Math Master

Algebra 2 & Geometry — 20 Core Problems

Name: ________________________ Date: ______________ Score: _____ / 20

Instructions: Answer all 3 sub-questions for each problem. Show all work in the space provided. Circle your answer choice. All 3 must be correct to earn full credit for the problem.

ALGEBRA 2 (Problems 1-10)

Problem 1 — Quadratic Formula & Discriminant

■ Key Concept:

- Quadratic Formula: $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$ for $ax^2 + bx + c = 0$
- Discriminant $D = b^2 - 4ac$:
- $D > 0$ -> Two distinct real roots
- $D = 0$ -> One repeated real root
- $D < 0$ -> Two complex (imaginary) roots

Q1. What is the discriminant of $3x^2 - 4x + 2 = 0$?

(A) -8 (B) 4

(C) 28 (D) 16

✓ Answer: (A) | $D = (-4)^2 - 4(3)(2) = 16 - 24 = -8$. $D < 0$, so no real roots.

Work space:

Q2. Solve $x^2 + 6x + 9 = 0$. What is the value of x?

(A) $x = 3$ (B) $x = -3$

(C) $x = \pm 3$ (D) $x = 9$

✓ Answer: (B) | $D = 36 - 36 = 0$, one repeated root. $x = -6/2 = -3$.

Work space:

Q3. Which equation has two complex (imaginary) roots?

(A) $x^2 - 5x + 6 = 0$ (B) $x^2 + 4 = 0$

(C) $x^2 - 9 = 0$ (D) $2x^2 - 3x = 0$

✓ Answer: (B) | $x^2 + 4 = 0$: $D = 0 - 16 = -16 < 0$, so roots are imaginary: $x = \pm 2i$.

Work space:

Problem 2 — Polynomial Division & Remainder Theorem

■ **Key Concept:**

- Remainder Theorem: when f(x) is divided by (x - c), the remainder = f(c).
- Factor Theorem: (x - c) is a factor of f(x) <=> f(c) = 0.

Q1. What is the remainder when f(x) = x^3 + 3x^2 - x + 2 is divided by (x - 2)?

(A) 10 (B) 16

(C) 20 (D) 24

✓ Answer: (C) | f(2) = 8 + 12 - 2 + 2 = 20.

Work space:

Q2. If (x + 1) is a factor of f(x) = x^3 + kx^2 + 2x - 4, what is k?

(A) k = 3 (B) k = -3

(C) k = 5 (D) k = 7

✓ Answer: (D) | f(-1) = 0: -1 + k - 2 - 4 = 0 => k = 7.

Work space:

Q3. Which value of c makes (x - c) a factor of p(x) = x^3 - 8?

(A) c = -2 (B) c = 4

(C) c = 2 (D) c = 8

✓ Answer: (C) | p(2) = 8 - 8 = 0, so (x - 2) is a factor.

Work space:

Problem 3 — Rational Exponents & Radical Expressions

■ **Key Concept:**

- a^(m/n) = n-th root of a^m = (n-th root of a)^m
- a^(1/2) = sqrt(a), a^(1/3) = cube root of a, a^(-n) = 1/a^n

Q1. Simplify 16^(3/4).

(A) 4 (B) 6

(C) 8 (D) 12

✓ Answer: (C) | 16^(3/4) = (4th root of 16)^3 = 2^3 = 8.

Work space:

Q2. Which expression equals the cube root of $x^4 * x^2$?

(A) x^2

(B) $x^{(4/3)}$

(C) $x^{(2/3)}$

(D) $x^{(8/3)}$

✓ Answer: (A) | Cube root of $x^6 = x^{(6/3)} = x^2$.

Work space:

Q3. Simplify $x^{(3/2)} * x^{(1/2)} / x^{(1/2)}$.

(A) x

(B) $x^{(3/4)}$

(C) $x^{(3/2)}$

(D) x^2

✓ Answer: (C) | Num: $x^{(3/2+1/2)} = x^2$. Then $x^2 / x^{(1/2)} = x^{(3/2)}$.

Work space:

Problem 4 — Logarithms & Log Properties

■ **Key Concept:**

- $\log_b(x) = y$ means $b^y = x$
- $\log(mn) = \log m + \log n$
- $\log(m/n) = \log m - \log n$
- $\log(m^k) = k * \log m$
- Change of base: $\log_b(a) = \ln(a) / \ln(b)$

Q1. Solve $\log_3(x) = 4$.

