

# MATH MASTER

## Algebra 2 & Geometry

Common Mistakes Workbook • Self-Study Edition

■ 10 Algebra 2 Problems

■ 10 Geometry Problems

—■ 3 Sub-problems each

■ Concept + Examples

Name: \_\_\_\_\_  
\_\_\_\_\_

Date: \_\_\_\_\_  
\_\_\_\_\_

Score: \_\_\_\_\_ / 20

Time: \_\_\_\_\_  
\_\_\_\_\_

**Instructions:** Each question has a concept review, worked example, and 3 multiple-choice sub-problems. Circle your answer (A, B, C, or D) for each part. All 3 parts must be correct to earn full credit for the question. Answers and explanations are provided at the end of the workbook.

### QUICK ANSWER GRID

Q#	Topic	Part 1	Part 2	Part 3	Pts
1	Quadratic Functions	_____	_____	_____	/3
2	Polynomial Factoring	_____	_____	_____	/3
3	Rational Exponents & R	_____	_____	_____	/3
4	Exponential & Logarith	_____	_____	_____	/3
5	Systems of Equations	_____	_____	_____	/3
6	Complex Numbers	_____	_____	_____	/3
7	Sequences & Series	_____	_____	_____	/3
8	Radical & Rational Equ	_____	_____	_____	/3
9	Function Transformatio	_____	_____	_____	/3
10	Conic Sections (Circle	_____	_____	_____	/3
11	Parallel Lines & Trans	_____	_____	_____	/3
12	Triangle Congruence	_____	_____	_____	/3
13	Similar Triangles & Pr	_____	_____	_____	/3
14	Circle Theorems	_____	_____	_____	/3
15	Area & Volume Formulas	_____	_____	_____	/3

<b>16</b>	Pythagorean Theorem &	_____	_____	_____	/3
<b>17</b>	Transformations	_____	_____	_____	/3
<b>18</b>	Trigonometry (SOH-CAH-	_____	_____	_____	/3
<b>19</b>	Coordinate Geometry &	_____	_____	_____	/3
<b>20</b>	Geometric Proofs & Qua	_____	_____	_____	/3

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1

## Quadratic Functions

ALG

## ■ CORE CONCEPT

A quadratic  $f(x) = ax^2 + bx + c$  has vertex at  $x = -b/(2a)$ . Vertex form:  $f(x) = a(x-h)^2 + k$ . Discriminant  $\Delta = b^2 - 4ac$ : if  $>0$  two real roots;  $=0$  one root;  $<0$  no real roots.

Formula: Vertex:  $x = -b/(2a)$  | Discriminant:  $\Delta = b^2 - 4ac$

## ■ WORKED EXAMPLE

Find vertex of  $f(x) = 2x^2 - 8x + 3$ :

$$h = -(-8)/(2 \cdot 2) = 2 \quad | \quad k = 2(4) - 16 + 3 = -5$$

Vertex = (2, -5)

== ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** What is the vertex of  $f(x) = x^2 - 6x + 5$ ?

A) (3, -4)

B) (-3, 4)

C) (3, 4)

D) (6, 5)

Answer: \_\_\_\_\_

**Part 2.** How many real solutions does  $x^2 - 4x + 5 = 0$  have?

A) 0 real solutions

B) 1 real solution

C) 2 real solutions

D) Cannot determine

Answer: \_\_\_\_\_

**Part 3.** Axis of symmetry of  $f(x) = 3x^2 + 12x - 1$  is  $x = ?$

A)  $x = -2$ B)  $x = 2$ C)  $x = -4$ D)  $x = 4$ 

Answer: \_\_\_\_\_

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2

## Polynomial Factoring

ALG

## ■ CORE CONCEPT

Key factoring methods: GCF, difference of squares  $a^2 - b^2 = (a+b)(a-b)$ , perfect square trinomials, and grouping. Always factor out GCF first.

