Math Power Up! ■

Algebra 2 & Geometry — 20 Core Problems

Focused on: Ratios, Graphs, Functions • From Basics to Exam-Level

Name: _____ Date: _____ Score: _____ / 20

Q01 Ratio Word Problem [Algebra 2]

Topic: Ratios & Proportions

■ **Key Concept**

Ratio is a comparison of two quantities.

If a:b = m:n, then a/b = m/n (cross multiply to solve).

Key tip: Always re-read which quantity goes on top!

■ **Example**

Example: Ratio of boys to girls is 3:5. If there are 24 boys, how many girls?

3/5 = 24/g => 3g = 120 => g = 40

■ **Problem:**

The ratio of red balls to blue balls is 4:7. If there are 28 blue balls, how many red balls are there?

Step 1: Set up the proportion (let r = red balls):

(A) 4/7 = r/28

(B) 7/4 = r/28

(C) 4/7 = 28/r

(D) r/4 = 7/28

Answer: _____ Correct Answer: (A)

Step 2: Cross multiply:

(A) 7r = 112

(B) 4r = 196

(C) 7r = 28

(D) 4r = 28

Answer: _____ Correct Answer: (A)

Step 3: Solve for r:

(A) 49

(B) 12

(C) 16

(D) 21

Answer: _____ Correct Answer: (C)

Work Space:

Q02 Vertex Form of a Parabola [Algebra 2]

Topic: Quadratic Functions – Vertex Form

Key Concept

Vertex form: y = a(x - h)^2 + k

- Vertex is at (h, k)

- a > 0: opens up (minimum k); a < 0: opens down (maximum k)

Common mistake: Students write vertex as (-h, k) — watch the sign!

Example

Example: y = 2(x - 3)^2 + 5

Vertex = (3, 5), opens upward, minimum value = 5

Problem:

Given f(x) = -3(x + 2)^2 - 1, identify the vertex and direction of opening.

Step 1: Rewrite in y = a(x - h)^2 + k form. What is h?

(A) h = 2

(B) h = -2

(C) h = 1

(D) h = -1

Answer: _____ Correct Answer: (B)

Step 2: What is the vertex?

(A) (2, -1)

(B) (-2, 1)

(C) (-2, -1)

(D) (2, 1)

Answer: _____ Correct Answer: (C)

Step 3: Direction and max/min?

(A) Opens up, min y=-1

(B) Opens down, max y=-1

(C) Opens up, max y=-2

(D) Opens down, min y=-1

Answer: _____ Correct Answer: (B)

Work Space:

Q03 Finding the Domain of a Rational Function [Algebra 2]

Topic: Rational Functions – Domain

■ **Key Concept**

Rational function: f(x) = p(x) / q(x)

Domain excludes all x where the denominator = 0.

Common mistake: Forgetting to check ALL factors of the denominator.

■ **Example**

Example: f(x) = 1 / (x^2 - 4)

x^2 - 4 = 0 => (x-2)(x+2) = 0 => x = 2 or x = -2

Domain: all reals except x = 2 and x = -2

■ **Problem:**

Find the domain of f(x) = (x + 3) / (x^2 - x - 6)

Step 1: Factor the denominator x^2 - x - 6:

(A) (x-3)(x+2)
(B) (x+3)(x-2)
(C) (x-6)(x+1)
(D) (x-2)(x-3)

Answer: _____ Correct Answer: (A)

Step 2: Which x values are excluded?

(A) x=3 and x=-2
(B) x=-3 and x=2
(C) x=6 and x=-1
(D) x=3 only

Answer: _____ Correct Answer: (A)

Step 3: Write the domain:

(A) All reals
(B) All reals except x=3
(C) All reals except x=3 and x=-2
(D) All reals except x=-3 and x=2

Answer: _____ Correct Answer: (C)

■ **Work Space:**

Q04 Exponential Growth & Decay [Algebra 2]

Topic: Exponential Functions

■ **Key Concept**

f(x) = a * b^x

- b > 1: growth; 0 < b < 1: decay

- Growth formula: A = P(1+r)^t

- Decay formula: A = P(1-r)^t

■ **Example**

Example: $1000 at 5% annual growth for 3 years:

A = 1000(1.05)^3 ≈ $1,157.63

■ **Problem:**

A bacteria colony starts with 500 cells and doubles every hour. How many cells after 4 hours?