(A) $x = 12$

(B) $x = 81$

(C) $x = 64$

(D) $x = 27$

✓ Answer: (B) | $\log_3(x) = 4 \Rightarrow x = 3^4 = 81$.

Work space:

Q2. Which is equal to $\log(100) + \log(0.01)$?

(A) 0

(B) 1

(C) 2

(D) -2

✓ Answer: (A) | $\log(100) = 2$, $\log(0.01) = -2$. Sum = 0.

Work space:

Q3. Solve $2^{(x+1)} = 32$.

(A) $x = 3$

(B) $x = 4$

(C) x = 5

(D) x = 6

✓ Answer: (B) | 32 = 2^5, so x+1 = 5, x = 4.

Work space:

Problem 5 — Complex Numbers

■ Key Concept:

- Imaginary unit: i = sqrt(-1), i^2 = -1
- Complex number: a + bi
- Powers cycle: i^1=i, i^2=-1, i^3=-i, i^4=1
- Conjugate of (a+bi) is (a-bi)

Q1. What is i^23?

(A) 1

(B) -1

(C) i

(D) -i

✓ Answer: (D) | 23 = 4*5 + 3, so i^23 = i^3 = -i.

Work space:

Q2. Simplify (2 + 3i) + (4 - i).

(A) 6 + 2i

(B) 6 - 2i

(C) 8 + 2i

(D) 6 + 4i

✓ Answer: (A) | Real: 2+4=6. Imaginary: 3i-i=2i. Answer: 6+2i.

Work space:

Q3. What is 1/i simplified?

(A) i

(B) -i

(C) 1

(D) -1

✓ Answer: (B) | 1/i * i/i = i/i^2 = i/(-1) = -i.

Work space:

Problem 6 — Arithmetic & Geometric Sequences

■ Key Concept:

- Arithmetic: a_n = a_1 + (n-1)d (d = common difference)
- Geometric: a_n = a_1 * r^(n-1) (r = common ratio)
- Geometric sum: S_n = a_1 * (1 - r^n) / (1 - r)

Q1. In the geometric sequence 2, 6, 18, ..., what is the 5th term?

(A) 54 (B) 108

(C) 162 (D) 216

✓ Answer: (C) | r=3. a_5 = 2*3^4 = 2*81 = 162.

Work space:

Q2. What is the sum of the first 5 terms of 1, 2, 4, 8, ...?

(A) 15 (B) 31

(C) 32 (D) 63

✓ Answer: (B) | S_5 = 1*(1-2^5)/(1-2) = (1-32)/(-1) = 31.

Work space:

Q3. Arithmetic sequence: a_3 = 11, a_7 = 23. What is d?

(A) 2 (B) 3

(C) 4 (D) 6

✓ Answer: (B) | a_7 - a_3 = 4d => 12 = 4d => d = 3.

Work space:

Problem 7 — Systems of Equations (3 Variables)

■ **Key Concept:**

- To solve 3 equations with 3 unknowns:
- 1. Eliminate one variable from two pairs of equations
- 2. Solve the resulting 2-variable system
- 3. Back-substitute to find all variables

Q1. x+y+z=10, x-y=2, z=3. What is y?

(A) y=2 (B) y=2.5

(C) y=3 (D) y=4

✓ Answer: (B) | z=3 => x+y=7. With x-y=2: 2x=9 => x=4.5, y=2.5.

Work space:

Q2. Solve: 2x+y=7, x-y=2. Solution (x,y)?

(A) (2,3) (B) (-3,2)

(C) (3,1) (D) (1,5)

✓ Answer: (C) | Add: 3x=9 => x=3. Then y=7-6=1.

Work space:

Q3. Which method eliminates a variable by adding multiples of equations?

(A) Substitution
(B) Graphing
(C) Elimination
(D) Decomposition

✓ Answer: (C) | The Elimination method multiplies equations so adding them cancels one variable.

Work space:

Problem 8 — Inverse & Composite Functions

■ **Key Concept:**

- Inverse f^(-1)(x): swap x and y, then solve for y.
- Composition: (f o g)(x) = f(g(x))
- A function has an inverse only if it passes the Horizontal Line Test.

Q1. f(x)=x^2+1, g(x)=x-1. Find (f o g)(3).

(A) 4
(B) 5
(C) 8
(D) 9

✓ Answer: (B) | g(3)=2. f(2)=4+1=5.

Work space:

Q2. Find f^(-1)(7) if f(x) = 3x - 2.

(A) 2
(B) 3
(C) 4
(D) 5

✓ Answer: (B) | 3x-2=7 => 3x=9 => x=3.

