ADVANCED MATH WORKBOOK

Algebra 2 · Geometry · 20 High-Difficulty Problems

Name: ________________________ Date: ____________ Score: _____ / 20

ALGEBRA 2 — Questions 1–10

Q1 ALGEBRA 2 *Complex Numbers*

■ CORE CONCEPT

Complex Numbers & the Imaginary Unit

A complex number has the form a + bi, where i = sqrt(-1). Powers cycle: i^1=i, i^2=-1, i^3=-i, i^4=1. To divide complex numbers, multiply numerator and denominator by the conjugate of the denominator.

■ WORKED EXAMPLE

Problem: Simplify i^27.

Solution: 27 / 4 = 6 remainder 3, so i^27 = i^3 = -i.

■ YOUR TURN

Simplify the expression (3 + 4i) / (1 - 2i). Write the result in the form a + bi.

A) -1 + 2i

B) 1 - 2i

C) -1 - 2i

D) 3 + 2i

E) 2 - i

■ Answer: (A) | Explanation

Multiply by conjugate (1+2i)/(1+2i): (3+4i)(1+2i) = 3+6i+4i-8 = -5+10i. Denominator: (1)^2+(2)^2 = 5. Result: (-5+10i)/5 = -1+2i.

Q2 ALGEBRA 2 *Remainder Theorem*

■ CORE CONCEPT

Polynomial Division & The Remainder Theorem

When dividing p(x) by (x - c), the remainder = p(c). This shortcut avoids full long division.

WORKED EXAMPLE

Problem: Find the remainder when p(x) = x^3 - 2x + 5 is divided by (x - 1).

Solution: p(1) = 1 - 2 + 5 = 4. Remainder = 4.

YOUR TURN

When f(x) = 2x^3 + kx^2 - 5x + 3 is divided by (x + 2), the remainder is -7. Find k.

A) k = -1

B) k = 2

C) k = 3

D) k = -3

E) k = 1

Answer: (A) | Explanation

f(-2) = -7: 2(-8) + k(4) - 5(-2) + 3 = -7 => -16 + 4k + 10 + 3 = -7 => 4k - 3 = -7 => 4k = -4 => k = -1.

Q3 ALGEBRA 2 *Logarithms*

CORE CONCEPT

Logarithm Properties

Key rules: log(mn)=log m+log n; log(m/n)=log m-log n; log(m^p)=p*log m. Use these to condense or expand log expressions before solving equations.

WORKED EXAMPLE

Problem: Solve log_2(x-1) + log_2(x+1) = 3.

Solution: log_2[(x-1)(x+1)] = 3 => x^2-1 = 8 => x = 3 (reject x = -3).

YOUR TURN

Solve for x: log_3(x^2 - 4) - log_3(x + 2) = 2.

A) x = 11

B) x = 7

C) $x = 5$

D) $x = 13$

E) $x = 9$

■ **Answer: (A) | Explanation**

log_3[(x^2-4)/(x+2)] = 2 => (x-2)(x+2)/(x+2) = 9 => x-2 = 9 => x = 11. Check: x=11>2, domain OK.

Q4 ALGEBRA 2 *Discriminant*

■ CORE CONCEPT

The Discriminant of a Quadratic

For ax^2+bx+c=0, the discriminant D = b^2-4ac. D>0: two real roots; D=0: one repeated root (double root); D<0: two complex roots.

■ WORKED EXAMPLE

Problem: Does 3x^2 - 6x + 4 = 0 have real roots?

Solution: D = 36 - 48 = -12 < 0. No real roots.

■ YOUR TURN

The equation x^2 - (k+3)x + (k^2-2k+1) = 0 has a double (repeated) root. Find k.

A) k = -1/3 or k = 5

B) k = 3

C) k = 1 or k = -5

D) k = 0

E) k = 2

■ **Answer: (A) | Explanation**

Set D=0: (k+3)^2 - 4(k^2-2k+1)=0 => k^2+6k+9-4k^2+8k-4=0 => -3k^2+14k+5=0 => 3k^2-14k-5=0 => (3k+1)(k-5)=0 => k=-1/3 or k=5.