Step 1: Identify P (initial), b (base), t (time):

(A) P=500, b=2, t=4

(B) P=500, b=4, t=2

(C) P=2, b=500, t=4

(D) P=4, b=2, t=500

Answer: _____ Correct Answer: (A)

Step 2: Write the equation:

(A) 500 * 4^2

(B) 500 * 2^4

(C) 2 * 500^4

(D) 4 * 500^2

Answer: _____ Correct Answer: (B)

Step 3: Calculate the answer:

(A) 4,000

(B) 2,000

(C) 8,000

(D) 16,000

Answer: _____ Correct Answer: (C)

■ **Work Space:**

Q05 Solving Logarithmic Equations [Algebra 2]

Topic: Logarithmic Functions

■ **Key Concept**

log_b(x) = y means b^y = x

Key rules:

log_b(mn) = log_b(m) + log_b(n)

log_b(m/n) = log_b(m) - log_b(n)

log_b(m^n) = n * log_b(m)

WARNING: log(a+b) ≠ log(a) + log(b) !!

■ **Example**

Example: Solve log_2(x) = 5

2^5 = x => x = 32

■ **Problem:**

Solve for x: log_3(x - 2) = 4

Step 1: Convert to exponential form:

(A) 3^4 = x-2

(B) 4^3 = x-2

(C) 3^x = 4-2

(D) x^3 = 4-2

Answer: _____ Correct Answer: (A)

Step 2: Calculate 3^4:

(A) 9

(B) 12

(C) 81

(D) 64

Answer: _____ Correct Answer: (C)

Step 3: Solve for x:

(A) x=79

(B) x=83

(C) x=77

(D) x=85

Answer: _____ Correct Answer: (B)

—■ Work Space:

Q06 Solving Systems — Substitution/Elimination [Algebra 2]

Topic: Systems of Equations

■ **Key Concept**

System of equations: solve for values satisfying all equations.

Methods: Substitution, Elimination, Graphing.

Tip: Always substitute back to check your answer!

■ **Example**

Example: x + y = 5 and 2x - y = 1

Add: 3x = 6 => x=2, y=3

■ **Problem:**

Solve the system: 3x + 2y = 16 and x - y = 2

Step 1: From equation 2, express x in terms of y:

(A) x = y+2
(B) x = y-2
(C) x = 2-y
(D) x = 2y

Answer: _____ Correct Answer: (A)

Step 2: Substitute into equation 1; solve for y:

(A) y=10
(B) y=2
(C) y=6
(D) y=4

Answer: _____ Correct Answer: (B)

Step 3: Find x and write the solution:

(A) (4, 2)
(B) (3, 2)
(C) (5, 2)
(D) (0, 2)

Answer: _____ Correct Answer: (A)

Work Space:

Q07 Finding Zeros of Polynomials [Algebra 2]

Topic: Polynomial Functions – Zeros

■ **Key Concept**

Zeros (roots) are x-values where f(x) = 0.

1. Factor the polynomial

2. Zero Product Property: if ab = 0, then a=0 or b=0

Note: A zero with multiplicity 2 means the graph TOUCHES (doesn't cross) the x-axis.

■ **Example**

Example: f(x) = x^2 - 5x + 6 = (x-2)(x-3)

Zeros at x = 2 and x = 3

■ **Problem:**

Find all zeros of f(x) = x^3 - 4x^2 + 4x

Step 1: Factor out the GCF:

(A) x(x^2-4x+4)
(B) x^2(x-4+4)
(C) 4x(x^2-x+1)
(D) x(x^2+4x-4)

Answer: _____ Correct Answer: (A)

Step 2: Factor the trinomial x^2 - 4x + 4:

(A) (x-4)(x-1)
(B) (x-2)(x+2)
(C) (x-2)^2
(D) (x+2)^2

Answer: _____ Correct Answer: (C)

Step 3: List all zeros with multiplicity:

(A) x=0, x=2 (mult. 2)
(B) x=0, x=2, x=-2
(C) x=0 (mult.2), x=2
(D) x=4, x=0

Answer: _____ Correct Answer: (A)

■ **Work Space:**

Q08 Finding Inverse Functions [Algebra 2]

Topic: Inverse Functions

■ **Key Concept**

To find the inverse f^(-1)(x):

1. Replace f(x) with y

2. Swap x and y

3. Solve for y

Tip: Domain of f becomes range of f^(-1)!