Work space:

Q3. Which condition must be true for a function to have an inverse?

(A) It must be quadratic
(B) Pass the Horizontal Line Test
(C) Defined for all reals
(D) Domain = Range

✓ Answer: (B) | A function has an inverse only if it is one-to-one (Horizontal Line Test).

Work space:

Problem 9 — Exponential Growth & Decay

■ **Key Concept:**

- Growth: A = A_0 * (1+r)^t
- Decay: A = A_0 * (1-r)^t
- Continuous: A = A_0 * e^(kt)
- Half-life: A = A_0 * (1/2)^(t/h)

Q1. Decay 10%/yr, start 500g, after 2 yrs?

(A) 400g (B) 405g

(C) 450g (D) 410g

✓ Answer: (B) | A = 500*(0.9)^2 = 500*0.81 = 405g.

Work space:

Q2. Half-life 4 yrs, start 80g. After 8 yrs?

(A) 40g (B) 20g

(C) 10g (D) 5g

✓ Answer: (B) | 8 yrs = 2 half-lives: 80 -> 40 -> 20g.

Work space:

Q3. A = 200*e^(0.03t). What is the initial amount?

(A) 0.03 (B) e

(C) 200 (D) 203

✓ Answer: (C) | At t=0: A = 200*e^0 = 200.

Work space:

Problem 10 — Binomial Theorem

■ **Key Concept:**

- (a+b)^n = sum C(n,k) * a^(n-k) * b^k
- C(n,k) = n! / [k! * (n-k)!]
- The (k+1)-th term is C(n,k)*a^(n-k)*b^k.

Q1. Coefficient of x^2 in (x+3)^4?

(A) 36 (B) 54

(C) 108 (D) 81

✓ Answer: (B) | C(4,2)*x^2*3^2 = 6*9*x^2 = 54x^2.

Work space:

Q2. Find the 3rd term of (2x-1)^5.

(A) 80x^3 (B) -80x^3

(C) 40x^4 (D) 80x^4

✓ Answer: (A) | k=2: C(5,2)*(2x)^3*(-1)^2 = 10*8x^3*1 = 80x^3.

Work space:

Q3. What is C(6,2)?

(A) 6 (B) 12

(C) 15 (D) 30

✓ Answer: (C) | C(6,2) = 6*5/(2*1) = 15.

Work space:

GEOMETRY (Problems 11-20)

Problem 11 — Triangle Congruence & Similarity

■ **Key Concept:**

- Congruence: SSS, SAS, ASA, AAS, HL
- Similarity: AA, SSS~, SAS~
- Similar triangles: proportional sides, equal angles
- AB/DE = BC/EF = CA/FD (if triangle ABC ~ triangle DEF)

Q1. Triangles ABC and DEF are similar. AB=6, DE=9, BC=8. Find EF.

(A) 10 (B) 12

(C) 14 (D) 16

✓ Answer: (B) | 6/9 = 8/EF => EF = 12.

Work space:

Q2. Two triangles have angles 50, 70, 60 degrees each. Conclusion?

(A) Congruent (B) Similar by AA

(C) Equilateral (D) Right triangles

✓ Answer: (B) | Equal angles => similar by AA postulate.

Work space:

Q3. SAS congruence requires:

(A) Two sides and included angle (B) Two angles and a side

(C) Three sides (D) Two sides and non-included angle

✓ Answer: (A) | SAS = Side-Angle-Side: two sides and the angle BETWEEN them.

Work space:

Problem 12 — Pythagorean Theorem & Special Triangles

■ **Key Concept:**

- Right triangle: a^2 + b^2 = c^2 (c = hypotenuse)
- 30-60-90 triangle: sides ratio 1 : sqrt(3) : 2
- 45-45-90 triangle: sides ratio 1 : 1 : sqrt(2)

Q1. Right triangle legs 9 and 12. Hypotenuse?

(A) 13 (B) 15

(C) 17 (D) 21

✓ Answer: (B) | c = sqrt(81+144) = sqrt(225) = 15.

Work space:

Q2. In 30-60-90 triangle, hypotenuse=10. Shorter leg?

(A) 3 (B) 5

(C) 5*sqrt(3) (D) 10*sqrt(3)

✓ Answer: (B) | Shorter leg = hyp/2 = 10/2 = 5.

Work space:

Q3. Which is a Pythagorean triple?

(A) 5,10,13 (B) 6,8,10

(C) 4,6,8 (D) 3,6,8

✓ Answer: (B) | 6^2+8^2=36+64=100=10^2. Yes!

