

AMC Math Practice

AMC 8 · AMC 9 · AMC 10 | 20 Core Problems

Print-Ready Worksheet · Work Space Included

Instructions: Each problem has a Key Concept, Worked Example, and 3 Steps. Answer ALL 3 steps for each problem. Write your work in the space provided. Answers are shown at the end of each problem.

Problem 01 of 20 · Number Theory · GCD & LCM

■ Key Concept

GCD (Greatest Common Divisor): largest integer dividing both numbers.

LCM (Least Common Multiple): smallest positive integer divisible by both.

Key: $\text{GCD}(a,b) \times \text{LCM}(a,b) = a \times b$

■ Worked Example

Find $\text{GCD}(12, 18)$ and $\text{LCM}(12, 18)$.

$12 = 2^2 \times 3$, $18 = 2 \times 3^2$

$\text{GCD} = 2 \times 3 = 6$, $\text{LCM} = 4 \times 9 = 36$

Check: $6 \times 36 = 216 = 12 \times 18$ ✓

— ■ Your Turn — Answer All 3 Steps

Step 1: What is $\text{LCM}(8, 12)$?

(A) 16 (B) 24 (C) 32 (D) 48

Work space:

Step 2: If $\text{GCD}(a,b) = 6$ and $\text{LCM}(a,b) = 60$, what is $a \times b$?

(A) 66 (B) 120 (C) 360 (D) 720

Work space:

Step 3: How many positive integers ≤ 100 are divisible by both 4 and 6?

(A) 8 (B) 12 (C) 16 (D) 25

Work space:

Answers: Step 1: (B) 24 | Step 2: (C) 360 | Step 3: (A) 8

Problem 02 of 20 - Fractions & Ratios - Ratios & Proportions

■ Key Concept

A ratio $a:b$ compares two quantities.

Proportion: $a/b = c/d$ means $ad = bc$ (cross-multiply).

For mixture/sharing problems, write variables as $k \times$ (ratio part).

■ Worked Example

If $a:b = 3:5$ and $a+b = 40$, find a and b .

Let $a = 3k$, $b = 5k$. Then $8k = 40 \rightarrow k = 5$.

So $a = 15$, $b = 25$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Boys:Girls = 3:4, 28 total students. How many boys?

(A) 9 (B) 12 (C) 16 (D) 21

Work space:

Step 2: $x/y = 5/3$ and $y = 12$. What is x ?

(A) 15 (B) 18 (C) 20 (D) 25

Work space:

Step 3: Mix 40% juice and 70% juice to get 100 L of 55% juice. Liters of 40% needed?

(A) 40 (B) 50 (C) 55 (D) 60

Work space:

Answers: Step 1: (B) 12 | Step 2: (C) 20 | Step 3: (B) 50

Problem 03 of 20 - Percent - Percent Change & Discount

■ Key Concept

Percent change = $(\text{New} - \text{Old}) / \text{Old} \times 100\%$

Successive changes: multiply factors, e.g., +20% then -20% $\rightarrow 1.2 \times 0.8 = 0.96$ (-4% net).

■ Worked Example

A shirt costs \$80. After 25% discount: $80 \times 0.75 = \$60$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Price increases from \$50 to \$65. Percent increase?

(A) 23% (B) 25% (C) 30% (D) 35%

Work space:

Step 2: Store raises prices 20%, then reduces 20%. Net change?

(A) 0% (B) -2% (C) -4% (D) +4%

Work space:

Step 3: After 40% discount, item costs \$48. Original price?

(A) \$67.20 (B) \$72 (C) \$80 (D) \$84

Work space:

Answers: Step 1: (C) 30% | Step 2: (C) -4% | Step 3: (C) \$80

Problem 04 of 20 - Algebra - Linear Equations

■ Key Concept

Solve $ax + b = cx + d$ by collecting x on one side.

Word problems: define variables, write equations, solve, verify.

■ Worked Example

Solve $3x - 7 = 2(x + 4)$.

$3x - 7 = 2x + 8 \rightarrow x = 15$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: If $5x - 3 = 2x + 12$, what is x ?