■ **Example**

Example: f(x) = 2x + 3

y = 2x+3 => swap: x = 2y+3 => y = (x-3)/2

f^(-1)(x) = (x-3)/2

■ **Problem:**

Find the inverse of f(x) = (3x - 1) / 2

Step 1: Swap x and y. What equation do you get?

(A) x = (3y-1)/2

(B) y = (3x-1)/2

(C) x = (3y+1)/2

(D) y = (2x-1)/3

Answer: _____ Correct Answer: (A)

Step 2: Multiply both sides by 2:

(A) 2x = 3y-1

(B) 2x = 3y+1

(C) x = 3y-2

(D) 2x+1 = 6y

Answer: _____ Correct Answer: (A)

Step 3: Solve for y to get f^(-1)(x):

(A) (2x-1)/3

(B) (2x+1)/3

(C) (x+1)/6

(D) 3(2x+1)

Answer: _____ Correct Answer: (B)

Work Space:

Q09 Arithmetic vs Geometric Sequences [Algebra 2]

Topic: Sequences & Series

■ **Key Concept**

Arithmetic: a_n = a_1 + (n-1)d (d = common difference)

Geometric: a_n = a_1 * r^(n-1) (r = common ratio)

Sum (arithmetic): S_n = n/2 * (a_1 + a_n)

Common mistake: Using n instead of (n-1) in the exponent!

■ **Example**

Example: Geometric 2, 6, 18, 54... r=3

5th term: a_5 = 2 * 3^(5-1) = 2 * 81 = 162

■ **Problem:**

Find the 8th term of the geometric sequence: 5, 15, 45, 135, ...

Step 1: Identify a_1 and r:

(A) a_1=5, r=3

(B) a_1=5, r=10

(C) a_1=15, r=3

(D) a_1=5, r=5

Answer: _____ Correct Answer: (A)

Step 2: Write the formula for the 8th term:

(A) 5 * 3^8

(B) 5 * 3^7

(C) 5 * 8^3

(D) 8 * 3^5

Answer: _____ Correct Answer: (B)

Step 3: Calculate 5 * 3^7:

(A) 5,465

(B) 10,935

(C) 2,187

(D) 8,748

Answer: _____ Correct Answer: (B)

—■ **Work Space:**

Q10 Vertical & Horizontal Asymptotes [Algebra 2]

Topic: Rational Functions – Asymptotes

■ **Key Concept**

For f(x) = p(x)/q(x):

- Vertical asymptote (VA): where q(x) = 0 (after simplification)

- Horizontal asymptote (HA):

* deg(p) < deg(q): HA y = 0

* deg(p) = deg(q): HA y = leading coefficients ratio

* deg(p) > deg(q): No HA (oblique asymptote)

■ **Example**

Example: f(x) = (2x+1)/(x-3)

VA: x=3 | HA: y=2

■ **Problem:**

Find the vertical and horizontal asymptotes of f(x) = (3x^2 - 1) / (x^2 - 4)

Step 1: Find VA(s) — factor denominator:

(A) x=2 and x=-2

(B) x=4 and x=-4

(C) x=1 and x=-1

(D) x=2 only

Answer: _____ Correct Answer: (A)

Step 2: Compare degrees of numerator and denominator:

(A) Both degree 2 — same degree

(B) top > bottom — no HA

(C) top < bottom — HA at 0

(D) Not relevant

Answer: _____ Correct Answer: (A)

Step 3: Find the horizontal asymptote:

(A) y=3

(B) y=0

(C) y=1/4

(D) No HA

Answer: _____ Correct Answer: (A)

Work Space:

Q11 Similar Triangle Side Ratios [Geometry]

Topic: Similar Triangles & Proportions

■ **Key Concept**

Two triangles are similar (~) if corresponding angles are equal and corresponding sides are proportional.