Formula:  $a^2 - b^2 = (a+b)(a-b)$  |  $a^2 + 2ab + b^2 = (a+b)^2$

### ■ WORKED EXAMPLE

Factor:  $x^3 - 8$  (difference of cubes)

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$x^3 - 8 = (x-2)(x^2+2x+4)$$

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Factor completely:  $x^2 - 16$

A)  $(x-4)(x+4)$

B)  $(x-4)^2$

C)  $(x+4)^2$

D)  $(x-8)(x+2)$

Answer: \_\_\_\_\_

**Part 2.** Which is a factor of  $x^2 - 5x + 6$ ?

A)  $(x - 2)$

B)  $(x + 2)$

C)  $(x - 6)$

D)  $(x + 3)$

Answer: \_\_\_\_\_

**Part 3.** Factor:  $2x^2 + 7x + 3$

A)  $(2x+1)(x+3)$

B)  $(2x+3)(x+1)$

C)  $(x+7)(x+3)$

D)  $(2x-1)(x-3)$

Answer: \_\_\_\_\_

# # 3

## Rational Exponents & Radicals

ALG

### ■ CORE CONCEPT

Rational exponents:  $a^{(m/n)} = (\text{nth-root of } a)^m$ . Laws:  $a^m \cdot a^n = a^{(m+n)}$ ;  $(a^m)^n = a^{(mn)}$ ;  $a^{(-n)} = 1/a^n$ ;  $a^{(1/n)} = \text{nth root of } a$ .

Formula:  $a^{(m/n)} = \text{nth-root}(a^m)$  |  $a^m \cdot a^n = a^{(m+n)}$

### ■ WORKED EXAMPLE

Simplify  $x^{(2/3)} \cdot x^{(1/3)}$ :

$$= x^{(2/3 + 1/3)} = x^{(3/3)} = x^1 = x$$

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Simplify:  $8^{(2/3)}$

A) 4

B) 2

C) 6

D) 16

Answer: \_\_\_\_\_

**Part 2.** Simplify:  $x^{3/4} * x^{1/4}$

A)  $x$

B)  $x^{4/4}$

C)  $x^2$

D)  $x^{3/16}$

Answer: \_\_\_\_\_

**Part 3.** Rewrite  $\sqrt{x^5}$  using rational exponents

A)  $x^{5/2}$

B)  $x^{2/5}$

C)  $5x$

D)  $x^{5/2}$

Answer: \_\_\_\_\_

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## Exponential & Logarithmic Functions

ALG

### ■ CORE CONCEPT

Exponential and logarithm are inverses:  $\log_a(b)=c$  means  $a^c=b$ . Properties:  $\log(xy)=\log x+\log y$ ;  $\log(x/y)=\log x - \log y$ ;  $\log(x^n)=n*\log x$ . Change of base:  $\log_a(b)=\log(b)/\log(a)$ .

Formula:  $\log_a(b) = c \Leftrightarrow a^c = b$  | Change of base:  $\log_a(b) = \ln(b)/\ln(a)$

### ■ WORKED EXAMPLE

Solve  $2^x = 16$ :

$$2^x = 2^4 \Rightarrow x = 4$$

$$\text{Or: } x = \log_2(16) = \log 16 / \log 2 = 4$$

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Solve:  $\log_2(x) = 5$

A) 32

B) 10

C) 25

D) 64

Answer: \_\_\_\_\_

**Part 2.** Which equals  $\log(6)$ ?

A)  $\log(2) + \log(3)$

B)  $\log(2) * \log(3)$

C)  $\log(9) - \log(3)$

D)  $\log(12) - \log(6)$

Answer: \_\_\_\_\_

**Part 3.** Solve:  $3^{(x-1)} = 27$

A)  $x = 4$

B)  $x = 3$

C)  $x = 2$

D)  $x = 1$

Answer: \_\_\_\_\_

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## Systems of Equations

ALG

### ■ CORE CONCEPT

Solve systems by substitution, elimination, or matrices. Types: unique solution (intersecting lines), no solution (parallel), infinite solutions (same line). Cramer's Rule uses determinants.

Formula: Substitution: solve one variable, plug in. | Elimination: add/subtract to eliminate.

### ■ WORKED EXAMPLE

$$2x + y = 7 \text{ and } x - y = 2$$

$$\text{Add: } 3x = 9, \quad x = 3$$

$$\text{Sub: } 3 - y = 2, \quad y = 1 \Rightarrow (3, 1)$$

— ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Solve:  $x + y = 6$  and  $x - y = 2$

A) (4, 2)

B) (2, 4)

C) (3, 3)

D) (5, 1)

Answer: \_\_\_\_\_

**Part 2.** Which system has NO solution?

A)  $2x+y=3$  and  $4x+2y=8$

B)  $x+y=5$  and  $x-y=1$

C)  $2x-y=4$  and  $x+y=5$

D)  $x=3$  and  $y=2$

Answer: \_\_\_\_\_

**Part 3.** If  $3x + 2y = 12$  and  $x = 2$ , find  $y$

A)  $y = 3$

B)  $y = 6$

C)  $y = 2$

D)  $y = 4$

Answer: \_\_\_\_\_

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6

## Complex Numbers

ALG

### ■ CORE CONCEPT

Complex numbers:  $a + bi$  where  $i = \sqrt{-1}$ ,  $i^2 = -1$ . Add/subtract: combine like terms. Multiply: FOIL and replace  $i^2$  with  $-1$ . Divide: multiply by conjugate.  $|a+bi| = \sqrt{a^2+b^2}$ .

Formula:  $i^2 = -1$  |  $(a+bi)(a-bi) = a^2 + b^2$  | conjugate of  $(a+bi) = (a-bi)$

### ■ WORKED EXAMPLE

Multiply:  $(2+3i)(1-i)$

$$= 2 - 2i + 3i - 3i^2 = 2 + i - 3(-1) = 5 + i$$

== ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Simplify:  $(3+2i) + (1-4i)$

A)  $4 - 2i$

B)  $4 + 6i$

C)  $2 - 2i$

D)  $2 + 6i$

Answer: \_\_\_\_\_

**Part 2.** What is  $i^4$  equal to?

A) 1

B)  $-1$

C)  $i$

D)  $-i$

Answer: \_\_\_\_\_

**Part 3.** Multiply:  $(2+i)(2-i)$

A) 5

B) 3

C)  $4 - i^2$

D) 4

Answer: \_\_\_\_\_

# # 7

## Sequences & Series

ALG

### ■ CORE CONCEPT

Arithmetic:  $a_n = a_1 + (n-1)d$ ; Sum  $S_n = n/2(a_1+a_n)$ . Geometric:  $a_n = a_1 r^{(n-1)}$ ; Sum  $S_n = a_1(1-r^n)/(1-r)$ . Infinite geometric ( $|r| < 1$ ):  $S = a_1/(1-r)$ .

Formula: Arith:  $a_n = a_1 + (n-1)d$  | Geom:  $a_n = a_1 r^{(n-1)}$  | Inf:  $S = a_1/(1-r)$

### ■ WORKED EXAMPLE

Find 10th term of 3, 7, 11, 15...

$$d=4, a_1=3$$

$$a_{10} = 3 + (10-1) \cdot 4 = 3 + 36 = 39$$

== ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Find the 8th term of: 5, 9, 13, 17...

A) 33

B) 29

C) 37

D) 25

Answer: \_\_\_\_\_

**Part 2.** Sum of infinite series: 12, 6, 3, 1.5...

A) 24

B) 18

C) 12

D) 36

Answer: \_\_\_\_\_

**Part 3.** Common ratio in: 4, 12, 36, 108...

A) 3

B) 4

C) 8

D) 2

Answer: \_\_\_\_\_

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## Radical & Rational Equations

ALG

### ■ CORE CONCEPT

Radical equations: isolate radical, square both sides, ALWAYS check for extraneous solutions. Rational equations: multiply both sides by LCD; exclude values making denominator zero.

Formula:  $\sqrt{\text{expr}} = k \Rightarrow \text{expr} = k^2$  (CHECK!) | Rational: multiply by LCD

### ■ WORKED EXAMPLE

Solve  $\sqrt{x+3} = 4$ :

Square:  $x+3 = 16 \Rightarrow x = 13$

Check:  $\sqrt{16} = 4$  VALID

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Solve:  $\sqrt{2x-1} = 3$

A)  $x = 5$

B)  $x = 4$

C)  $x = 3$

D)  $x = 2$

Answer: \_\_\_\_\_

**Part 2.** Which value is extraneous in  $\sqrt{x+4} = x-2$ ?

A)  $x = 0$

B)  $x = 5$

C)  $x = 3$

D)  $x = 4$

Answer: \_\_\_\_\_

**Part 3.** Solve:  $3/(x-2) = 6$

A)  $x = 5/2$

B)  $x = 4$

C)  $x = 3$

D)  $x = 2$

Answer: \_\_\_\_\_

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## Function Transformations

ALG

### ■ CORE CONCEPT

For  $g(x)=a*f(b(x-h))+k$ :  $h$  shifts right,  $-h$  shifts left;  $k$  shifts up/down; negative  $a$  reflects over  $x$ -axis;  $|a|>1$  stretches vertically;  $|b|>1$  compresses horizontally.

Formula:  $g(x) = a*f(b(x-h))+k$  |  $h$ : horizontal shift |  $k$ : vertical shift

### ■ WORKED EXAMPLE

$g(x) = -(x-2)^2 + 3$  vs  $f(x) = x^2$ :

- Shift right 2, up 3

- Reflect over  $x$ -axis

==■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.**  $g(x) = f(x-3) + 2$ . This is  $f(x)$  shifted:

A) Right 3, up 2

B) Left 3, up 2

C) Right 3, down 2

D) Left 3, down 2

Answer: \_\_\_\_\_

**Part 2.**  $f(x)=x^2$ . What is  $g(x) = -f(x) + 4$ ?

A) Reflect  $x$ -axis, up 4

B) Reflect  $y$ -axis, up 4

C) Reflect  $x$ -axis, down 4

D) No reflection, up 4

Answer: \_\_\_\_\_

**Part 3.**  $g(x) = 2f(x)$  compared to  $f(x)$ :

A) Vertical stretch by 2

B) Horizontal stretch by 2

C) Shift up 2

D) Shift right 2

Answer: \_\_\_\_\_

## ■ CORE CONCEPT

Circle:  $(x-h)^2+(y-k)^2=r^2$ , center  $(h,k)$ , radius  $r$ . Complete the square to convert general form. Parabola:  $y=a(x-h)^2+k$  opens up ( $a>0$ ) or down ( $a<0$ ).

Formula: Circle:  $(x-h)^2+(y-k)^2=r^2$  | Parabola:  $y=a(x-h)^2+k$

## ■ WORKED EXAMPLE

$$(x-2)^2+(y+3)^2=25$$

Center:  $(2, -3)$  Radius: 5

== ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Center of  $(x+4)^2 + (y-1)^2 = 9$ ?

A)  $(-4, 1)$

B)  $(4, -1)$

C)  $(-4, -1)$

D)  $(4, 1)$

Answer: \_\_\_\_\_

**Part 2.** Radius of  $x^2+y^2-6x+8y-11=0$ ?

A) 6

B) 36

C)  $\sqrt{11}$

D)  $\sqrt{47}$

Answer: \_\_\_\_\_

**Part 3.**  $y = 2(x-1)^2 + 3$  opens:

A) Upward

B) Downward

C) Left

D) Right

Answer: \_\_\_\_\_

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## Parallel Lines & Transversals

GEO

### ■ CORE CONCEPT

Transversal cutting parallel lines: alternate interior angles are EQUAL; corresponding angles are EQUAL; co-interior (same-side) angles are SUPPLEMENTARY (sum=180 degrees).

Formula: Alt interior = equal | Corresponding = equal | Co-interior sum = 180 degrees

### ■ WORKED EXAMPLE

Parallel lines, transversal creates 65 degrees:

Alt. interior = 65 degrees

Co-interior =  $180 - 65 = 115$  degrees

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Two parallel lines, transversal creates 70 degrees. Alternate interior angle = ?

- |                |                |
|----------------|----------------|
| A) 70 degrees  | B) 110 degrees |
| C) 180 degrees | D) 35 degrees  |

Answer: \_\_\_\_\_

**Part 2.** Co-interior angles:  $3x$  degrees and 57 degrees. Parallel lines. Find  $x$ .

- |       |        |
|-------|--------|
| A) 41 | B) 57  |
| C) 90 | D) 123 |

Answer: \_\_\_\_\_

**Part 3.** Corresponding angles:  $(2x+10)$  degrees and  $(x+40)$  degrees. Find  $x$ .

- |       |       |
|-------|-------|
| A) 30 | B) 20 |
| C) 50 | D) 10 |

Answer: \_\_\_\_\_

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## Triangle Congruence

GEO

### ■ CORE CONCEPT

Congruence shortcuts: SSS (3 sides), SAS (2 sides + included angle), ASA (2 angles + included side), AAS (2 angles + non-included side), HL (right triangles: hyp+leg). SSA and AAA are NOT valid.

Formula: SSS | SAS | ASA | AAS | HL (right triangles only)

### ■ WORKED EXAMPLE

Two triangles share a side, both have two equal angles.

=> Two angles + included side => ASA

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** All three sides of two triangles are equal. They are congruent by:

- |        |        |
|--------|--------|
| A) SSS | B) SAS |
| C) ASA | D) AAS |

Answer: \_\_\_\_\_

**Part 2.** Which is NOT a valid congruence theorem?

- |        |        |
|--------|--------|
| A) SSA | B) SAS |
| C) ASA | D) AAS |

Answer: \_\_\_\_\_

**Part 3.** Right triangles:  $AB=DE$  (hyp),  $BC=EF$  (leg). Congruence by:

- |        |        |
|--------|--------|
| A) HL  | B) SSA |
| C) SAS | D) ASA |

Answer: \_\_\_\_\_

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## Similar Triangles & Proportions

GEO

### ■ CORE CONCEPT

Similarity by: AA (two equal angles), SSS~ (all sides proportional), SAS~ (two proportional sides + equal included angle).  
Similar triangles: angles equal, sides proportional. Area ratio =  $k^2$ .

Formula: AA~ | SSS~ | SAS~ | Area ratio =  $k^2$  | Perimeter ratio =  $k$

### ■ WORKED EXAMPLE

Triangle ABC ~ Triangle DEF, ratio 3:1

$AB=9$ ,  $DE=3$ ,  $AC=15$

=>  $DF = 15/3 = 5$

—■ YOUR TURN — Solve All 3 Parts (circle your answer):



A) 90 degrees

B) 180 degrees

C) 45 degrees

D) 60 degrees

Answer: \_\_\_\_\_

**Part 3.** Two tangents from point P, PA=8. PB = ?

A) 8

B) 16

C) 4

D) Cannot determine

Answer: \_\_\_\_\_

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## Area & Volume Formulas

GEO

### ■ CORE CONCEPT

Volume: Cylinder  $V = \pi r^2 h$ ; Cone  $V = (1/3)\pi r^2 h$ ; Sphere  $V = (4/3)\pi r^3$ . Area: Circle  $A = \pi r^2$ ; Triangle  $= (1/2)b h$ ; Sector  $= (\theta/360)\pi r^2$ .

Formula: Cylinder:  $\pi r^2 h$  | Cone:  $(1/3)\pi r^2 h$  | Sphere:  $(4/3)\pi r^3$

### ■ WORKED EXAMPLE

Volume of cone:  $r=3, h=4$

$$V = (1/3)\pi(3^2) \cdot 4 = (1/3)\pi \cdot 36 = 12\pi$$

— ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Volume of cylinder with  $r=5$  and  $h=8$ :

A)  $200\pi$

B)  $80\pi$

C)  $100\pi$

D)  $40\pi$

Answer: \_\_\_\_\_

**Part 2.** Sphere with radius 3. Volume = ?

A)  $36\pi$

B)  $108\pi$

C)  $12\pi$

D)  $27\pi$

Answer: \_\_\_\_\_

**Part 3.** Cone and cylinder: equal  $r$  and  $h$ . Volume of cone is what fraction of cylinder?

A)  $1/3$

B)  $1/2$

C)  $2/3$

D)  $1/4$

Answer: \_\_\_\_\_



### ■ WORKED EXAMPLE

Rotate (3,2) by 90 deg CCW:

$$(x, y) \rightarrow (-y, x)$$

$$(3, 2) \rightarrow (-2, 3)$$

— ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Reflect (3,-5) over x-axis:

A) (3, 5)

B) (-3, 5)

C) (-3, -5)

D) (5, 3)

Answer: \_\_\_\_\_

**Part 2.** Rotate (4,2) by 180 degrees:

A) (-4, -2)

B) (2, -4)

C) (-2, 4)

D) (4, 2)

Answer: \_\_\_\_\_

**Part 3.** Dilation scale factor 3 maps (2,1) to:

A) (6, 3)

B) (5, 4)

C) (3, 2)

D) (2, 3)

Answer: \_\_\_\_\_

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8

## Trigonometry (SOH-CAH-TOA)

GEO

### ■ CORE CONCEPT

In a right triangle:  $\sin(\theta) = O/H$ ,  $\cos(\theta) = A/H$ ,  $\tan(\theta) = O/A$ . Special values:  $\sin 30 = 1/2$ ,  $\cos 30 = \sqrt{3}/2$ ,  $\sin 45 = \cos 45 = \sqrt{2}/2$ ,  $\sin 60 = \sqrt{3}/2$ ,  $\cos 60 = 1/2$ .

Formula:  $\sin = O/H$  |  $\cos = A/H$  |  $\tan = O/A$  | SOH-CAH-TOA

### ■ WORKED EXAMPLE

Right triangle: opposite=5, hypotenuse=13

$$\sin(A) = 5/13$$

$$A = \sin^{-1}(5/13) \text{ approx } 22.6 \text{ deg}$$

— ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** In right triangle ABC, AB (hyp)=10, BC (opp)=6. Find sin A.

A)  $\frac{3}{5}$

B)  $\frac{4}{5}$

C)  $\frac{3}{4}$

D)  $\frac{5}{3}$

Answer: \_\_\_\_\_

**Part 2.** Value of  $\cos 60$  degrees:

A)  $\frac{1}{2}$

B)  $\frac{\sqrt{3}}{2}$

C)  $\frac{\sqrt{2}}{2}$

D) 1

Answer: \_\_\_\_\_

**Part 3.**  $\tan(\theta) = \frac{3}{4}$  and hyp = 5. Side opposite  $\theta = ?$

A) 3

B) 4

C) 5

D)  $\sqrt{7}$

Answer: \_\_\_\_\_

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9

## Coordinate Geometry & Midpoints

GEO

### ■ CORE CONCEPT

Midpoint:  $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ . Slope  $m = \frac{y_2-y_1}{x_2-x_1}$ . Parallel: equal slopes. Perpendicular: slopes are negative reciprocals,  $m_1 \cdot m_2 = -1$ . Point-slope:  $y - y_1 = m(x - x_1)$ .

Formula: Midpoint =  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$  | Perpendicular:  $m_1 \cdot m_2 = -1$

### ■ WORKED EXAMPLE

Midpoint of (2,4) and (8,10):

$$M = \left(\frac{2+8}{2}, \frac{4+10}{2}\right) = (5, 7)$$

== ■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** Midpoint of (-2, 6) and (4, -2):

A) (1, 2)

B) (2, 1)

C) (-1, 2)

D) (1, -2)

Answer: \_\_\_\_\_

**Part 2.** Line has slope 3. A perpendicular line has slope:

A)  $-\frac{1}{3}$

B)  $\frac{1}{3}$

C) -3

D) 3

Answer: \_\_\_\_\_

**Part 3.** Equation through (2,1) with slope 4:

A)  $y = 4x - 7$

B)  $y = 4x + 7$

C)  $y = 4x - 9$

D)  $y = 4x + 1$

Answer: \_\_\_\_\_

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0

## Geometric Proofs & Quadrilaterals

GEO

### ■ CORE CONCEPT

Parallelogram: opposite sides equal and parallel, diagonals bisect each other. Rectangle: all angles 90 deg, equal diagonals. Rhombus: all sides equal, perpendicular diagonals. Square: all of above. Midsegment of triangle = half the base.

Formula: Parallelogram: opp sides equal & parallel | Rhombus diagonals: perpendicular

### ■ WORKED EXAMPLE

Parallelogram ABCD, diagonal AC=12.

Diagonals bisect each other:

Each half = 6

—■ YOUR TURN — Solve All 3 Parts (circle your answer):

**Part 1.** In parallelogram ABCD, AB=8, BC=5. What is CD?

A) 8

B) 5

C) 13

D) 3

Answer: \_\_\_\_\_

**Part 2.** Quadrilateral: four equal sides and all 90-degree angles. It is a:

A) Square

B) Rhombus

C) Rectangle

D) Trapezoid

Answer: \_\_\_\_\_

**Part 3.** Midsegment of a triangle is parallel to the base and is:

A) Half the base

B) Equal to the base

C) Twice the base

D) Perpendicular to base

Answer: \_\_\_\_\_

# ANSWER KEY & EXPLANATIONS

## #1 Quadratic Functions

ALG

**Part 1:** Answer = A  $h = 6/2 = 3; k = 9 - 18 + 5 = -4$ . Vertex (3, -4).

**Part 2:** Answer = A  $\Delta = 16 - 20 = -4 < 0$ . No real solutions.

**Part 3:** Answer = A  $x = -b/(2a) = -12/6 = -2$

## #2 Polynomial Factoring

ALG

**Part 1:** Answer = A Difference of squares:  $x^2 - 4^2 = (x-4)(x+4)$

**Part 2:** Answer = A  $x^2 - 5x + 6 = (x-2)(x-3)$ . Answer: (x-2)

**Part 3:** Answer = A Find factors of 6 summing to 7: 6, 1. Split:  $2x^2 + 6x + x + 3 = (2x+1)(x+3)$

## #3 Rational Exponents & Radicals

ALG

**Part 1:** Answer = A  $8^{1/3} = \text{cube-root}(8) = 2$ . Then  $2^2 = 4$ .

**Part 2:** Answer = A Add exponents:  $3/4 + 1/4 = 1$ . Answer:  $x^1 = x$

**Part 3:** Answer = A  $\sqrt{x^5} = (x^5)^{1/2} = x^{5/2}$

## #4 Exponential & Logarithmic Functions

ALG

**Part 1:** Answer = A  $\log_2(x) = 5$  means  $2^5 = x$ . So  $x = 32$ .

**Part 2:** Answer = A Product rule:  $\log(2 \cdot 3) = \log(2) + \log(3)$

**Part 3:** Answer = A  $27 = 3^3$ , so  $x-1=3$ ,  $x=4$ .

## #5 Systems of Equations

ALG

**Part 1:** Answer = A Add:  $2x=8$ ,  $x=4$ .  $y = 6-4 = 2$ .

**Part 2:** Answer = A  $4x+2y=8$  simplifies to  $2x+y=4$ , parallel to  $2x+y=3$ . No solution.

**Part 3:** Answer = A  $3(2)+2y=12 \Rightarrow 6+2y=12 \Rightarrow y=3$

## #6 Complex Numbers

ALG

**Part 1:** Answer = A Real:  $3+1=4$ . Imaginary:  $2-4=-2$ . Answer:  $4-2i$

**Part 2:** Answer = A  $i^1=i$ ,  $i^2=-1$ ,  $i^3=-i$ ,  $i^4=1$ . Pattern repeats every 4.

**Part 3:** Answer = A  $(a+b)(a-b)=a^2+b^2$ . So  $4+1=5$ . ( $i^2=-1$ , so  $-i^2=+1$ )

## #7 Sequences & Series

ALG

**Part 1:** Answer = A  $d=4$ ,  $a_1=5$ .  $a_8 = 5+7(4) = 5+28 = 33$

**Part 2:** Answer = A  $r=1/2, a_1=12. S = 12/(1-0.5) = 24$

**Part 3:** Answer = A  $12/4=3, 36/12=3. Common ratio r=3$

## #8 Radical & Rational Equations

ALG

**Part 1:** Answer = A *Square:  $2x-1=9 \Rightarrow 2x=10 \Rightarrow x=5$ . Check:  $\sqrt{9}=3$  Valid*

**Part 2:** Answer = A *Solutions are  $x=0$  and  $x=5$ . Check  $x=0$ :  $\sqrt{4}=2$ , but  $0-2=-2$ . Extraneous!*

**Part 3:** Answer = A  $3 = 6(x-2) \Rightarrow 3=6x-12 \Rightarrow 6x=15 \Rightarrow x=5/2$

## #9 Function Transformations

ALG

**Part 1:** Answer = A  *$f(x-h)$ : right by  $h$ .  $+2$ : up. So right 3, up 2.*

**Part 2:** Answer = A *Negative sign reflects over  $x$ -axis.  $+4$  shifts up.*

**Part 3:** Answer = A *Multiplying outside = vertical stretch by factor 2.*

## #10 Conic Sections (Circles & Parabolas)

ALG

**Part 1:** Answer = A *Standard form:  $h=-4, k=1$ . Center  $(-4, 1)$ .*

**Part 2:** Answer = A *Complete square:  $(x-3)^2+(y+4)^2=9+16+11=36. r=\sqrt{36}=6$*

**Part 3:** Answer = A  *$a=2>0$  means parabola opens upward.*

## #11 Parallel Lines & Transversals

GEO

**Part 1:** Answer = A *Alternate interior angles are equal. Answer: 70 degrees.*

**Part 2:** Answer = A  $3x + 57 = 180 \Rightarrow 3x = 123 \Rightarrow x = 41$

**Part 3:** Answer = A *Equal:  $2x+10 = x+40 \Rightarrow x = 30$*

## #12 Triangle Congruence

GEO

**Part 1:** Answer = A *Three pairs of equal sides  $\Rightarrow$  SSS (Side-Side-Side)*

**Part 2:** Answer = A *SSA (two sides, non-included angle) is NOT valid.*

**Part 3:** Answer = A *Right triangles with equal hypotenuse and leg  $\Rightarrow$  HL theorem.*

## #13 Similar Triangles & Proportions

GEO

**Part 1:** Answer = A *Scale factor:  $6/9=2/3. DF = 4*(3/2) = 6$*

**Part 2:** Answer = A *Area ratio =  $k^2. 16/64=1/4=(1/2)^2$ . Side ratio = 1:2*

**Part 3:** Answer = A *Both have same two angles  $\Rightarrow$  AA similarity*

## #14 Circle Theorems

GEO

**Part 1:** Answer = A *Inscribed =  $(1/2) \times$  central =  $120/2 = 60$  degrees*

**Part 2:** Answer = A Semicircle = 180 degree arc. Inscribed =  $180/2 = 90$  degrees (Thales Theorem)

**Part 3:** Answer = A Two tangents from external point are EQUAL in length.  $PB = 8$

## #15 Area & Volume Formulas

GEO

**Part 1:** Answer = A  $V = \pi r^2 h = \pi \cdot 25 \cdot 8 = 200\pi$

**Part 2:** Answer = A  $V = (4/3)\pi(3^3) = (4/3)\pi \cdot 27 = 36\pi$

**Part 3:** Answer = A  $V_{\text{cone}} = (1/3)\pi r^2 h$  vs  $V_{\text{cyl}} = \pi r^2 h$ . Ratio =  $1/3$

## #16 Pythagorean Theorem & Distance

GEO

**Part 1:** Answer = A  $c^2 = 64 + 225 = 289$ ,  $c = 17$ . (8-15-17 triple!)

**Part 2:** Answer = A In 30-60-90: shorter leg  $x$ , hyp =  $2x$ .  $2 \cdot 4 = 8$

**Part 3:** Answer = A  $d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

## #17 Transformations

GEO

**Part 1:** Answer = A Reflect over  $x$ -axis:  $(x,y) \rightarrow (x,-y)$ .  $(3,-5) \rightarrow (3,5)$

**Part 2:** Answer = A 180 deg rotation:  $(x,y) \rightarrow (-x,-y)$ .  $(4,2) \rightarrow (-4,-2)$

**Part 3:** Answer = A Multiply each coordinate by 3:  $(2 \cdot 3, 1 \cdot 3) = (6,3)$

## #18 Trigonometry (SOH-CAH-TOA)

GEO

**Part 1:** Answer = A  $\sin = \text{opposite/hypotenuse} = 6/10 = 3/5$

**Part 2:** Answer = A  $\cos 60 = 1/2$  (memorize the special angle table!)

**Part 3:** Answer = A  $\tan = \text{opp/adj} = 3/4$ ; with hyp=5 (3-4-5 triple), opposite=3

## #19 Coordinate Geometry & Midpoints

GEO

**Part 1:** Answer = A  $M = ((-2+4)/2, (6-2)/2) = (2/2, 4/2) = (1, 2)$

**Part 2:** Answer = A Perpendicular slope =  $-1/m = -1/3$

**Part 3:** Answer = A  $y-1=4(x-2) \Rightarrow y=4x-8+1 = 4x-7$

## #20 Geometric Proofs & Quadrilaterals

GEO

**Part 1:** Answer = A Opposite sides of parallelogram are equal.  $CD = AB = 8$

**Part 2:** Answer = A Equal sides (rhombus) + 90-degree angles (rectangle) = Square

**Part 3:** Answer = A Midsegment Theorem: parallel to and HALF the length of the base