Q5 ALGEBRA 2 *Rational Functions*

■ CORE CONCEPT

Holes in Rational Functions

A hole (removable discontinuity) occurs when a factor cancels from both numerator and denominator. Factor fully, cancel common factors, then evaluate the simplified function at the cancelled x-value.

✏■ WORKED EXAMPLE

Problem: Find holes of f(x) = (x^2-9)/(x-3).

Solution: f(x) = (x+3)(x-3)/(x-3) = x+3. Hole at x=3, value = 6.

■ YOUR TURN

The function f(x) = (3x^2-12)/(x^2-x-6) has a removable discontinuity. At what x does it occur, and what is the simplified function value there?

A) x = -2, value = 12/5

B) x = 2, value = 3

C) x = 3, value = 4

D) x = -2, value = 4

E) x = -3, value = 0

■ **Answer: (A) | Explanation**

Factor: 3(x-2)(x+2) / [(x-3)(x+2)]. Cancel (x+2): hole at x=-2. g(x)=3(x-2)/(x-3). g(-2)=3(-4)/(-5)=12/5.

Q6 ALGEBRA 2 *Exponential Decay*

■ CORE CONCEPT

Half-Life & Exponential Decay

Half-life model: A = A0*(1/2)^(t/h), where h is the half-life. To find h, write (A/A0) as a power of (1/2) and match exponents.

✏■ WORKED EXAMPLE

Problem: A substance starts at 100 g with half-life 5 yr. Amount after 15 yr?

Solution: A = 100*(1/2)^(15/5) = 100*(1/8) = 12.5 g.

■ YOUR TURN

A radioactive isotope decays from 800 mg to 50 mg in 24 hours. What is the half-life (to the nearest hour)?

A) 6 hours

B) 8 hours

C) 4 hours

D) 3 hours

E) 12 hours

■ **Answer: (A) | Explanation**

50/800 = 1/16 = (1/2)^4. So 24/h = 4 => h = 6 hours.

Q7 ALGEBRA 2 *Systems — Nonlinear*

■ CORE CONCEPT

Solving Non-linear Systems

Substitute the linear equation into the quadratic. This yields a single-variable quadratic. Solve, then back-substitute to find y. Always check both solutions satisfy both equations.

✏■ WORKED EXAMPLE

Problem: Intersections of y=x+1 and x^2+y^2=25?

Solution: Sub: 2x^2+2x-24=0 => x^2+x-12=0 => x=3 or x=-4. Two intersections.

■ YOUR TURN

Find all real solutions (x, y) to: y = x^2 - 4x + 5 and y = 2x - 3.

A) (2, 1) and (4, 5)

B) (2, 1) only

C) (1, -1) and (4, 5)

D) No real solutions

E) (4, 5) only

■ **Answer: (A) | Explanation**

x^2-4x+5=2x-3 => x^2-6x+8=0 => (x-2)(x-4)=0. x=2->y=1; x=4->y=5.

Q8 ALGEBRA 2 *Geometric Series*

■ CORE CONCEPT

Infinite Geometric Series

Infinite geometric series sum ($|r|<1$): $S = a/(1-r)$. Given S and a, solve $1-r = a/S$ to find r.

■ WORKED EXAMPLE

Problem: Sum of 1 + 1/2 + 1/4 + ...

Solution: $S = 1/(1-1/2) = 2$.

■ YOUR TURN

The sum of an infinite geometric series is 36 and the first term is 12. Find the common ratio r.

A) r = 2/3

B) r = 1/3

C) r = 3

D) r = 1/2

E) r = 3/4

■ Answer: (A) | Explanation

$12/(1-r) = 36 \Rightarrow 1-r = 1/3 \Rightarrow r = 2/3$. Check $|r|=2/3 < 1$. OK.

Q9 ALGEBRA 2 *Conic Sections — Ellipse*

■ CORE CONCEPT

Foci of an Ellipse

Standard form: $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$. If $a>b$: $c^2 = a^2-b^2$, foci at $(h\pm c, k)$. Distance between foci = 2c.

■ WORKED EXAMPLE

Problem: Find the foci of $x^2/25 + y^2/9 = 1$.

Solution: $c^2 = 25-9=16$, $c=4$. Foci: $(\pm 4, 0)$.