Similarity criteria: AA, SSS~, SAS~

Key: AB/DE = BC/EF = AC/DF

■ **Example**

Example: ■ABC ~ ■DEF, AB=4, DE=6, BC=5. Find EF.

4/6 = 5/EF => EF = 7.5

■ **Problem:**

■PQR ~ ■XYZ. PQ = 8, XY = 12, QR = 10. Find YZ.

Step 1: Set up the correct proportion:

(A) 8/12 = 10/YZ

(B) 8/10 = 12/YZ

(C) 12/8 = YZ/10

(D) A and C both correct

Answer: _____ Correct Answer: (D)

Step 2: Cross multiply:

(A) 8*YZ = 120

(B) 12*YZ = 80

(C) 8*YZ = 80

(D) YZ = 120/8

Answer: _____ Correct Answer: (A)

Step 3: Solve for YZ:

(A) YZ=16

(B) YZ=15

(C) YZ=14

(D) YZ=18

Answer: _____ Correct Answer: (B)

—■ **Work Space:**

Q12 Arc Length and Sector Area [Geometry]

Topic: Circle – Arc Length & Sector Area

■ **Key Concept**

For circle with radius r and central angle theta (degrees):

- Arc length: L = (theta/360) * 2*pi*r

- Sector area: A = (theta/360) * pi*r^2

Common mistake: Forgetting to divide angle by 360 first!

■ **Example**

Example: r=6, theta=90 degrees

L = (90/360) * 2*pi*6 = (1/4) * 12*pi = 3*pi

■ **Problem:**

A circle has radius 10 cm and central angle 120°. Find the arc length and sector area (in terms of pi).

Step 1: What fraction of the circle is 120 degrees?

(A) 1/4
(B) 1/3
(C) 2/3
(D) 1/2

Answer: _____ Correct Answer: (B)

Step 2: Find the arc length:

(A) 10*pi
(B) 20*pi/3
(C) 20*pi
(D) 10*pi/3

Answer: _____ Correct Answer: (B)

Step 3: Find the sector area:

(A) 200*pi/3
(B) 100*pi
(C) 100*pi/3
(D) 50*pi/3

Answer: _____ Correct Answer: (C)

—■ **Work Space:**

Q13 Pythagorean Theorem & Special Triangles [Geometry]

Topic: Pythagorean Theorem

■ **Key Concept**

Right triangle: a^2 + b^2 = c^2 (c = hypotenuse)

Special triangles:

- 45-45-90: legs = x, hyp = x*sqrt(2)

- 30-60-90: short leg=x, long leg=x*sqrt(3), hyp=2x

■ **Example**

Example: 45-45-90 triangle, leg=5. Hypotenuse = 5*sqrt(2)

■ **Problem:**

In a 30-60-90 triangle, the hypotenuse is 14. Find both legs.

Step 1: If hypotenuse = 2x = 14, what is x?

(A) x=7

(B) x=14

(C) x=28

(D) x=7*sqrt(2)

Answer: _____ Correct Answer: (A)

Step 2: What is the short leg (opposite 30 degrees)?

(A) 7*sqrt(3)

(B) 14

(C) 7

(D) 7*sqrt(2)

Answer: _____ Correct Answer: (C)

Step 3: What is the long leg (opposite 60 degrees)?

(A) 7

(B) 7*sqrt(3)

(C) 14*sqrt(3)

(D) 7*sqrt(2)

Answer: _____ Correct Answer: (B)

—■ **Work Space:**

Q14 Volume Formulas — Cylinder, Cone, Sphere [Geometry]

Topic: Volume of 3D Figures

■ **Key Concept**

Key volume formulas:

- Cylinder: V = pi*r^2*h
- Cone: V = (1/3)*pi*r^2*h
- Sphere: V = (4/3)*pi*r^3

Trick: Cone = 1/3 of Cylinder (same r and h)!

■ **Example**

Example: Cone with r=3, h=4

V = (1/3)*pi*(9)*(4) = 12*pi

■ **Problem:**

A cone and a cylinder both have radius 6 and height 9. What is the ratio of cone's volume to cylinder's volume?

