MATH MASTERY

Pre-Algebra & Geometry

20 High-Difficulty Problems	Core Concepts + Worked Examples
Step-by-Step Solutions	Most Commonly Missed Topics

Self-Study Edition · Complete with Solutions

Table of Contents

Section I — Pre-Algebra

PRE-ALGEBRA · ORDER OF OPERATIONS

01 PEMDAS with Nested Expressions

Concept Review

The Order of Operations (PEMDAS) defines the evaluation sequence: Parentheses → Exponents → Multiplication/Division (L to R) → Addition/Subtraction (L to R). The most common error is adding before multiplying.

```
PEMDAS: P -> E -> M/D (L to R) -> A/S (L to R)
```

Worked Example

Evaluate: 3 + 4 x 2 - (6 - 1)

```
Step 1 (Parentheses): (6-1) = 5
Step 2 (Multiply): 4 x 2 = 8
Step 3 (Add/Sub L to R): 3 + 8 - 5 = 6
```

◆ ■ ◆

Your Turn — Problem 1

Evaluate: 5 + 3^2 x (4 - 1) - 8 / 2

A (A) 28 ← Answer

B (B) 30

C (C) 34

D (D) 18

Solution & Explanation

```
Step 1 (Parentheses): (4-1) = 3
Step 2 (Exponent): 3^2 = 9
Step 3 (Multiply): 9 x 3 = 27
Step 4 (Divide): 8 / 2 = 4
Step 5 (Add/Sub): 5 + 27 - 4 = 28
```

Answer: 28. Common error: skipping the division step or adding before multiplying.

PRE-ALGEBRA · FRACTIONS & RATIOS

02 Complex Fraction Simplification

Concept Review

To simplify a complex fraction (fraction within a fraction): multiply by the reciprocal of the denominator. Rule: (a/b) / (c/d) = (a/b) x (d/c) = ad/bc.

```
(a/b) / (c/d) = (a x d) / (b x c)
```

Worked Example

Simplify: (3/4) / (9/16)

```
(3/4) x (16/9) = 48/36
Simplify: 48/36 = 4/3
```

◆ ■ ◆

Your Turn — Problem 2

Simplify: (5/6) / (15/8)

A	(A) 4/9	← Answer
B	(B) 9/4	
C	(C) 25/48	
D	(D) 1/2	

Solution & Explanation

```
(5/6) x (8/15) = 40/90
GCF of 40 and 90 is 10
40/90 = 4/9
```

Multiply by the reciprocal, then reduce. 40 / 10 = 4, 90 / 10 = 9.

PRE-ALGEBRA · INTEGERS & ABSOLUTE VALUE

03 Combining Absolute Value Expressions

Concept Review

Absolute value |x| = distance from zero, always >= 0. Key: |-a| = |a|. Common mistake: writing |-3| = -3 (wrong — it equals 3). Always evaluate the expression inside before applying the absolute value.

|x| = x if x >= 0 ; |x| = -x if x < 0

Worked Example

Evaluate: |-7| + |3 - 10| - |-2|

```
|-7| = 7, |3-10| = |-7| = 7, |-2| = 2
7 + 7 - 2 = 12
```

◆ ■ ◆

Your Turn — Problem 3

Evaluate: |-5| x |-3| - |-8 + 3|

A (A) 10 ← Answer

B (B) 20

C (C) 15

D (D) -10

Solution & Explanation

```
|-5| = 5, |-3| = 3
5 x 3 = 15
|-8 + 3| = |-5| = 5
15 - 5 = 10
```

Evaluate inside absolute values first. 15 - 5 = 10.

PRE-ALGEBRA · PERCENTAGES

04 Multi-Step Percentage Change

Concept Review

Sequential percentage changes use multipliers. A 15% increase means x 1.15; a 10% decrease means x 0.90. Apply one at a time — a +20% then -20% does NOT return to the original value.

```
New Value = Original x (1 + r1) x (1 + r2)
```

Worked Example

$40 increases 25%, then decreases 20%. Final price?

```
$40 x 1.25 = $50
$50 x 0.80 = $40.00
```

◆ ■ ◆

Your Turn — Problem 4

A laptop costs $800. It increases 15%, then decreases 10%. Final price?

A (A) $828 ← Answer

B (B) $800

C (C) $720

D (D) $840

Solution & Explanation

```
$800 x 1.15 = $920
$920 x 0.90 = $828
```

Sequential: +15% then -10% gives a net change of +3.5%, not +5%.

PRE-ALGEBRA · LINEAR EQUATIONS

05 Variables on Both Sides

Concept Review

To solve: (1) Distribute if needed. (2) Move all variable terms to one side. (3) Move constants to the other. (4) Divide by the coefficient. Always verify by substituting back.

```
ax + b = cx + d => (a-c)x = d-b => x = (d-b)/(a-c)
```

Worked Example

Solve: 5x - 3 = 2x + 9

```
5x - 2x = 9 + 3
3x = 12
x = 4
```

◆ ■ ◆

Your Turn — Problem 5

Solve for x: 3(2x - 4) = 2(x + 5) + 6

A	(A) x = 7	← Answer
B	(B) x = 4	
C	(C) x = 6	
D	(D) x = 5	

Solution & Explanation

```
Distribute: 6x - 12 = 2x + 10 + 6
6x - 12 = 2x + 16
4x = 28
x = 7
```

Distribute first, then collect like terms. Check: 3(14-4)=30, 2(12)+6=30 ✓

PRE-ALGEBRA · EXPONENTS & POWERS

06 Laws of Exponents

Concept Review

Product rule: a^m * a^n = a^(m+n). Quotient rule: a^m / a^n = a^(m-n). Power rule: (a^m)^n = a^(mn). Zero: a^0 = 1. Negative: a^(-n) = 1/a^n.

```
a^m * a^n = a^(m+n) | a^m / a^n = a^(m-n)
```

Worked Example

Simplify: x^5 * x^(-2) / x^3

```
x^(5+(-2)) = x^3
x^3 / x^3 = x^0 = 1
```

◆ ■ ◆

Your Turn — Problem 6

Simplify: 2^4 x 2^(-1) / 2^2

A (A) 2 ← Answer

B (B) 4

C (C) 8

D (D) 1/2

Solution & Explanation

```
Product rule: 2^(4+(-1)) = 2^3
Quotient rule: 2^3 / 2^2 = 2^(3-2) = 2^1 = 2
```

Apply product rule first, then quotient rule. Result is 2.

PRE-ALGEBRA · INEQUALITIES

07 Inequality Sign Flipping

Concept Review

All regular algebra applies to inequalities EXCEPT: when you multiply or divide both sides by a negative number, you MUST flip the inequality sign. This is the most common inequality mistake.

```
-ax <= b => x >= -b/a (sign flips when dividing by -a)
```

Worked Example

Solve: -2x + 3 > 7

```
-2x > 4
x < -2 (flip sign: divided by -2)
```

◆ ■ ◆

Your Turn — Problem 7

Solve for x: -3x + 6 <= 18

A	(A) x >= -4	← Answer
B	(B) x <= -4	
C	(C) x >= 4	
D	(D) x <= 4	

Solution & Explanation

```
-3x <= 18 - 6
-3x <= 12
Divide by -3 (flip sign!): x >= -4
```

Dividing by -3 flips <= to >=. Answer: x >= -4.

PRE-ALGEBRA · PROPORTIONS & RATES

08 Cross-Multiplication

Concept Review

A proportion states two ratios are equal: a/b = c/d. Cross-multiply: ad = bc. For unit rates, find the value per single unit first, then scale. Always label your units.

```
a/b = c/d => a x d = b x c
```

Worked Example

A car travels 180 miles in 3 hours. How far in 7 hours?

```
180/3 = x/7
3x = 1260
x = 420 miles
```

◆ ■ ◆

Your Turn — Problem 8

If 5 notebooks cost $8.50, how much do 12 notebooks cost?

A (A) $20.40 ← Answer

B (B) $18.50

C (C) $22.00

D (D) $17.00

Solution & Explanation

```
Unit rate: $8.50 / 5 = $1.70 per notebook
$1.70 x 12 = $20.40
Or cross-multiply: 5x = 8.50 x 12 = 102, x = $20.40
```

Unit rate method: find cost per 1, then multiply by 12.

PRE-ALGEBRA · SQUARE ROOTS & RADICALS

09 Simplifying Radicals

Concept Review

To simplify sqrt(n): factor out the largest perfect square. Rule: sqrt(a*b) = sqrt(a) * sqrt(b). Identify perfect squares: 4, 9, 16, 25, 36, 49, 64, 81, 100...

```
sqrt(a^2 * b) = a * sqrt(b) where b has no perfect square factors
```

Worked Example

Simplify: sqrt(72)

```
72 = 36 x 2
sqrt(72) = sqrt(36) x sqrt(2) = 6*sqrt(2)
```

◆ ■ ◆

Your Turn — Problem 9

Simplify: sqrt(180)

A	(A) 6*sqrt(5)	← Answer
B	(B) 9*sqrt(5)	
C	(C) 6*sqrt(10)	
D	(D) 3*sqrt(20)	

Solution & Explanation

```
Find largest perfect square: 180 = 36 x 5
sqrt(180) = sqrt(36) x sqrt(5)
= 6*sqrt(5)
```

36 is the largest perfect square factor of 180. sqrt(36) = 6.

PRE-ALGEBRA · SYSTEMS OF EQUATIONS

10 Word Problem — Two Variables

Concept Review

Define two variables, write two equations, then solve using substitution or elimination. Substitution: solve one equation for one variable, substitute into the other. Always check your answer in both original equations.

System: { ax + by = c ; dx + ey = f }

Worked Example

Two numbers sum to 20 and differ by 4. Find both.

```
x + y = 20 and x - y = 4
Add equations: 2x = 24, x = 12
y = 20 - 12 = 8
```

◆ ■ ◆

Your Turn — Problem 10

Apples cost $0.75 and oranges $1.25. Jake buys 10 fruits for $9.50 total. How many apples?

A (A) 6 ← Answer

B (B) 4

C (C) 5

D (D) 7

Solution & Explanation

```
Let a = apples, r = oranges
a + r = 10 and 0.75a + 1.25r = 9.50
Substitute r = 10 - a:
0.75a + 1.25(10-a) = 9.50
-0.50a = -3.00, a = 6
```

Substitution: replace r with (10-a), then solve. 6 apples + 4 oranges = $4.50 + $5.00 = $9.50 ✓

Section II — Geometry

GEOMETRY · ANGLES & LINES

11 Parallel Lines & Transversal

Concept Review

When a transversal crosses parallel lines: Alternate interior angles are EQUAL. Corresponding angles are EQUAL. Co-interior (same-side interior) angles are SUPPLEMENTARY (sum = 180 degrees).

```
Alt. interior: angle3 = angle6 | Co-interior: angle3 + angle5 = 180 deg
```

Worked Example

Co-interior angle is 65 degrees. Find the other co-interior angle.

```
Co-interior angles are supplementary
65 + x = 180
x = 115 degrees
```

◆ ■ ◆

Your Turn — Problem 11

Alternate interior angles measure (3x + 10) deg and (5x - 30) deg. Find x.

A	(A) x = 20	← Answer
B	(B) x = 15	
C	(C) x = 25	
D	(D) x = 10	

Solution & Explanation

```
Alternate interior angles are equal:
3x + 10 = 5x - 30
40 = 2x
x = 20
```

Set alternate interior angles equal, then solve. Check: 3(20)+10=70, 5(20)-30=70 ✓

GEOMETRY · TRIANGLE PROPERTIES

12 Exterior Angle Theorem

Concept Review

Exterior Angle Theorem: An exterior angle of a triangle = the SUM of the two non-adjacent (remote) interior angles. Also: all interior angles of a triangle sum to 180 degrees.

```
Angle_exterior = Angle_A + Angle_B (remote interior angles)
```

Worked Example

Triangle has interior angles 40 deg and 65 deg. Find the exterior angle at the third vertex.

```
Direct: 40 + 65 = 105 degrees (exterior angle)
Or: third interior = 180-40-65=75; exterior = 180-75=105
```

◆ ■ ◆

Your Turn — Problem 12

Two angles: (2x + 15) deg and (3x - 5) deg. Exterior angle at third vertex = 130 deg. Find x.

A (A) x = 24 ← Answer

B (B) x = 20

C (C) x = 22

D (D) x = 18

Solution & Explanation

```
Exterior = sum of remote interior angles:
(2x + 15) + (3x - 5) = 130
5x + 10 = 130
5x = 120, x = 24
```

Exterior angle = sum of the two remote interior angles. Check: (48+15)+(72-5) = 63+67 = 130 ✓

GEOMETRY · PYTHAGOREAN THEOREM

13 Pythagorean Triples & 3D Diagonal

Concept Review

For right triangles: a^2 + b^2 = c^2 (c = hypotenuse). Common triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25. For a 3D box diagonal: d = sqrt(l^2 + w^2 + h^2).

```
a^2 + b^2 = c^2 | d_3D = sqrt(l^2 + w^2 + h^2)
```

Worked Example

Find the hypotenuse of a triangle with legs 5 and 12.

```
5^2 + 12^2 = 25 + 144 = 169
c = sqrt(169) = 13 (5-12-13 triple)
```

◆ ■ ◆

Your Turn — Problem 13

A rectangular box is 3 cm x 4 cm x 12 cm. Find the space diagonal.

A (A) 13 cm ← Answer

B (B) 19 cm

C (C) sqrt(144) cm

D (D) 15 cm

Solution & Explanation

```
d = sqrt(3^2 + 4^2 + 12^2)
= sqrt(9 + 16 + 144)
= sqrt(169) = 13 cm
```

3^2 + 4^2 = 25 = 5^2 (recognize 3-4-5), then 5^2 + 12^2 = 169 = 13^2.

GEOMETRY · CIRCLES

14 Arc Length & Sector Area

Concept Review

For central angle theta (degrees) in a circle of radius r: Arc Length = (theta/360) x 2*pi*r. Sector Area = (theta/360) x pi*r^2. Think of theta/360 as the fraction of the full circle.

```
L = (theta/360) x 2*pi*r | A_sector = (theta/360) x pi*r^2
```

Worked Example

Find arc length of a 90 degree sector, radius = 8.

```
L = (90/360) x 2*pi*8 = (1/4) x 16*pi = 4*pi
```

◆ ■ ◆

Your Turn — Problem 14

Circle with radius 6. Find the area of a 120 degree sector. (Leave in terms of pi)

A (A) 12*pi ← **Answer**

B (B) 6*pi

C (C) 36*pi

D (D) 24*pi

Solution & Explanation

```
A = (120/360) x pi x 6^2
= (1/3) x 36*pi
= 12*pi
```

*120/360 = 1/3. One-third of the full circle area (36*pi) = 12*pi.*

15

GEOMETRY · AREA & PERIMETER

Composite Shapes

Concept Review

For composite shapes: break into simple parts (rectangles, triangles, circles). Add or subtract areas. Common trap: forgetting to subtract interior holes, or using slant height instead of perpendicular height.

```
A_composite = A1 +/- A2 +/- A3 ...
```

Worked Example

An L-shape: 10x8 rectangle with a 4x3 cut from corner. Area?

```
A_big = 10 x 8 = 80
A_cut = 4 x 3 = 12
A_L = 80 - 12 = 68
```

◆ ■ ◆

Your Turn — Problem 15

Square with side 10 cm has a circle of radius 3 cm cut from its center. Remaining area? (pi = 3.14)

A (A) 71.74 cm^2 ← Answer

B (B) 84.26 cm^2

C (C) 28.26 cm^2

D (D) 91 cm^2

Solution & Explanation

```
A_square = 10^2 = 100 cm^2
A_circle = pi x 3^2 = 9 x 3.14 = 28.26 cm^2
Remaining = 100 - 28.26 = 71.74 cm^2
```

*Subtract the circular hole. 100 - 9*pi = 100 - 28.26 = 71.74 cm^2.*

GEOMETRY · SIMILAR TRIANGLES

16 Proportional Sides & Scale Factor

Concept Review

Similar triangles (~) have equal corresponding angles and proportional sides. AA Similarity: two matching angles prove similarity. Scale factor k: ratio of any pair of corresponding sides. Perimeters also scale by k.

```
a1/a2 = b1/b2 = c1/c2 = k (scale factor)
```

Worked Example

Triangles ABC ~ DEF. AB=6, BC=8, DE=9. Find EF.

```
6/9 = 8/EF
EF = (8 x 9)/6 = 12
```

◆ ■ ◆

Your Turn — Problem 16

Two similar triangles have sides in ratio 3:5. Smaller perimeter = 36 cm. Find larger perimeter.

A (A) 60 cm ← Answer

B (B) 45 cm

C (C) 54 cm

D (D) 75 cm

Solution & Explanation

```
Scale factor = 5/3
P_large = 36 x (5/3) = 60 cm
```

Perimeters scale by the same ratio as side lengths.

GEOMETRY · VOLUME & SURFACE AREA

17 Cylinder vs Cone Volume

Concept Review

Volume: Cylinder = pi*r^2*h. Cone = (1/3)*pi*r^2*h. Sphere = (4/3)*pi*r^3. Key insight: a cone holds exactly 1/3 the volume of a cylinder with the same base and height.

```
V_cone = (1/3) x V_cylinder (same r and h)
```

Worked Example

Volume of a cone with r=3 and h=7.

```
V = (1/3) x pi x 9 x 7 = 21*pi
```

◆ ■ ◆

Your Turn — Problem 17

Cylinder and cone share r=5 cm and h=9 cm. Difference in volumes? (in terms of pi)

A	(A) 150*pi cm^3	← Answer
B	(B) 75*pi cm^3	
C	(C) 225*pi cm^3	
D	(D) 50*pi cm^3	

Solution & Explanation

```
V_cyl = pi x 25 x 9 = 225*pi
V_cone = (1/3) x 225*pi = 75*pi
Difference = 225*pi - 75*pi = 150*pi cm^3
```

*Cone = 1/3 of cylinder. Difference = 2/3 of cylinder volume = 150*pi.*

GEOMETRY · COORDINATE GEOMETRY

18 Midpoint, Distance & Slope

Concept Review

Distance: d = sqrt((x2-x1)^2 + (y2-y1)^2). Midpoint: M = ((x1+x2)/2, (y1+y2)/2). Slope: m = (y2-y1)/(x2-x1). Parallel lines: equal slopes. Perpendicular: slopes are negative reciprocals.

```
d = sqrt((x2-x1)^2 + (y2-y1)^2) M = ((x1+x2)/2, (y1+y2)/2)
```

Worked Example

Distance between (1,2) and (4,6).

```
d = sqrt((4-1)^2 + (6-2)^2) = sqrt(9+16) = sqrt(25) = 5
```

Your Turn — Problem 18

Points A(2,3) and B(8,11). Find the midpoint M.

A	(A) M = (5, 7)	← **Answer**
B	(B) M = (4, 6)	
C	(C) M = (6, 8)	
D	(D) M = (3, 5)	

Solution & Explanation

```
M = ((2+8)/2, (3+11)/2)
= (10/2, 14/2)
= (5, 7)
```

Average the x-coordinates and y-coordinates separately.

GEOMETRY · SPECIAL RIGHT TRIANGLES

19 30-60-90 and 45-45-90

Concept Review

45-45-90: legs equal, hypotenuse = leg x sqrt(2). Ratio: x : x : x*sqrt(2). 30-60-90: short leg x 2 = hypotenuse; long leg = short x sqrt(3). Ratio: x : x*sqrt(3) : 2x.

```
30-60-90: x : x*sqrt(3) : 2x | 45-45-90: x : x : x*sqrt(2)
```

Worked Example

45-45-90 triangle has hypotenuse 8*sqrt(2). Find each leg.

```
x*sqrt(2) = 8*sqrt(2)
x = 8
```

◆ ■ ◆

Your Turn — Problem 19

In a 30-60-90 triangle, the short leg is 7. Find the hypotenuse and long leg.

A (A) 14 and 7*sqrt(3) ← Answer

B (B) 7*sqrt(2) and 7

C (C) 14 and 14*sqrt(3)

D (D) 7 and 7*sqrt(2)

Solution & Explanation

```
Short leg = x = 7
Hypotenuse = 2x = 14
Long leg = x*sqrt(3) = 7*sqrt(3)
```

*30-60-90 ratio is x : x*sqrt(3) : 2x. Memorize this — it saves major time on tests.*

GEOMETRY · QUADRILATERALS

20 Parallelograms & Trapezoids

Concept Review

Parallelogram: opposite sides parallel and equal, diagonals bisect each other. Area = base x height. Trapezoid: one pair of parallel sides. Area = (1/2)(b1 + b2) x h. Common error: using slant side instead of perpendicular height.

```
A_trapezoid = (1/2) x (b1 + b2) x h
```

Worked Example

Trapezoid with parallel sides 6 and 10, height 5. Area?

```
A = (1/2)(6+10) x 5 = (1/2)(16)(5) = 40
```

◆ ■ ◆

Your Turn — Problem 20

Trapezoid has parallel sides 9 cm and 15 cm, height 8 cm. Find the area.

A (A) 96 cm^2 ← Answer

B (B) 108 cm^2

C (C) 120 cm^2

D (D) 84 cm^2

Solution & Explanation

```
A = (1/2)(9 + 15) x 8
= (1/2)(24)(8)
= 12 x 8 = 96 cm^2
```

(b1+b2)/2 is the average base length. 24/2 = 12, then 12 x 8 = 96 cm^2.

Math Mastery

Pre-Algebra & Geometry Core Problem Set