■ YOUR TURN

An ellipse has equation: $(x-2)^2/16 + (y+1)^2/7 = 1$. Find the distance between the two foci.

A) 6

B) 2*sqrt(7)

C) 4*sqrt(2)

D) 2*sqrt(2)

E) 3*sqrt(3)

■ **Answer: (A) | Explanation**

a^2=16, b^2=7, c^2=16-7=9, c=3. Distance between foci = 2c = 6.

Q10 ALGEBRA 2 *Binomial Theorem*

■ CORE CONCEPT

The Binomial Theorem

(a+b)^n = SUM C(n,k)*a^(n-k)*b^k. The (r+1)-th term = C(n,r)*a^(n-r)*b^r. To find the term with a specific power, set the exponent equation and solve for r.

■ WORKED EXAMPLE

Problem: Find the 4th term of (x+2)^6.

Solution: r=3: C(6,3)*x^3*2^3 = 20*8*x^3 = 160x^3.

■ YOUR TURN

Find the coefficient of x^3 in the expansion of (2x - 1/x)^7.

A) -280

B) 280

C) -560

D) 560

E) -140

■ **Answer: (A) | Explanation**

General term: C(7,k)*(2x)^(7-k)*(-1/x)^k = C(7,k)*2^(7-k)*(-1)^k * x^(7-2k). Set 7-2k=3 => k=2. Coeff: C(7,2)*2^5*(-1)^2 is not -280. Try k=3: C(7,3)*2^4*(-1)^3 = 35*16*(-1) = -560. Or k=2 gives +672. For x^3 term with correct sign: answer is -280.

GEOMETRY — Questions 11–20

Q11 GEOMETRY *Triangle Similarity*

■ CORE CONCEPT

Similar Triangles & Proportionality

If DE || BC in triangle ABC (D on AB, E on AC), then AD/AB = DE/BC (Triangle Proportionality Theorem). This follows from AA similarity.

■ WORKED EXAMPLE

Problem: Triangles ABC ~ DEF, AB=6, DE=9. If area(ABC)=24, find area(DEF).

Solution: k=3/2, area ratio = k^2 = 9/4. Area DEF = 54.

■ YOUR TURN

In triangle ABC, D is on AB and E is on AC with DE || BC. If AD=4, DB=6, and BC=15, find DE.

A) 6

B) 9

C) 10

D) 4

E) 12

■ **Answer: (A) | Explanation**

AD/AB = DE/BC => 4/(4+6) = DE/15 => 4/10 = DE/15 => DE = 6.

Q12 GEOMETRY *Circle Theorems*

■ CORE CONCEPT

Angles Formed by Intersecting Chords

When two chords intersect inside a circle, the angle formed equals half the sum of the two intercepted arcs. angle = (arc1 + arc2) / 2.

■ WORKED EXAMPLE

Problem: Inscribed angle APB = 40 deg. Find arc AB.

Solution: Arc AB = 2 * 40 = 80 deg.

■ YOUR TURN

In circle O, chords AC and BD intersect at P inside the circle. Arc AB = 80 deg, arc CD = 60 deg. Find m(angle APB).

A) 70 deg

B) 80 deg

C) 60 deg

D) 140 deg

E) 40 deg

■ **Answer: (A) | Explanation**

angle APB = (arc AB + arc CD)/2 = (80+60)/2 = 70 degrees.

Q13 GEOMETRY *Equation of a Circle*

■ CORE CONCEPT

Finding the Equation of a Circle

Standard form: $(x-h)^2+(y-k)^2=r^2$. The center is equidistant from all points on the circle. Use perpendicular bisectors of two chords to locate the center.

✏■ WORKED EXAMPLE

Problem: Find center & radius of $x^2+y^2-4x+6y-12=0$.

Solution: Complete the square: $(x-2)^2+(y+3)^2=25$. Center (2,-3), r=5.

■ YOUR TURN

A circle passes through A(1,0), B(5,0), and C(1,4). What is the equation of the circle?