Step 1: Find the volume of the cylinder:

(A) 324*pi
(B) 108*pi
(C) 54*pi
(D) 216*pi

Answer: _____ Correct Answer: (A)

Step 2: Find the volume of the cone:

(A) 324*pi
(B) 216*pi
(C) 108*pi
(D) 72*pi

Answer: _____ Correct Answer: (C)

Step 3: What is the ratio cone : cylinder?

(A) 1:2
(B) 1:3
(C) 2:3
(D) 1:4

Answer: _____ Correct Answer: (B)

Work Space:

Q15 Midpoint, Distance, and Slope [Geometry]

Topic: Coordinate Geometry

■ **Key Concept**

Three key formulas:

- Midpoint: M = ((x1+x2)/2, (y1+y2)/2)
- Distance: d = sqrt((x2-x1)^2 + (y2-y1)^2)
- Slope: m = (y2-y1) / (x2-x1)

■ **Example**

Example: A(1,2), B(4,6)

d = sqrt(9+16) = sqrt(25) = 5

■ **Problem:**

Points A(-3, 4) and B(5, -2). Find distance AB and midpoint M.

Step 1: Find delta-x and delta-y:

(A) dx=8, dy=-6
(B) dx=-8, dy=6
(C) dx=2, dy=2
(D) dx=8, dy=6

Answer: _____ Correct Answer: (A)

Step 2: Calculate distance AB:

(A) sqrt(28)
(B) 2*sqrt(5)
(C) 10
(D) sqrt(100)=10

Answer: _____ Correct Answer: (D)

Step 3: Find midpoint M:

(A) (1, 1)
(B) (4, -3)
(C) (2, -1)
(D) (-4, 3)

Answer: _____ Correct Answer: (A)

■ **Work Space:**

Q16 Angles Formed by Parallel Lines & Transversal [Geometry]

Topic: Angle Relationships – Parallel Lines

■ **Key Concept**

When a transversal cuts two parallel lines:

- Corresponding angles: equal

- Alternate interior angles: equal

- Co-interior (same-side interior): supplementary (sum=180 degrees)

Tip: Draw a diagram — it's much easier to visualize!

■ **Example**

Example: Two parallel lines, one angle = 70 degrees

Co-interior angle = 180 - 70 = 110 degrees

■ **Problem:**

Two parallel lines are cut by a transversal. Alternate interior angles are (3x+20)° and (5x-40)°. Find x and the angle.

Step 1: Alternate interior angles are:

(A) Equal: 3x+20 = 5x-40

(B) Supplementary: sum=180

(C) Complementary: sum=90

(D) Differ by 90

Answer: _____ Correct Answer: (A)

Step 2: Solve for x:

(A) x=10

(B) x=20

(C) x=30

(D) x=15

Answer: _____ Correct Answer: (C)

Step 3: Find the angle measure:

(A) 80 degrees

(B) 100 degrees

(C) 110 degrees

(D) 120 degrees

Answer: _____ Correct Answer: (C)

Work Space:

Q17 Right Triangle Trigonometry [Geometry]

Topic: Trigonometry — SOH CAH TOA

■ **Key Concept**

SOH CAH TOA:

- sin(theta) = opposite / hypotenuse

- cos(theta) = adjacent / hypotenuse

- tan(theta) = opposite / adjacent

Memory trick: SOHCAHTOA

■ **Example**

Example: theta=30 degrees, hyp=10. Find opposite.

sin(30) = opp/10 => opp = 10*0.5 = 5

■ **Problem:**

Right triangle, theta=40 degrees, adjacent=12. Find the opposite side. (tan 40° ≈ 0.839)

Step 1: Which trig ratio uses opposite and adjacent?

(A) sin
(B) cos
(C) tan
(D) sec

Answer: _____ Correct Answer: (C)

Step 2: Set up the equation:

(A) tan40 = 12/opp
(B) tan40 = opp/12
(C) sin40 = opp/12
(D) cos40 = opp/12

Answer: _____ Correct Answer: (B)

Step 3: Solve for the opposite side:

(A) ≈14.31
(B) ≈10.07
(C) ≈9.18
(D) ≈12.84

Answer: _____ Correct Answer: (B)

■ Work Space:

Q18 Properties of Special Quadrilaterals [Geometry]

Topic: Quadrilaterals – Properties

■ Key Concept

- Rectangle: 4 right angles, opposite sides equal, diagonals equal
- Rhombus: all 4 sides equal, diagonals perpendicular bisectors
- Square: Rectangle + Rhombus (all properties!)
- Parallelogram: opposite sides parallel and equal; opposite angles equal

Key: Every square is a rectangle, but NOT every rectangle is a square!

■ Example

Example: Parallelogram ABCD, AB=8, BC=5, angle A=70 degrees

CD=8 (opposite sides), angle C=70 degrees (opposite angles)

■ Problem:

In parallelogram PQRS, angle P = (4x-10)° and angle R = (2x+30)°. Find x and angle P.

Step 1: In a parallelogram, angles P and R (opposite) are:

(A) Supplementary (sum=180)
(B) Equal (congruent)
(C) Complementary
(D) Right angles

Answer: _____ Correct Answer: (B)

Step 2: Solve 4x-10 = 2x+30:

(A) x=10
(B) x=15
(C) x=20
(D) x=25

Answer: _____ Correct Answer: (C)

Step 3: Find angle P:

(A) 70 degrees
(B) 80 degrees
(C) 90 degrees

(D) 100 degrees

Answer: _____ Correct Answer: (A)

Work Space:

Q19 Reflections, Rotations, and Translations [Geometry]

Topic: Transformations

■ **Key Concept**

- Translation by (a,b): (x,y) -> (x+a, y+b)
- Reflection over x-axis: (x,y) -> (x, -y)
- Reflection over y-axis: (x,y) -> (-x, y)
- Rotation 90 degrees CCW: (x,y) -> (-y, x)
- Rotation 180 degrees: (x,y) -> (-x, -y)

■ **Example**

Example: Reflect (3,-5) over y-axis

Result: (-3,-5) — only x changes sign!

■ **Problem:**

Point A(4, -3) is rotated 90° counterclockwise about the origin. What are the new coordinates?

Step 1: Rule for 90 degrees CCW rotation:

(A) (x,y)->(y,x)
(B) (x,y)->(-y,x)
(C) (x,y)->(-x,-y)
(D) (x,y)->(y,-x)

Answer: _____ Correct Answer: (B)

Step 2: Apply to A(4,-3): what is -y?

(A) -(-3)=3
(B) -(4)=-4
(C) 4
(D) -3

Answer: _____ Correct Answer: (A)

Step 3: Final coordinates of A':

(A) (3,4)
(B) (-3,4)
(C) (3,-4)
(D) (-4,-3)

Answer: _____ Correct Answer: (A)

—■ Work Space:

Q20 Triangle Congruence Criteria (SSS, SAS, ASA, AAS) [Geometry]

Topic: Triangle Congruence

■ **Key Concept**

Congruence shortcuts:

- SSS: all 3 sides equal

- SAS: 2 sides + included angle

- ASA: 2 angles + included side

- AAS: 2 angles + non-included side

WARNING: AAA and SSA do NOT prove congruence!

■ **Example**

Example: AB=DE, angle B=angle E, BC=EF

Side-Angle-Side (included angle) => SAS

■ **Problem:**

In ■ABC and ■DEF: angle A=angle D, angle B=angle E, and BC=EF. Which congruence criterion applies?

Step 1: We have 2 equal angles and 1 equal side. This suggests:

(A) SSS
(B) SAS
(C) ASA or AAS
(D) AAA

Answer: _____ Correct Answer: (C)

Step 2: Is BC the included side (between angle A and angle B)?

(A) Yes, BC is between angle A and angle B
(B) No, BC is NOT between both angles
(C) Depends
(D) BC is the hypotenuse

Answer: _____ Correct Answer: (B)

Step 3: Which criterion proves the triangles congruent?

(A) ASA

(B) SAS

(C) AAS

(D) SSS

Answer: _____ Correct Answer: (C)

Work Space: