HIGH SCHOOL MATHEMATICS

Math Mastery Workbook

Algebra 1 & Geometry

20 High-Frequency Exam Mistake Problems · With Concept Review & Answer Key

20 Problems	**10 Algebra 1**	**10 Geometry**	**Concept + Example per Q**

Self-Study Edition · Print & Practice

ALGEBRA 1 Problems 1 – 10

1 Algebra 1 · Linear Equations
Solving Equations with Fractions

■ CONCEPT & EXAMPLE

To solve equations with fractions, multiply every term by the LCD to eliminate fractions first.
Then apply inverse operations step by step.

✏ WORKED EXAMPLE
Solve x/4 + 3 = 7
Multiply by 4: x + 12 = 28
Subtract 12: x = 16

Solve for x: (2x - 4) / 3 = 6

A) x = 11	B) x = 13
C) x = 7	D) x = 10

✔ Answer: A) x = 11

Multiply both sides by 3: 2x - 4 = 18 → Add 4: 2x = 22 → x = 11
Common mistake: Dividing by 2 before clearing the denominator.

2 Algebra 1 · Slope & Linear Functions
Finding the Equation of a Line

■ CONCEPT & EXAMPLE

Slope-intercept form: y = mx + b (m = slope, b = y-intercept)
Slope between two points: m = (y■ - y■) / (x■ - x■)
Then substitute one known point to solve for b.

✏ WORKED EXAMPLE
Through (1, 4) and (3, 10):
m = (10-4)/(3-1) = 3; plug in (1,4): 4 = 3 + b → b = 1
Equation: y = 3x + 1

Find the equation of the line through (2, 5) and (4, 11).

A) y = 2x + 1	B) y = 3x - 1
C) y = 3x + 1	D) y = 2x + 3

✔ Answer: B) y = 3x - 1

m = (11-5)/(4-2) = 3; using (2,5): 5 = 6 + b → b = -1
Equation: y = 3x - 1
Common mistake: Finding b before computing slope.

3 Algebra 1 · Systems of Equations
Substitution Method

CONCEPT & EXAMPLE

Step 1: Solve one equation for one variable.

Step 2: Substitute that expression into the other equation.

Step 3: Solve, then back-substitute to find the other variable.

WORKED EXAMPLE

y = 2x and x + y = 9
Substitute: x + 2x = 9 → x = 3, y = 6

Solve the system: y = x - 1 and 2x + 3y = 12

A) (3, 2)	B) (4, 3)
C) (5, 4)	D) (2, 1)

✔ Answer: A) (3, 2)

Substitute y = x-1: 2x + 3(x-1) = 12 → 5x - 3 = 12 → x = 3, y = 2
Common mistake: Not distributing 3 to both terms in 3(x-1).

4 Algebra 1 · Inequalities
Flipping the Inequality Sign

CONCEPT & EXAMPLE

Solve inequalities like equations, BUT:

■ Flip the sign when multiplying or dividing by a NEGATIVE number!

Adding or subtracting never changes the sign direction.

WORKED EXAMPLE

Solve -2x > 10:
Divide by -2 and FLIP: x < -5 ✓

Solve: -3x + 6 > 15

A) x > -3	B) x < -3
C) x > 3	D) x < 3

✔ Answer: B) x < -3

Subtract 6: -3x > 9 → Divide by -3 and FLIP: x < -3
Common mistake: Forgetting to flip → writing x > -3 (wrong direction!).

5 — Algebra 1 · Polynomials

Perfect Square Expansion

CONCEPT & EXAMPLE

Perfect square formula: $(a + b)^2 = a^2 + 2ab + b^2$

■ NOT $a^2 + b^2$! The middle term 2ab is ALWAYS required.

This is the #1 most common algebra error on standardized tests.

WORKED EXAMPLE

$(x + 4)^2 = x^2 + 2(x)(4) + 16 = x^2 + 8x + 16$

Expand: $(2x + 5)^2$

A) $4x^2 + 25$	B) $4x^2 + 10x + 25$
C) $4x^2 + 20x + 25$	D) $4x^2 - 20x + 25$

✔ Answer: C) $4x^2 + 20x + 25$

$(2x+5)^2 = (2x)^2 + 2(2x)(5) + 5^2 = 4x^2 + 20x + 25$

Common mistake: Writing $4x^2 + 25$ — forgetting the middle term $2ab = 20x$.

6 — Algebra 1 · Factoring

Factoring Trinomials (AC Method)

CONCEPT & EXAMPLE

For $ax^2 + bx + c$ with $a \neq 1$:

1. Compute a·c 2. Find two factors of a·c summing to b

3. Split the middle term and factor by grouping.

WORKED EXAMPLE

$2x^2 + 7x + 3$: $a \cdot c = 6$; factors: 1 and 6 $(1+6=7)$

$2x^2 + x + 6x + 3 = x(2x+1) + 3(2x+1) = (x+3)(2x+1)$

Factor completely: $3x^2 - 10x - 8$

A) $(3x + 2)(x - 4)$	B) $(3x - 2)(x + 4)$
C) $(3x + 4)(x - 2)$	D) $(x - 4)(3x - 2)$

✔ Answer: A) $(3x + 2)(x - 4)$

$a \cdot c = -24$; factors summing to -10: 2 and -12

$3x^2 + 2x - 12x - 8 = x(3x+2) - 4(3x+2) = (x-4)(3x+2)$

Common mistake: Guessing without verifying via FOIL.

7	**Algebra 1 · Quadratic Equations** The Quadratic Formula

■ **CONCEPT & EXAMPLE**

For $ax^2 + bx + c = 0$: $x = (-b \pm \sqrt{(b^2-4ac)}) / 2a$

Discriminant $\Delta = b^2-4ac$:

$\Delta > 0 \rightarrow$ 2 solutions; $\Delta = 0 \rightarrow$ 1 solution; $\Delta < 0 \rightarrow$ no real solutions

WORKED EXAMPLE

$x^2 - 5x + 6 = 0$: $x = (5 \pm \sqrt{1}) / 2 \rightarrow x = 3$ or $x = 2$

Use the quadratic formula to solve: $2x^2 - 3x - 5 = 0$

A) $x = 5/2, -1$	B) $x = -5/2, 1$
C) $x = 5, -1$	D) $x = 3/2, -1$

✔ Answer: A) $x = 5/2, -1$

$a=2, b=-3, c=-5$; $\Delta = 9+40 = 49$

$x = (3 \pm 7)/4 \rightarrow x = 10/4 = 5/2$ or $x = -4/4 = -1$

Common mistake: Forgetting ± and finding only one solution.

8	**Algebra 1 · Functions** Evaluating Functions

■ **CONCEPT & EXAMPLE**

$f(x)$ = the output when input is x.

To evaluate: replace every x with the given value.

■ Key fact: $(-n)^2 = +n^2$ (squaring a negative gives POSITIVE)

WORKED EXAMPLE

$f(x) = 3x^2 - 2$: $f(-1) = 3(-1)^2 - 2 = 3 - 2 = 1$

If $g(x) = 2x^2 - 3x + 1$, find $g(-2)$.

A) 15	B) 9
C) 11	D) 7

✔ Answer: A) 15

$g(-2) = 2(-2)^2 - 3(-2) + 1 = 2(4) + 6 + 1 = 8 + 6 + 1 = 15$

Common mistake: Computing $(-2)^2 = -4$ instead of $+4$.

9 Algebra 1 · Radicals

Simplifying Radical Expressions

■ CONCEPT & EXAMPLE

Find the LARGEST perfect-square factor of n, then simplify.

Rule: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

Perfect squares: 4, 9, 16, 25, 36, 49, 64, 81, 100 ...

WORKED EXAMPLE

$\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$

Simplify: $\sqrt{180}$

A) $9\sqrt{5}$	B) $6\sqrt{5}$
C) $6\sqrt{30}$	D) $3\sqrt{20}$

✔ Answer: B) $6\sqrt{5}$

$180 = 36 \times 5 \rightarrow \sqrt{180} = \sqrt{36} \cdot \sqrt{5} = 6\sqrt{5}$

Common mistake: Using $\sqrt{4 \times 45} = 2\sqrt{45}$ but not simplifying $\sqrt{45}$ further.

10 Algebra 1 · Word Problems

Distance, Rate & Time

■ CONCEPT & EXAMPLE

Fundamental formula: $d = r \times t$

Opposite directions: combined rate = sum of speeds

Same direction: relative rate = difference of speeds

WORKED EXAMPLE

Two cars at 40 mph and 60 mph in opposite directions for 2 h:
$d = (40 + 60) \times 2 = 200$ miles apart

Two trains start from the same station in opposite directions at 55 mph and 70 mph. After how many hours are they 375 miles apart?

A) 2 hours	B) 3 hours
C) 2.5 hours	D) 4 hours

✔ Answer: B) 3 hours

Combined rate = 55 + 70 = 125 mph

$t = 375 / 125 = 3$ hours

Common mistake: Using only one train's speed instead of the combined rate.

11 Geometry · Angle Relationships
Parallel Lines & Transversals

■ CONCEPT & EXAMPLE

Alternate interior/exterior angles = EQUAL

Corresponding angles = EQUAL

Co-interior (same-side interior) angles = SUPPLEMENTARY (sum = 180°)

⟹ WORKED EXAMPLE

Co-interior angles: $x + y = 180°$

Alternate interior: $x = y$

Two parallel lines are cut by a transversal. Co-interior angles measure $(3x + 15)°$ and $(2x + 5)°$. Find x.

A) $x = 32$	B) $x = 34$
C) $x = 30$	D) $x = 36$

✔ Answer: A) $x = 32$

Co-interior angles sum to 180°:

$(3x+15) + (2x+5) = 180 \rightarrow 5x + 20 = 180 \rightarrow x = 32$

Common mistake: Setting them equal (alternate interior rule) instead of summing to 180°.

12 Geometry · Triangles
Exterior Angle Theorem

■ CONCEPT & EXAMPLE

Triangle Angle Sum: Interior angles sum to 180°.

Exterior Angle Theorem:

An exterior angle = sum of the two NON-ADJACENT interior angles.

⟹ WORKED EXAMPLE

Remote interior angles 40° and 75° → Exterior angle = 115°

In ■ABC, the exterior angle at C = 115°. If ∠A = 55°, find ∠B.

A) 50°	B) 60°
C) 70°	D) 65°

✔ Answer: B) 60°

Exterior $\angle C = \angle A + \angle B \rightarrow 115 = 55 + \angle B \rightarrow \angle B = 60°$

Common mistake: Including the exterior angle in the 180° triangle sum.

13 Geometry · Similarity

Similar Triangle Proportions

■ **CONCEPT & EXAMPLE**

Similar triangles have equal corresponding angles.

Corresponding sides are proportional: a/a' = b/b' = c/c'

AA shortcut: 2 pairs of equal angles → triangles are similar.

WORKED EXAMPLE

■ABC ~ ■DEF, ratio 1:2. AB = 5 → DE = 10

■PQR ~ ■XYZ. PQ = 8, QR = 12, XY = 6. Find YZ.

A) YZ = 9	B) YZ = 8
C) YZ = 10	D) YZ = 7

✔ Answer: A) YZ = 9

Corresponding sides: PQ↔XY, QR↔YZ

8/6 = 12/YZ → YZ = (12×6)/8 = 9

Common mistake: Inverting the ratio or matching non-corresponding sides.

14 Geometry · Pythagorean Theorem

Right Triangle Applications

■ **CONCEPT & EXAMPLE**

In a right triangle with legs a, b and hypotenuse c: $a^2 + b^2 = c^2$

The hypotenuse is always the longest side (opposite the right angle).

Common triples: 3-4-5, 5-12-13, 8-15-17

WORKED EXAMPLE

Legs 6 and 8: c = √(36 + 64) = √100 = 10

A ladder 13 ft long leans against a wall with its base 5 ft from the wall. How high up the wall does it reach?

A) 10 ft	B) 12 ft
C) 11 ft	D) 8 ft

✔ Answer: B) 12 ft

$a^2 + 5^2 = 13^2$ → $a^2 = 169 - 25 = 144$ → a = 12 ft

Common mistake: Computing $(13-5)^2 = 64$ → a = 8 ft (wrong!).

15	**Geometry · Circles** Sector Area & Arc Length

■ **CONCEPT & EXAMPLE**

For radius r and central angle θ°:

Arc length = (θ/360) · 2πr

Sector area = (θ/360) · πr^2

✏ **WORKED EXAMPLE**

r = 6, θ = 90°: fraction = 1/4

Arc = (1/4)·12π = 3π; Sector area = (1/4)·36π = 9π

Circle with radius 10 and central angle 72°. Find the sector area in terms of π.

A) 18π	B) 20π
C) 16π	D) 24π

✔ Answer: B) 20π

Fraction = 72/360 = 1/5

Area = (1/5) · $\pi(10)^2$ = 100π/5 = 20π

Common mistake: Using diameter (20) instead of radius (10) → 80π.

16	**Geometry · Coordinate Geometry** Midpoint & Distance Formulas

■ **CONCEPT & EXAMPLE**

Midpoint: M = ((x■+x■)/2, (y■+y■)/2)

Distance: d = √((x■-x■)² + (y■-y■)²)

(The distance formula is the Pythagorean Theorem in coordinate form.)

✏ **WORKED EXAMPLE**

Points (1,2) and (5,6): Midpoint = (3,4); Distance = 4√2

M(3, -1) is the midpoint of AB. If A = (-1, 5), find B.

A) (7, -7)	B) (1, 2)
C) (5, -3)	D) (4, -4)

✔ Answer: A) (7, -7)

3 = (-1+xB)/2 → xB = 7; -1 = (5+yB)/2 → yB = -7

B = (7, -7)

Common mistake: Adding coords instead of solving the midpoint equation.

17 Geometry · Area & Perimeter
Area of Composite Figures

■ CONCEPT & EXAMPLE

Add areas of attached regions; Subtract areas of removed regions.

Rectangle: $A = l \times w$

Triangle: $A = (1/2)\, b \cdot h$ ← the 1/2 is essential!

✏ WORKED EXAMPLE

Rectangle 8×5 with triangle (base=8, h=3) on top:

Total = 40 + (1/2)(8)(3) = 40 + 12 = 52

A 10×6 rectangle has a right triangle cut from one corner with legs 4 and 3. What is the remaining area?

A) 54	B) 48
C) 50	D) 56

✔ Answer: A) 54

Rectangle: 10 × 6 = 60

Triangle: (1/2)(4)(3) = 6

Remaining: 60 - 6 = 54

Common mistake: Using 4×3=12 without (1/2) → answer 48 (wrong!).

18 Geometry · 3D Geometry
Volume of Cones & Cylinders

■ CONCEPT & EXAMPLE

Cylinder: $V = \pi r^2 h$

Cone: $V = (1/3)\pi r^2 h$

■ A cone holds exactly 1/3 of a cylinder with the same base and height.

✏ WORKED EXAMPLE

Cylinder r=3, h=7: V = 63π

Same-size cone: V = 63π/3 = 21π

A cone has radius 5 and height 12. Find its volume in terms of π.

A) 100π	B) 300π
C) 60π	D) 200π

✔ Answer: A) 100π

$V = (1/3)\pi(5^2)(12) = (1/3)(300\pi) = 100\pi$

Common mistake: Forgetting 1/3 → writing 300π (that's the cylinder volume!).

19 Geometry · Transformations
Reflections & Rotations

■ CONCEPT & EXAMPLE

Reflect over x-axis: (x, y) → (x, -y)

Reflect over y-axis: (x, y) → (-x, y)

Rotate 90° CCW: (x, y) → (-y, x)

Rotate 180°: (x, y) → (-x, -y)

✏ WORKED EXAMPLE

(3, 4) rotated 90° CCW: (3, 4) → (-4, 3)

P(-2, 5) is reflected over the x-axis, then rotated 90° CCW about the origin. Find the final coordinates.

A) (5, -2)	B) (-5, 2)
C) (5, 2)	D) (-5, -2)

✔ Answer: A) (5, -2)

Step 1 — Reflect over x-axis: (-2, 5) → (-2, -5)
Step 2 — Rotate 90° CCW: (-y, x) = (5, -2)
Common mistake: Reversing the order of transformations.

20 Geometry · Quadrilaterals
Properties of Parallelograms

■ CONCEPT & EXAMPLE

In any parallelogram:

- Opposite angles are EQUAL
- Consecutive angles are SUPPLEMENTARY (sum = 180°)
- Opposite sides are parallel and equal

✏ WORKED EXAMPLE

If ∠A = 70°: ∠C = 70° (opposite); ∠B = ∠D = 110° (consecutive)

In parallelogram ABCD, ∠A = (3x + 10)° and ∠C = (5x - 30)°. Find ∠B.

A) 70°	B) 110°
C) 80°	D) 100°

✔ Answer: B) 110°

Opposite angles equal: 3x+10 = 5x-30 → 2x = 40 → x = 20
∠A = 70°; consecutive angles supplementary: ∠B = 180 - 70 = 110°
Common mistake: Setting consecutive angles equal instead of supplementary.