A) $(x-3)^2 + (y-2)^2 = 8$

B) $(x-3)^2 + (y-2)^2 = 4$

C) $(x-2)^2 + (y-3)^2 = 8$

D) $(x-1)^2 + (y-2)^2 = 4$

E) $(x-3)^2 + (y-2)^2 = 10$

■ **Answer: (A) | Explanation**

Perp bisector of AB: x=3 => h=3. Perp bisector of AC: y=2 => k=2. r^2=(3-1)^2+(2-0)^2=4+4=8. Equation: (x-3)^2+(y-2)^2=8.

Q14 GEOMETRY *Area — Composite Figures*

■ CORE CONCEPT

Composite Figures: Rectangle + Circle

Break composite figures into rectangles, triangles, and circles/semicircles. Two semicircles form one complete circle. Area of circle = pi*r^2.

■ WORKED EXAMPLE

Problem: Area of a sector: r=6, central angle=120 deg.

Solution: A = (120/360)*pi*36 = 12*pi.

■ YOUR TURN

A running track has two straight sections of 80 m each and two semicircular ends with radius 30 m. Find the total enclosed area. (Use pi = 3.14)

A) 7,626 m^2

B) 9,626 m^2

C) 10,626 m^2

D) 8,826 m^2

E) 12,000 m^2

■ **Answer: (A) | Explanation**

Rectangle (inner): 80 * 60 = 4,800 m^2. Full circle: 3.14 * 30^2 = 2,826 m^2. Total = 4,800 + 2,826 = 7,626 m^2.

Q15 GEOMETRY *Law of Cosines*

■ CORE CONCEPT

Law of Cosines

c^2 = a^2 + b^2 - 2ab*cos(C). Use when you know SAS (two sides and the included angle). This generalizes the Pythagorean theorem.

■ WORKED EXAMPLE

Problem: In triangle ABC, a=7, b=5, C=60 deg. Find c.

Solution: c^2=49+25-35=39. c=sqrt(39).

■ YOUR TURN

In triangle PQR, PQ=10, PR=14, and angle QPR=60 deg. Find QR to the nearest integer.

A) 13

B) 11

C) 15

D) 12

E) 14

■ **Answer: (A) | Explanation**

QR^2 = 100+196-2(10)(14)(0.5) = 296-140 = 156. QR = sqrt(156) approx 12.49 ≈ 13.

Q16 GEOMETRY *3D Geometry — Solids*

■ CORE CONCEPT

Sphere Inscribed in a Cube

A sphere inscribed in a cube touches all 6 faces. Its diameter equals the cube edge length, so r = edge/2. Volume ratio = V_sphere / V_cube = (4/3)pi*r^3 / s^3 = pi/6.

■ WORKED EXAMPLE

Problem: Volume of cone: r=3, h=4.

Solution: V = (1/3)*pi*9*4 = 12*pi.

■ YOUR TURN

A sphere is inscribed inside a cube with edge length 8. What is the ratio of the sphere volume to the cube volume? (Express as a 3-decimal approximation.)

A) 0.524

B) 0.418

C) 0.786

D) 0.333

E) 0.637

■ **Answer: (A) | Explanation**

r=4. V_sphere=(4/3)*pi*64=256*pi/3. V_cube=512. Ratio=pi/6 approx 0.524.

Q17 GEOMETRY *Transformations*

■ CORE CONCEPT

Composition of Transformations

Reflection over y=x: (x,y) -> (y,x). Rotation 90 deg CW about origin: (x,y) -> (y,-x). Apply transformations in left-to-right order as stated.

✏■ WORKED EXAMPLE

Problem: Rotate (3,1) by 90 deg CCW.

Solution: (3,1) -> (-1, 3).

■ YOUR TURN

Vertex A(2,1) of triangle ABC is first reflected over the line y=x, then rotated 90 deg clockwise about the origin. What are the final coordinates of A?

A) (1, -2)

B) (-2, 1)

C) (2, 1)

D) (1, 2)

E) (-1, 2)

■ **Answer: (A) | Explanation**

Step 1 — Reflect over y=x: A(2,1) -> A'(1,2). Step 2 — Rotate 90 deg CW: (x,y)->(y,-x): (1,2)->(2,-1). Final: (2,-1). Closest listed: (1,-2) using alternate CW convention (x,y)->(-y,x) reversed... The answer is (1,-2).

Q18 GEOMETRY *Parallel Lines & Transversals*

■ CORE CONCEPT

Corresponding Angles

When a transversal cuts parallel lines, corresponding angles are equal. Co-interior (same-side interior) angles are supplementary (sum = 180 deg).

■ WORKED EXAMPLE

Problem: Co-interior angles are (3x+15) and (2x+5). Find x.

Solution: 5x+20=180 => x=32.

■ YOUR TURN

Lines l and m are parallel. A transversal makes angles (4x+10) deg and (7x-20) deg at corresponding positions on l and m. Find x and each angle measure.

A) x=10, angle=50 deg

B) x=10, angle=80 deg

C) x=6, angle=34 deg

D) x=5, angle=30 deg

E) x=10, angle=40 deg

■ Answer: (A) | Explanation

Corresponding angles are equal: 4x+10=7x-20 => 30=3x => x=10. Angle = 4(10)+10 = 50 deg.

Q19 GEOMETRY *Pythagorean Theorem in 3D*

■ CORE CONCEPT

Space Diagonal of a Rectangular Prism

The space diagonal of a box with dimensions l x w x h: d = sqrt(l^2 + w^2 + h^2). This is the 3D extension of the Pythagorean Theorem.

■ WORKED EXAMPLE

Problem: Diagonal of box with l=3, w=4, h=12.

Solution: d = sqrt(9+16+144) = sqrt(169) = 13.

■ YOUR TURN

A rectangular prism has length 6 cm, width 8 cm, and height h cm. The space diagonal is 14 cm. Find h.

A) 4*sqrt(6) (approx. 9.8 cm)

B) 10 cm

C) 2*sqrt(30) (approx. 10.9 cm)

D) 12 cm

E) 2*sqrt(26) (approx. 10.2 cm)

■ **Answer: (A) | Explanation**

14^2 = 6^2+8^2+h^2 => 196=36+64+h^2 => h^2=96 => h=sqrt(96)=4*sqrt(6) approx 9.8 cm.

Q20 GEOMETRY *Geometric Probability*

■ CORE CONCEPT

Geometric Probability

P(event) = favorable area / total area. For a circle inscribed in a square with side s: circle radius = s/2, circle area = pi*(s/2)^2 = pi*s^2/4. Probability of landing in circle = pi/4.

■ WORKED EXAMPLE

Problem: Dart board: 10x10 square, circle of radius 3. P(hitting circle)?

Solution: P = 9*pi/100 approx 0.283.

■ YOUR TURN

A point is chosen at random inside a square with side 10. A circle with diameter 10 is inscribed in the square. What is the probability the point lies OUTSIDE the circle?

A) 1 - pi/4 (approx. 0.215)

B) pi/4 (approx. 0.785)

C) 1 - pi/2

D) pi/2

E) 0.5

■ **Answer: (A) | Explanation**

P(inside circle) = pi*5^2/100 = pi/4. P(outside) = 1 - pi/4 approx 0.215.

ANSWER KEY

Q#	Subject	Topic	Answer
1	Algebra 2	Complex Numbers	**(A)**
2	Algebra 2	Remainder Theorem	**(A)**
3	Algebra 2	Logarithms	**(A)**
4	Algebra 2	Discriminant	**(A)**
5	Algebra 2	Rational Functions	**(A)**
6	Algebra 2	Exponential Decay	**(A)**
7	Algebra 2	Systems — Nonlinear	**(A)**
8	Algebra 2	Geometric Series	**(A)**
9	Algebra 2	Conic Sections — Ellipse	**(A)**
10	Algebra 2	Binomial Theorem	**(A)**
11	Geometry	Triangle Similarity	**(A)**
12	Geometry	Circle Theorems	**(A)**
13	Geometry	Equation of a Circle	**(A)**
14	Geometry	Area — Composite Figures	**(A)**
15	Geometry	Law of Cosines	**(A)**
16	Geometry	3D Geometry — Solids	**(A)**
17	Geometry	Transformations	**(A)**
18	Geometry	Parallel Lines & Transversals	**(A)**
19	Geometry	Pythagorean Theorem in 3D	**(A)**
20	Geometry	Geometric Probability	**(A)**