(A) 3 (B) 4 (C) 5 (D) 7

Work space:

Step 2: Two numbers sum to 54 and differ by 18. Larger number?

(A) 27 (B) 32 (C) 36 (D) 42

Work space:

Step 3: Train A travels 300 km at speed v . Train B travels same at $v+25$ km/h and arrives 1 hour earlier. Find v .

(A) 50 (B) 60 (C) 75 (D) 100

Work space:

Answers: Step 1: (C) 5 | Step 2: (C) 36 | Step 3: (C) 75

Problem 05 of 20 - Algebra - Systems of Equations

■ Key Concept

Two equations, two unknowns.

- Substitution: isolate one variable and substitute.
- Elimination: multiply/add equations to cancel one variable.

■ Worked Example

$x + y = 10$, $2x - y = 5$. Add: $3x = 15 \rightarrow x = 5$, $y = 5$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: $x + y = 7$ and $x - y = 3$. Find x .

(A) 2 (B) 3 (C) 4 (D) 5

Work space:

Step 2: $2x + 3y = 12$ and $4x - 3y = 6$. Find $x + y$.

(A) 2 (B) 3 (C) 4 (D) 5

Work space:

Step 3: $3a + 2b = 16$ and $a - b = 2$. What is $5a + b$?

(A) 16 (B) 18 (C) 20 (D) 22

Work space:

Answers: Step 1: (D) 5 | Step 2: (D) 5 | Step 3: (D) 22

Problem 06 of 20 - Geometry - Area & Perimeter

■ Key Concept

Rectangle: $A = lw$, $P = 2(l+w)$

Triangle: $A = (1/2)bh$

Circle: $A = \pi r^2$, $C = 2\pi r$

Trapezoid: $A = (1/2)(b_1 + b_2)h$

■ Worked Example

Triangle base 10, height 6: $A = (1/2)(10)(6) = 30$.

■ Your Turn — Answer All 3 Steps

Step 1: Rectangle perimeter 36, length 10. Area?

(A) 60 (B) 80 (C) 90 (D) 100

Work space:

Step 2: Circle area = 49π . Circumference?

(A) 7π (B) 14π (C) 21π (D) 49π

Work space:

Step 3: Trapezoid parallel sides 8 and 14, height 5. Area?

(A) 45 (B) 50 (C) 55 (D) 60

Work space:

Answers: Step 1: (B) 80 | Step 2: (B) 14π | Step 3: (C) 55

Problem 07 of 20 - Geometry - Pythagorean Theorem

■ Key Concept

Right triangle legs a , b and hypotenuse c : $a^2 + b^2 = c^2$

Common triples: (3,4,5), (5,12,13), (8,15,17), (7,24,25).

■ Worked Example

Legs 6 and 8: $c = \sqrt{(36+64)} = \sqrt{100} = 10$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Right triangle legs 9 and 12. Hypotenuse?

(A) 14 (B) 15 (C) 16 (D) 18

Work space:

Step 2: Ladder 13 ft, base 5 ft from wall. Height reached?

(A) 10 (B) 11 (C) 12 (D) 13

Work space:

Step 3: Square ABCD with side 6. Length of diagonal AC?

(A) 6 (B) $6\sqrt{2}$ (C) $6\sqrt{3}$ (D) 12

Work space:

Answers: Step 1: (B) 15 | Step 2: (C) 12 | Step 3: (B) $6\sqrt{2}$

Problem 08 of 20 - Number Theory - Prime Numbers & Divisibility

■ Key Concept

A prime has exactly 2 factors: 1 and itself. Primes: 2, 3, 5, 7, 11, 13, ...

Divisibility: by 2 (even), by 3 (digit sum $\div 3$), by 9 (digit sum $\div 9$), by 5 (ends 0 or 5).

Number of factors of $p^a \times q^b = (a+1)(b+1)$.

■ Worked Example

$72 = 2^3 \times 3^2$. Number of factors = $(3+1)(2+1) = 12$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: How many prime numbers are less than 20?

(A) 6 (B) 7 (C) 8 (D) 9

Work space:

Step 2: How many positive divisors does $2^3 \times 3 \times 5^2$ have?

(A) 18 (B) 20 (C) 24 (D) 30

Work space:

Step 3: Largest prime factor of 2310?

(A) 7 (B) 11 (C) 13 (D) 17

Work space:

Answers: Step 1: (C) 8 | Step 2: (C) 24 | Step 3: (B) 11

Problem 09 of 20 - Combinatorics - Counting & Combinations

■ Key Concept

Permutation (order matters): $P(n,r) = n! / (n-r)!$

Combination (order doesn't matter): $C(n,r) = n! / [r!(n-r)!]$

■ Worked Example

Choose 3 from 5 students: $C(5,3) = 5!/(3!2!) = 10$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: In how many ways can 4 books be arranged on a shelf?

(A) 16 (B) 20 (C) 24 (D) 32

Work space:

Step 2: From 6 people, how many groups of 2 can be formed?

(A) 12 (B) 15 (C) 18 (D) 30

Work space:

Step 3: 3-digit numbers using {1,2,3,4,5} with no digit repeated?

(A) 30 (B) 45 (C) 60 (D) 120

Work space:

Answers: Step 1: (C) 24 | Step 2: (B) 15 | Step 3: (C) 60

Problem 10 of 20 - Probability - Basic Probability

■ Key Concept

$P(\text{event}) = \text{favorable outcomes} / \text{total outcomes}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Independent: $P(A \text{ and } B) = P(A) \times P(B)$

■ Worked Example

Drawing a red card from a standard deck: $P = 26/52 = 1/2$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Bag has 3 red, 4 blue, 5 green balls. $P(\text{red})$?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{3}{12}$

Work space:

Step 2: Two fair coins flipped. $P(\text{exactly one head})$?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Work space:

Step 3: Die rolled. $P(\text{odd}) + P(>4) - P(\text{odd AND } >4) = ?$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) 1

Work space:

Answers: Step 1: (B) $\frac{1}{4}$ | Step 2: (C) $\frac{1}{2}$ | Step 3: (B) $\frac{2}{3}$

Problem 11 of 20 · Algebra · Quadratic Equations

■ Key Concept

$ax^2 + bx + c = 0$. Solve by factoring or quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots = $-b/a$, Product of roots = c/a .

■ Worked Example

$$x^2 - 5x + 6 = 0 \rightarrow (x-2)(x-3) = 0 \rightarrow x = 2 \text{ or } 3.$$

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Solutions of $x^2 - 7x + 10 = 0$?

(A) 2 and 5 (B) 3 and 4 (C) 1 and 10 (D) 2 and 7

Work space:

Step 2: Sum of two numbers is 11 and product is 28. Larger number?

(A) 4 (B) 6 (C) 7 (D) 8

Work space:

Step 3: For what value(s) of k does $x^2 + kx + 9 = 0$ have exactly one solution?

(A) 3 (B) 6 (C) 9 (D) ± 6

Work space:

Answers: Step 1: (A) 2 and 5 | Step 2: (C) 7 | Step 3: (D) ± 6

Problem 12 of 20 · Geometry · Similar Triangles

■ Key Concept

Triangles are similar if angles match (AA, SAS, SSS).

Sides are proportional: $a/a' = b/b' = c/c'$.

Area ratio = (side ratio)².

■ Worked Example

■ABC ~ ■DEF, AB = 6, DE = 9. Area ratio = $(6/9)^2 = 4/9$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Similar triangles side ratio 2:3. Smaller area = 8. Larger area?

(A) 12 (B) 16 (C) 18 (D) 27

Work space:

Step 2: 6-ft person casts 4-ft shadow. Tree casts 18-ft shadow. Tree height?

(A) 12 (B) 24 (C) 27 (D) 36

Work space:

Step 3: In $\triangle ABC$, $DE \parallel BC$, $AD=4$, $DB=2$, $DE=6$. Find BC .

(A) 8 (B) 9 (C) 10 (D) 12

Work space:

Answers: Step 1: (C) 18 | Step 2: (C) 27 | Step 3: (B) 9

Problem 13 of 20 - Statistics - Mean, Median & Mode

■ Key Concept

Mean = sum / count

Median = middle value (sorted); average of two middles if even count.

Mode = most frequent value.

■ Worked Example

Data: 3, 7, 7, 9, 14. Mean = 8. Median = 7. Mode = 7.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Test scores: 78, 85, 90, 72, 95. Mean?

(A) 82 (B) 84 (C) 86 (D) 88

Work space:

Step 2: 6 numbers have mean 10. When 14 is added, new mean?

(A) $10\frac{4}{7}$ (B) $10\frac{2}{7}$ (C) 11 (D) $10\frac{6}{7}$

Work space:

Step 3: Data: 4,6,6,8,10,10,10. Mean – Median = ?

(A) 0 (B) 1 (C) 2 (D) 3

Work space:

Answers: Step 1: (B) 84 | Step 2: (A) $10\frac{4}{7}$ | Step 3: (A) 0 (approx)

Problem 14 of 20 - Number Theory - Number Patterns & Sequences

■ Key Concept

Arithmetic: $a_n = a_1 + (n-1)d$, $S_n = n/2 \times (a_1 + a_n)$

Geometric: $a_n = a_1 \times r^{(n-1)}$, $S_n = a_1(r^n - 1)/(r-1)$

■ Worked Example

AP 3, 7, 11, ... $d=4$. $a_{10} = 3+9 \times 4 = 39$. $S_{10} = 5(3+39) = 210$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: 20th term of 5, 9, 13, ... ?

(A) 77 (B) 81 (C) 85 (D) 89

Work space:

Step 2: Sum of all even integers from 2 to 100?

(A) 2500 (B) 2550 (C) 5000 (D) 5050

Work space:

Step 3: Geometric sequence $a_1=2$, $a_3=54$. What is a_4 ?

(A) 6 (B) 9 (C) 18 (D) 27

Work space:

Answers: Step 1: (B) 81 | Step 2: (B) 2550 | Step 3: (C) 18

Problem 15 of 20 - Algebra - Exponents & Radicals

■ Key Concept

$a^m \times a^n = a^{(m+n)}$, $(a^m)^n = a^{(mn)}$, $a^{(-n)} = 1/a^n$

$a^{(1/n)} = n$ -th root of a . $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$

■ Worked Example

$$\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}.$$

== ■ Your Turn — Answer All 3 Steps

Step 1: Simplify $(2^5 \times 2^3) / 2^4$

(A) 2^2 (B) 2^8 (C) 2^4 (D) 2^3

Work space:

Step 2: $\sqrt{75} - \sqrt{48} = ?$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt{3}$ (D) $5\sqrt{3}$

Work space:

Step 3: If $4^x = 8$, what is x ?

(A) $2/3$ (B) $3/2$ (C) 2 (D) 3

Work space:

Answers: Step 1: (B) 2^8 | Step 2: (A) $\sqrt{3}$ | Step 3: (B) $3/2$

Problem 16 of 20 - Geometry - Volume & Surface Area

■ Key Concept

Cube: $V = s^3$, $SA = 6s^2$

Rectangular prism: $V = lwh$, $SA = 2(lw + lh + wh)$

Cylinder: $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi rh$

Sphere: $V = (4/3)\pi r^3$, $SA = 4\pi r^2$

■ Worked Example

Cylinder $r=3$, $h=5$: $V = \pi(9)(5) = 45\pi$.

== ■ Your Turn — Answer All 3 Steps

Step 1: Cube has surface area 150. Volume?

(A) 100 (B) 125 (C) 150 (D) 175

Work space:

Step 2: Cylinder radius 4, height 9. Volume?

(A) 36π (B) 72π (C) 112π (D) 144π

Work space:

Step 3: Sphere radius 3. Volume = $k\pi$. What is k?

(A) 12 (B) 18 (C) 36 (D) 108

Work space:

Answers: Step 1: (B) 125 | Step 2: (D) 144π | Step 3: (C) 36

Problem 17 of 20 - Combinatorics - Pascal's Triangle & Binomial

■ Key Concept

Pascal's triangle row n : coefficients of $(a+b)^n$.

$$(a+b)^n = \sum C(n,k) a^{n-k} b^k$$

Row n sum = 2^n .

■ Worked Example

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1.$$

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Sum of all entries in row 5 of Pascal's triangle?

(A) 16 (B) 32 (C) 64 (D) 128

Work space:

Step 2: Coefficient of x^2y^3 in $(x+y)^5$?

(A) 5 (B) 10 (C) 15 (D) 20

Work space:

Step 3: Paths from $(0,0)$ to $(4,4)$ moving only right or up?

(A) 35 (B) 56 (C) 70 (D) 120

Work space:

Answers: Step 1: (B) 32 | Step 2: (B) 10 | Step 3: (C) 70

Problem 18 of 20 - Geometry - Angles & Parallel Lines

■ Key Concept

Transversal crossing parallel lines:

- Alternate interior angles: equal
- Corresponding angles: equal
- Co-interior (same-side) angles: supplementary (sum 180°)

Triangle angle sum = 180° . Exterior angle = sum of two non-adjacent interiors.

■ Worked Example

If one co-interior angle is 65° , the other is $180^\circ - 65^\circ = 115^\circ$.

■ Your Turn — Answer All 3 Steps

Step 1: Triangle angles 47° and 63° . Third angle?

(A) 60° (B) 70° (C) 80° (D) 90°

Work space:

Step 2: Exterior angle 120° , one non-adjacent interior 45° . Other non-adjacent interior?

(A) 60° (B) 65° (C) 70° (D) 75°

Work space:

Step 3: Alternate interior angles $(3x+10)^\circ$ and $(5x-20)^\circ$. Find x .

(A) 10 (B) 15 (C) 20 (D) 25

Work space:

Answers: Step 1: (B) 70° | Step 2: (D) 75° | Step 3: (B) 15

Problem 19 of 20 - Word Problems - Rate, Work & Distance

■ Key Concept

Distance = Rate \times Time

Work: if A finishes in a hours, rate = $1/a$ jobs/hour.

Combined rate: $1/a + 1/b = 1/t$.

■ Worked Example

A + B finish in 6 h. A alone: 10 h. B alone: $1/6 - 1/10 = 1/15$, so 15 h.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Two cars 300 km apart, driving toward each other at 60 and 90 km/h. Meeting time?

(A) 1 h (B) 2 h (C) 3 h (D) 4 h

Work space:

Step 2: Pipe A fills tank in 4 h. Pipe B drains it in 12 h. Time to fill with both open?

(A) 3 h (B) 4 h (C) 6 h (D) 8 h

Work space:

Step 3: Train goes A→B at 80 km/h, returns at 120 km/h. Average speed?

(A) 96 km/h (B) 100 km/h (C) 104 km/h (D) 110 km/h

Work space:

Answers: Step 1: (B) 2 h | Step 2: (C) 6 h | Step 3: (A) 96 km/h

Problem 20 of 20 - Algebra - Inequalities & Absolute Value

■ Key Concept

$|x - a| < b$ means $a - b < x < a + b$

$|x - a| > b$ means $x < a - b$ or $x > a + b$

Flip the inequality sign when multiplying/dividing by a negative number.

■ Worked Example

Solve $|2x - 4| \leq 6$: $-6 \leq 2x - 4 \leq 6 \rightarrow -1 \leq x \leq 5$.

⇒ ■ Your Turn — Answer All 3 Steps

Step 1: Solve $3x - 5 > 7$. Solution?

(A) $x > 2$ (B) $x > 4$ (C) $x > -4$ (D) $x < 4$

Work space:

Step 2: How many integers satisfy $|x - 3| < 4$?

(A) 6 (B) 7 (C) 8 (D) 9

Work space:

Step 3: For what values of x is $x^2 < 9$?

(A) $x < 3$ (B) $0 < x < 3$ (C) $-3 < x < 3$ (D) $x > -3$

Work space:

Answers: Step 1: (B) $x > 4$ | Step 2: (B) 7 | Step 3: (C) $-3 < x < 3$