Work space:

Problem 13 — Circle Theorems (Angles & Arcs)

■ **Key Concept:**

- Inscribed angle = (1/2) * intercepted arc

- Central angle = intercepted arc
- Angle in semicircle = 90 degrees
- Cyclic quadrilateral: opposite angles sum to 180 degrees

Q1. Central angle 120 deg, radius 6. Arc length?

(A) 2*pi (B) 4*pi
(C) 6*pi (D) 8*pi

✓ Answer: (B) | Arc = r*theta = 6*(2*pi/3) = 4*pi.

Work space:

Q2. Inscribed angle intercepts 140 deg arc. Angle value?

(A) 35 deg (B) 70 deg
(C) 140 deg (D) 280 deg

✓ Answer: (B) | Inscribed angle = (1/2)*140 = 70 deg.

Work space:

Q3. Cyclic quadrilateral, one angle 75 deg. Opposite angle?

(A) 75 deg (B) 90 deg
(C) 105 deg (D) 115 deg

✓ Answer: (C) | Opposite angles supplementary: 180-75 = 105 deg.

Work space:

Problem 14 — Area & Perimeter of Polygons

■ Key Concept:

- Triangle: A = (1/2)*b*h
- Trapezoid: A = (1/2)*(b1+b2)*h
- Heron's: A = sqrt[s(s-a)(s-b)(s-c)], s=(a+b+c)/2
- Regular hexagon: A = (3*sqrt(3)/2)*s^2

Q1. Trapezoid: parallel sides 5,9, height 4. Area?

(A) 20 (B) 24
(C) 28 (D) 32

✓ Answer: (C) | A = (1/2)*(5+9)*4 = (1/2)*56 = 28.

Work space:

Q2. Regular hexagon, side=4. Area using A=(3*sqrt(3)/2)*s^2?

(A) 24*sqrt(3)
(B) 16*sqrt(3)
(C) 12*sqrt(3)
(D) 8*sqrt(3)

✓ Answer: (A) | A = (3*sqrt(3)/2)*16 = 24*sqrt(3).

Work space:

Q3. Heron's formula: triangle sides 5,12,13. Area?

(A) 20
(B) 30
(C) 40
(D) 60

✓ Answer: (B) | s=15. A=sqrt(15*10*3*2)=sqrt(900)=30.

Work space:

Problem 15 — Volume & Surface Area of Solids

■ **Key Concept:**

- Sphere: V=(4/3)*pi*r^3, SA=4*pi*r^2
- Cylinder: V=pi*r^2*h, SA=2*pi*r^2+2*pi*r*h
- Cone: V=(1/3)*pi*r^2*h, SA=pi*r^2+pi*r*l
- Pyramid: V=(1/3)*B*h

Q1. Cone: radius=3, height=4. Volume?

(A) 12*pi
(B) 36*pi
(C) 48*pi
(D) 16*pi

✓ Answer: (A) | V = (1/3)*pi*9*4 = 12*pi.

Work space:

Q2. Cylinder: radius=2, height=5. Lateral surface area?

(A) 10*pi
(B) 20*pi
(C) 25*pi
(D) 40*pi

✓ Answer: (B) | Lateral SA = 2*pi*r*h = 2*pi*2*5 = 20*pi.

Work space:

Q3. Rectangular pyramid: base 6x4, height=9. Volume?

(A) 36
(B) 72
(C) 108
(D) 216

✓ Answer: (B) | V = (1/3)*(24)*9 = 72.

Work space:

Problem 16 — Coordinate Geometry

■ Key Concept:

- Distance: d = sqrt[(x2-x1)^2 + (y2-y1)^2]
- Midpoint: M = ((x1+x2)/2, (y1+y2)/2)
- Slope: m = (y2-y1)/(x2-x1)
- Perpendicular slopes: m1 * m2 = -1

Q1. Midpoint of (-2,4) and (6,-2)?

(A) (2,1) (B) (4,1)

(C) (2,-1) (D) (1,2)

✓ Answer: (A) | M = ((-2+6)/2, (4-2)/2) = (2,1).

Work space:

Q2. Slope perpendicular to 3/4?

(A) 3/4 (B) -3/4

(C) 4/3 (D) -4/3

✓ Answer: (D) | Perpendicular slope = negative reciprocal = -4/3.

Work space:

Q3. Distance from (0,0) to (-3,4)?

(A) 4 (B) 5

(C) 6 (D) 7

✓ Answer: (B) | d = sqrt(9+16) = sqrt(25) = 5.

Work space:

Problem 17 — Transformations

■ Key Concept:

- Reflect over x-axis: (x,y) -> (x,-y)
- Reflect over y-axis: (x,y) -> (-x,y)
- Rotate 90 deg CCW: (x,y) -> (-y,x)
- Rotate 180 deg: (x,y) -> (-x,-y)
- Translate (a,b): (x,y) -> (x+a, y+b)

Q1. Image of (5,-2) after reflection over y-axis?

(A) (5,2) (B) (-5,-2)

(C) (-5,2) (D) (2,-5)

✓ Answer: (B) | Reflect over y-axis: (x,y)->(-x,y). So (5,-2)->(-5,-2).

Work space:

Q2. Image of (-1,3) after 180 deg rotation?

(A) (1,-3) (B) (-1,-3)

(C) (-3,1) (D) (3,1)

✓ Answer: (A) | 180 deg rotation: (x,y)->(-x,-y). So (-1,3)->(1,-3).

Work space:

Q3. (2,5) translated by (-3,4). New coordinates?

(A) (-1,9) (B) (5,1)

(C) (-5,9) (D) (1,-9)

✓ Answer: (A) | (2+(-3), 5+4) = (-1,9).

Work space:

Problem 18 — Trigonometric Ratios (SOH-CAH-TOA)

■ **Key Concept:**

- sin(theta) = opp/hyp, cos(theta) = adj/hyp, tan(theta) = opp/adj
- sin^2(theta) + cos^2(theta) = 1
- sin(30)=1/2, cos(60)=1/2, tan(45)=1
- tan(30) = sqrt(3)/3, tan(60) = sqrt(3)

Q1. sin(theta)=5/13. What is cos(theta)?

(A) 5/12 (B) 12/13

(C) 13/12 (D) 5/13

✓ Answer: (B) | adj = sqrt(169-25)=12. cos=12/13.

Work space:

Q2. What is tan(30 deg)?

(A) sqrt(3)/2 (B) 1/2

(C) sqrt(3)/3 (D) sqrt(3)

✓ Answer: (C) | tan(30) = sin(30)/cos(30) = (1/2)/(sqrt(3)/2) = 1/sqrt(3) = sqrt(3)/3.

Work space:

Q3. Ladder 10m, angle 60 deg with ground. Height reached?

(A) 5m
(B) 5*sqrt(3) m
(C) 10*sqrt(3) m
(D) 5*sqrt(2) m

✓ Answer: (B) | Height = 10*sin(60) = 10*(sqrt(3)/2) = 5*sqrt(3) m.

Work space:

Problem 19 — Parallel Lines & Transversals

■ **Key Concept:**

- Corresponding angles: equal
- Alternate interior angles: equal
- Alternate exterior angles: equal
- Co-interior (same-side interior): supplementary (sum=180 deg)

Q1. Alternate interior angle is 72 deg. Other alternate interior angle?

(A) 18 deg
(B) 72 deg
(C) 108 deg
(D) 144 deg

✓ Answer: (B) | Alternate interior angles are equal when lines are parallel.

Work space:

Q2. Corresponding angles: (3x+20) and (5x-10). Find x.

(A) x=10
(B) x=15
(C) x=20
(D) x=25

✓ Answer: (B) | 3x+20=5x-10 => 30=2x => x=15.

Work space:

Q3. Co-interior angles: (2x+10) and (3x-20). Find x.

(A) x=36
(B) x=38
(C) x=40
(D) x=42

✓ Answer: (B) | 2x+10+3x-20=180 => 5x-10=180 => 5x=190 => x=38.

Work space:

Problem 20 — Angles in Polygons

■ **Key Concept:**

- Sum of interior angles of n-gon: (n-2)*180 degrees
- Each interior angle of regular n-gon: (n-2)*180/n
- Sum of exterior angles of any convex polygon: 360 degrees

Q1. Sum of interior angles of a pentagon?

(A) 360 deg
(B) 450 deg
(C) 540 deg
(D) 720 deg

✓ Answer: (C) | (5-2)*180 = 3*180 = 540 deg.

Work space:

Q2. Each interior angle of regular polygon is 150 deg. Sides?

(A) 8
(B) 10
(C) 12
(D) 15

✓ Answer: (C) | Exterior=30 deg. n=360/30=12.

Work space:

Q3. Exterior angle of regular polygon = 40 deg. Sum of interior angles?

(A) 900 deg
(B) 1080 deg
(C) 1260 deg
(D) 1440 deg

✓ Answer: (C) | n=360/40=9. Sum=(9-2)*180=7*180=1260 deg.

Work space: