, 2):

$$M = ((2+8)/2, (6+2)/2) = (5, 4)$$

**Answer: Midpoint = (5, 4)**

**PRACTICE QUESTIONS:**

**Q1. What is the midpoint of (1, 3) and (7, 9)?**

- A. (3, 5)
- B. (4, 6)
- C. (6, 6)
- D. (8, 12)

Work / Notes:

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**Q2. What is the distance between (0, 0) and (3, 4)?**

- A. 3
- B. 4
- C. 5
- D. 7

Work / Notes:

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**Q3. The midpoint of segment AB is (5, 3). If A = (2, 1), what is B?**

- A. (8, 5)
- B. (7, 4)
- C. (3, 2)
- D. (10, 6)

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B —  $M = ((1+7)/2, (3+9)/2) = (4, 6)$ .

Q2: Answer C —  $d = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ .

Q3: Answer A —  $(2+x)/2 = 5 \Rightarrow x = 8$ .  $(1+y)/2 = 3 \Rightarrow y = 5$ . B = (8, 5).

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**Problem 19 | Congruent Triangles [Congruence]**

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**KEY CONCEPT:**

Two triangles are congruent ( $\cong$ ) if they have the same size and shape.

SSS: All 3 sides equal | SAS: 2 sides and included angle equal

ASA: 2 angles and included side equal | AAS: 2 angles and non-included side

HL: Hypotenuse-leg (right triangles only)

**EXAMPLE PROBLEM & SOLUTION:**

Triangle ABC is congruent to Triangle DEF by SAS means:

AB = DE, angle A = angle D, and AC = DF

**Answer: Two sides and the included angle are equal.**

**PRACTICE QUESTIONS:**

**Q1. Two triangles share a common side. Each triangle has two equal sides adjacent to that common side. Which postulate proves congruence?**

- A. ASA

- B. SSS
- C. AAS
- D. SAS

Work / Notes:

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**Q2. In triangles ABC and DEF: angle A = angle D, angle B = angle E, and AB = DE. What postulate proves congruence?**

- A. SAS
- B. AAS
- C. ASA
- D. HL

Work / Notes:

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**Q3. Which is NOT a valid triangle congruence shortcut?**

- A. SSS
- B. SAS
- C. AAA
- D. AAS

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer B — Three pairs of equal sides => SSS congruence.

Q2: Answer C — Two equal angles and the included side => ASA.

Q3: Answer C — AAA proves SIMILARITY, not congruence. Triangles can be similar but different sizes.

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**Problem 20 | Transformations [Transformations]**

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**KEY CONCEPT:**

Types of rigid transformations (preserve shape and size):

Translation: Slide  $(x+a, y+b)$  | Reflection: Flip over a line

Rotation: Turn around a point | Dilation: Scale — changes size (NOT rigid!)

**EXAMPLE PROBLEM & SOLUTION:**

Reflect  $(3, 5)$  over the x-axis:

Rule:  $(x, y) \Rightarrow (x, -y)$

**Answer:  $(3, -5)$**

**PRACTICE QUESTIONS:**

**Q1. Point A(4, -2) is translated by vector (-3, 5). What are the new coordinates?**

- A. (1, 3)
- B. (7, -7)
- C. (-1, 3)
- D. (1, 7)

Work / Notes:

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**Q2. Point P(2, 5) is reflected over the y-axis. What are the new coordinates?**

- A. (2, -5)
- B. (-2, 5)
- C. (-2, -5)
- D. (5, 2)

Work / Notes:

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**Q3. Which transformation changes the SIZE of a figure?**

- A. Translation
- B. Reflection
- C. Rotation
- D. Dilation

Work / Notes:

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**ANSWER KEY & EXPLANATIONS:**

Q1: Answer A —  $x: 4+(-3)=1$ ,  $y: -2+5=3$ . New coordinates: (1, 3).

Q2: Answer B — Reflection over y-axis:  $(x,y) \Rightarrow (-x,y)$ . So  $(2,5) \Rightarrow (-2,5)$ .

Q3: Answer D — Only dilation changes size. The others are rigid motions that preserve size and shape.

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