

# ■ Math Mastery Workbook

*Algebra 1 · Geometry*

*20 Core Problems — Most Commonly Missed Topics*

<b>Level</b>	High School (US Curriculum)
<b>Subjects</b>	Algebra 1 (Q1–10) + Geometry (Q11–20)
<b>Format</b>	Concept → Example → Practice Problem → Full Solution
<b>Difficulty</b>	★★ to ★★★★★ (common misconceptions focus)
<b>Name</b>	_____
<b>Date</b>	_____
<b>Score</b>	_____ / 20

## How to Use This Workbook

1. Read the Core Concept and Worked Example for each problem.
2. Attempt the practice question before looking at the solution.
3. Circle your answer choice, then check the Explanation below.
4. Mark ✓ if correct, ✗ if incorrect — review wrong problems at the end.

## Answer Key

Q#	Topic	Answer	Q#	Topic	Answer
1	Quadratic Equations	A	11	Pythagorean Theorem	B
2	Systems of Equations	C	12	Triangle Congruence	D
3	Inequalities	B	13	Circle Theorems	C
4	Functions	B	14	Similar Triangles	D
5	Polynomials	C	15	Area & Perimeter	B
6	Slope & Lines	B	16	Coordinate Geometry	A
7	Quadratic Formula	B	17	Angles & Parallel Lines	C
8	Exponent Rules	A	18	Volume	A
9	Absolute Value	D	19	Triangle Angle Theorems	B
10	Rational Expressions	A	20	Trigonometry	B

## ALGEBRA 1 — Questions 1–10

Q01

Quadratic Equations

★★★

Factoring

### ■ CORE CONCEPT

A quadratic equation has the form  $ax^2 + bx + c = 0$ . To solve by factoring, find two numbers that multiply to  $ac$  and add to  $b$ .

If  $x^2 + bx + c = (x+p)(x+q)$ , then  $p+q = b$  and  $p \cdot q = c$

### —■ WORKED EXAMPLE

Solve  $x^2 - 7x + 12 = 0$

Find two numbers that multiply to 12 and add to  $-7$ : those are  $-3$  and  $-4$ . So  $(x-3)(x-4)=0 \rightarrow x=3$  or  $x=4$ .

### ■ Solve: $x^2 - 5x - 14 = 0$ . Which values of $x$ satisfy the equation?

- (A)  $x = 7$  and  $x = -2$
- (B)  $x = -7$  and  $x = 2$
- (C)  $x = 7$  and  $x = 2$
- (D)  $x = -5$  and  $x = 14$
- (E)  $x = 5$  and  $x = -2$

### ■ SOLUTION — Correct Answer: (A) $x = 7$ and $x = -2$

Factor: find two numbers that multiply to  $-14$  and add to  $-5$ : they are  $-7$  and  $+2$ . So  $(x-7)(x+2)=0$ , giving  $x=7$  or  $x=-2$ .

Q02

Systems of Equations

★★★

Substitution

■ CORE CONCEPT

A system of linear equations has two equations with the same variables. Substitution method: solve one equation for one variable, substitute into the other.

**System:**  $y=mx+b$  and  $ax+by=c$  → substitute expression for  $y$  into second equation

■ WORKED EXAMPLE

Solve:  $y = 2x + 1$  and  $3x + y = 16$

Substitute  $y=2x+1$  into  $3x+y=16$ :  $3x+(2x+1)=16 \rightarrow 5x=15 \rightarrow x=3, y=7$ .

■ Solve:  $y = 3x - 2$  and  $2x + y = 13$ . What is  $x + y$ ?

- (A)  $x+y = 7$
- (B)  $x+y = 10$
- (C)  $x+y = 13$
- (D)  $x+y = 8$
- (E)  $x+y = 9$

■ SOLUTION — Correct Answer: (C)  $x+y = 13$

Substitute  $y=3x-2$  into  $2x+y=13$ :  $5x=15 \rightarrow x=3$ . Then  $y=7$ . So  $x+y=13$ .

Q03

Inequalities

★★

Linear Inequality

■ CORE CONCEPT

When solving a linear inequality, if you multiply or divide both sides by a negative number, you must FLIP the inequality sign.

If  $-2x > 6$ , divide by  $-2$  and flip:  $x < -3$

■ WORKED EXAMPLE

Solve:  $-3x + 5 > 14$

$-3x > 9 \rightarrow$  divide by  $-3$ , flip sign:  $x < -3$ .

■ Which value of  $x$  satisfies  $-4x + 3 \geq 19$ ?

- (A)  $x = -3$
- (B)  $x = -4$
- (C)  $x = 4$
- (D)  $x = 5$
- (E)  $x = -5$

■ SOLUTION — Correct Answer: (B)  $x = -4$

$-4x + 3 \geq 19 \rightarrow -4x \geq 16 \rightarrow x \leq -4$ . Check  $x = -4$ :  $-4(-4) + 3 = 19 \geq 19 \checkmark$

Q04

Functions

★★★

Function Evaluation

■ CORE CONCEPT

A function assigns exactly one output to each input.  $f(x)$  means "the value of  $f$  at input  $x$ ." To find  $f(a)$ , substitute  $a$  for every  $x$  in the formula.

If  $f(x) = 2x^2 - 3x + 1$ , then  $f(a) = 2a^2 - 3a + 1$

— ■ WORKED EXAMPLE

If  $g(x) = x^2 - 4$ , find  $g(-3)$

$$g(-3) = (-3)^2 - 4 = 9 - 4 = 5$$

■ If  $f(x) = 3x^2 - 2x + 5$ , what is  $f(-2)$ ?

- (A)  $f(-2) = 17$
- (B)  $f(-2) = 21$
- (C)  $f(-2) = 13$
- (D)  $f(-2) = 9$
- (E)  $f(-2) = 25$

■ SOLUTION — Correct Answer: (B)  $f(-2) = 21$

$$f(-2) = 3(-2)^2 - 2(-2) + 5 = 3(4) + 4 + 5 = 12 + 4 + 5 = 21$$

Q05

Polynomials

★★★

Difference of Squares

■ CORE CONCEPT

Difference of Squares pattern:  $a^2 - b^2 = (a+b)(a-b)$ . Look for two perfect squares being subtracted.

$$a^2 - b^2 = (a+b)(a-b)$$

— ■ WORKED EXAMPLE

Factor:  $25x^2 - 16$

$$(5x)^2 - 4^2 = (5x+4)(5x-4)$$

■ Which expression is equivalent to  $9x^2 - 49$ ?

- (A)  $(3x-7)^2$
- (B)  $(9x-7)(x+7)$
- (C)  $(3x+7)(3x-7)$
- (D)  $(3x-49)(x+1)$
- (E)  $(9x+49)(x-1)$

■ SOLUTION — Correct Answer: (C)  $(3x+7)(3x-7)$

$$9x^2 - 49 = (3x)^2 - 7^2 = (3x+7)(3x-7) \text{ by the difference of squares pattern.}$$

Q06

Slope & Lines

★★

Slope-Intercept

■ CORE CONCEPT

Slope-intercept form:  $y = mx + b$ , where  $m$  is the slope and  $b$  is the y-intercept. Parallel lines have equal slopes; perpendicular lines have slopes where  $m \cdot m = -1$ .

Slope:  $m = (y - y) / (x - x)$

■ WORKED EXAMPLE

Find the equation of the line through (2, 5) with slope 3

$y - 5 = 3(x - 2) \rightarrow y = 3x - 1$

■ A line passes through (1, 4) and (3, 10). What is its equation?

- (A)  $y = 2x + 3$
- (B)  $y = 3x + 1$
- (C)  $y = 4x - 1$
- (D)  $y = 2x + 1$
- (E)  $y = 3x - 2$

■ SOLUTION — Correct Answer: (B)  $y = 3x + 1$

Slope:  $m = (10 - 4) / (3 - 1) = 6 / 2 = 3$ . Use point (1,4):  $y - 4 = 3(x - 1) \rightarrow y = 3x + 1$ .

Q07

Quadratic Formula

★★★★

Discriminant

■ CORE CONCEPT

The quadratic formula solves any  $ax^2+bx+c=0$ :  $x = (-b \pm \sqrt{b^2-4ac}) / (2a)$ . The discriminant  $b^2-4ac$  tells number of real solutions.

$$x = (-b \pm \sqrt{b^2-4ac}) / (2a)$$

— ■ WORKED EXAMPLE

Solve  $2x^2 - 3x - 2 = 0$

$$x = (3 \pm \sqrt{9+16})/4 = (3 \pm 5)/4. \text{ So } x = 2 \text{ or } x = -1/2.$$

■ Solve  $3x^2 - 7x + 2 = 0$  using the quadratic formula.

- (A)  $x = 3$  and  $x = 2/3$
- (B)  $x = 2$  and  $x = 1/3$
- (C)  $x = 7$  and  $x = 2$
- (D)  $x = 1/3$  and  $x = -2$
- (E)  $x = 3$  and  $x = -2/3$

■ SOLUTION — Correct Answer: (B)  $x = 2$  and  $x = 1/3$

$a=3, b=-7, c=2$ . Discriminant =  $49-24 = 25$ .  $x = (7 \pm 5)/6$ . So  $x=2$  or  $x=1/3$ .

Q08

Exponent Rules

★★★

Simplification

■ CORE CONCEPT

Key rules:  $a^m \cdot a^n = a^{(m+n)}$ ;  $(a^m)^n = a^{(mn)}$ ;  $a^m/a^n = a^{(m-n)}$ ;  $a^{(-n)} = 1/a^n$ ;  $a^0 = 1$ .

$$(x^a \cdot y^b)^n = x^{(an)} \cdot y^{(bn)}$$

— ■ WORKED EXAMPLE

Simplify:  $(x^5 \cdot x^{(-2)}) / x^3$

$$x^{(5+(-2))} / x^3 = x^3/x^3 = x^0 = 1$$

■ Simplify:  $((2x^3)^2 \cdot x^{(-4)}) / (4x)$

- (A)  $x$
- (B)  $4x^2$
- (C)  $2x$
- (D)  $x^{(-1)}$
- (E)  $4x$

■ SOLUTION — Correct Answer: (A)  $x$

$(2x^3)^2 = 4x^6$ . So  $(4x^6 \cdot x^{(-4)}) / (4x) = 4x^2 / (4x) = x$ .

Q09

Absolute Value

★★★

Absolute Value  
Equation

■ CORE CONCEPT

Absolute value equation  $|ax+b|=c$  ( $c \geq 0$ ) gives two cases:  $ax+b=c$  or  $ax+b=-c$ .

$$|x| = c \rightarrow x = c \text{ or } x = -c$$

—■ WORKED EXAMPLE

Solve  $|2x - 3| = 7$

$2x-3=7 \rightarrow x=5$  or  $2x-3=-7 \rightarrow x=-2$ . Solutions:  $x=5$  or  $x=-2$ .

■ Solve  $|3x + 6| = 15$ . What is the sum of all solutions?

- (A) Sum = 3 only
- (B) Sum = -7 only
- (C) Sum = +4
- (D) Sum = -4
- (E) Sum = 0

■ SOLUTION — Correct Answer: (D) Sum = -4

Case 1:  $3x+6=15 \rightarrow x=3$ . Case 2:  $3x+6=-15 \rightarrow x=-7$ . Sum =  $3+(-7) = -4$ .

Q10

Rational Expressions

★★★★

Simplification

■ CORE CONCEPT

To simplify a rational expression, factor numerator and denominator, then cancel common factors. Only cancel FACTORS (multiplication), not terms (addition).

$$(x^2-9)/(x^2+x-6) = (x+3)(x-3)/((x+3)(x-2)) = (x-3)/(x-2)$$

■ WORKED EXAMPLE

Simplify  $(x^2-4)/(x^2-x-2)$

$$(x+2)(x-2)/((x-2)(x+1)) = (x+2)/(x+1) \text{ for } x \neq 2.$$

■ Simplify:  $(x^2 - 16) / (x^2 + 2x - 8)$

- (A)  $(x-4)/(x-2)$
- (B)  $(x+4)/(x+2)$
- (C)  $(x-4)/(x+2)$
- (D)  $(x+4)/(x-2)$
- (E)  $(x-16)/(x-8)$

■ SOLUTION — Correct Answer: (A)  $(x-4)/(x-2)$

Factor:  $(x+4)(x-4) / ((x+4)(x-2))$ . Cancel  $(x+4)$ :  $(x-4)/(x-2)$  for  $x \neq -4$ .

GEOMETRY — Questions 11–20

Q11

Pythagorean Theorem

★★★

Right Triangles

■ CORE CONCEPT

In a right triangle:  $a^2 + b^2 = c^2$  where  $c$  is the hypotenuse. Common triples: (3,4,5), (5,12,13), (8,15,17).

$a^2 + b^2 = c^2$  ( $c = \text{hypotenuse}$ )

—■ WORKED EXAMPLE

Find the hypotenuse with legs 6 and 8

$c^2 = 36+64 = 100 \rightarrow c = 10$

■ A right triangle has legs 9 and 12. What is the hypotenuse?

- (A) 14
- (B) 15
- (C) 21
- (D) 13
- (E)  $\sqrt{225} = 15$  (same as B)

■ SOLUTION — Correct Answer: (B) 15

$c^2 = 81+144 = 225 \rightarrow c = 15$ . This is a (3,4,5) triple scaled by 3.

Q12

Triangle Congruence

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Congruence Postulates

■ CORE CONCEPT

Triangles are congruent if matching sides/angles are equal. Postulates: SSS, SAS, ASA, AAS, HL. Note: SSA is NOT valid.

SSS · SAS · ASA · AAS · HL → Congruent triangles

—■ WORKED EXAMPLE

$AB=DE$ ,  $BC=EF$ , angle  $B = \text{angle } E$ . What postulate?

Two sides + included angle → SAS.

■ Two triangles have two angles and the NON-included side congruent. Which postulate applies?

- (A) SSS
- (B) SAS
- (C) ASA
- (D) AAS
- (E) HL

■ SOLUTION — Correct Answer: (D) AAS

Two angles and a non-included side → AAS. If the side were between the two angles, it would be ASA.

Q13

Circle Theorems

★★★★

Inscribed Angles

■ CORE CONCEPT

An inscribed angle equals HALF the central angle (or intercepted arc) that subtends the same arc.

$$\text{Inscribed angle} = (1/2) \times \text{intercepted arc}$$

—■ WORKED EXAMPLE

If an arc measures  $100^\circ$ , what is the inscribed angle?

$$\text{Inscribed angle} = (1/2)(100^\circ) = 50^\circ$$

■ A central angle measures  $124^\circ$ . What is the inscribed angle intercepting the same arc?

- (A)  $248^\circ$
- (B)  $124^\circ$
- (C)  $62^\circ$
- (D)  $56^\circ$
- (E)  $90^\circ$

■ SOLUTION — Correct Answer: (C)  $62^\circ$

$$\text{Inscribed angle} = \text{half the central angle} = 124^\circ/2 = 62^\circ.$$

Q14

Similar Triangles

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Proportions

■ CORE CONCEPT

Similar triangles have proportional corresponding sides. Set up a proportion with matching sides and cross-multiply to solve.

$$\text{side}\blacksquare/\text{side}\blacksquare = \text{side}\blacksquare/\text{side}\blacksquare \rightarrow \text{cross-multiply}$$

—■ WORKED EXAMPLE

■  $ABC \sim DEF$  with  $AB=6$ ,  $BC=9$ ,  $DE=4$ . Find  $EF$ .

$$6/4 = 9/EF \rightarrow EF = 6.$$

■ ■  $PQR \sim XYZ$ .  $PQ=8$ ,  $QR=12$ ,  $XY=6$ . Find  $YZ$ .

- (A)  $YZ = 10$
- (B)  $YZ = 16$
- (C)  $YZ = 8$
- (D)  $YZ = 9$
- (E)  $YZ = 6$

■ SOLUTION — Correct Answer: (D)  $YZ = 9$

$$PQ/XY = QR/YZ \rightarrow 8/6 = 12/YZ \rightarrow YZ = 72/8 = 9.$$

# Q15

Area & Perimeter

★★★

Composite Figures

## ■ CORE CONCEPT

For composite figures, divide into simpler shapes, find each area, then add or subtract. Rectangle:  $A=lw$ ; Triangle:  $A=\frac{1}{2}bh$ ; Circle:  $A=\pi r^2$ .

**Composite area = sum of individual areas**

## ■ WORKED EXAMPLE

Rectangle  $10 \times 6$  with semicircle (diameter 6) removed.

Rectangle: 60; Semicircle:  $\frac{1}{2}\pi(3)^2 = 4.5\pi$ . Area =  $60 - 4.5\pi \approx 45.86$ .

## ■ A rectangle $8 \times 5$ with a triangle (base 8, height 3) on top. Total area?

- (A) 64
- (B) 52
- (C) 60
- (D) 56
- (E) 48

## ■ SOLUTION — Correct Answer: (B) 52

Rectangle:  $8 \times 5 = 40$ . Triangle:  $\frac{1}{2}(8)(3) = 12$ . Total:  $40 + 12 = 52$ .

# Q16

Coordinate Geometry

★★★

Midpoint & Distance

## ■ CORE CONCEPT

Midpoint:  $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$ . Distance:  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

## ■ WORKED EXAMPLE

Midpoint of  $(2, -3)$  and  $(8, 5)$

$M = ((2+8)/2, (-3+5)/2) = (5, 1)$

## ■ Midpoint of AB is $M(4, 7)$ . If $A=(1, 3)$ , find B.

- (A) (7, 11)
- (B) (5, 10)
- (C) (3, 4)
- (D) (8, 14)
- (E) (6, 9)

## ■ SOLUTION — Correct Answer: (A) (7, 11)

$4 = (1 + x_B)/2 \rightarrow x_B = 7$ ;  $7 = (3 + y_B)/2 \rightarrow y_B = 11$ . So  $B = (7, 11)$ .

Q17

Angles & Parallel Lines

★★★

Transversals

■ CORE CONCEPT

When a transversal crosses parallel lines: alternate interior angles are EQUAL; corresponding angles are EQUAL; co-interior angles are SUPPLEMENTARY (sum =  $180^\circ$ ).

Alternate interior  $\angle$ s = equal | Co-interior  $\angle$ s =  $180^\circ$

■ WORKED EXAMPLE

Co-interior angle is  $65^\circ$ . Find the other co-interior angle.

$$180^\circ - 65^\circ = 115^\circ$$

■ Two parallel lines cut by a transversal. Alternate exterior angles:  $(5x+15)^\circ$  and  $(3x+45)^\circ$ . Find

x.

- (A)  $x = 10$
- (B)  $x = 12$
- (C)  $x = 15$
- (D)  $x = 18$
- (E)  $x = 20$

■ SOLUTION — Correct Answer: (C)  $x = 15$

Alternate exterior angles are equal:  $5x+15=3x+45 \rightarrow 2x=30 \rightarrow x=15$ .

Q18

Volume

★★★

3D Solids

■ CORE CONCEPT

Volume formulas: Cylinder:  $V = \pi r^2 h$ ; Cone:  $V = \frac{1}{3} \pi r^2 h$ ; Sphere:  $V = \frac{4}{3} \pi r^3$ ; Prism:  $V = lwh$ .

**Cone:**  $V = (1/3)\pi r^2 h$ ; **Cylinder:**  $V = \pi r^2 h$

— ■ WORKED EXAMPLE

*Cone with radius 3 and height 7.*

$$V = (1/3)\pi(9)(7) = 21\pi \approx 65.97$$

■ **Cylinder (radius 4, height 9) with a cone (same base & height) inside. Volume between them?**

- (A)  $96\pi$
- (B)  $144\pi$
- (C)  $48\pi$
- (D)  $192\pi$
- (E)  $72\pi$

■ **SOLUTION — Correct Answer: (A)  $96\pi$**

Cylinder:  $\pi(16)(9) = 144\pi$ . Cone:  $(1/3)(144\pi) = 48\pi$ . Difference:  $96\pi$ .

Q19

Triangle Angle Theorems

★★

Exterior Angle

■ CORE CONCEPT

Triangle Angle Sum: all interior angles =  $180^\circ$ . Exterior Angle Theorem: exterior angle = sum of the two remote interior angles.

**Exterior angle = angle A + angle B (remote interior)**

— ■ WORKED EXAMPLE

*Triangle angles  $40^\circ$  and  $75^\circ$ . Find the exterior angle at third vertex.*

$$\text{Exterior angle} = 40^\circ + 75^\circ = 115^\circ$$

■ **Two interior angles are  $(2x+10)^\circ$  and  $(3x-5)^\circ$ . Exterior angle at third vertex =  $115^\circ$ . Find x.**

- (A)  $x = 20$
- (B)  $x = 22$
- (C)  $x = 18$
- (D)  $x = 24$
- (E)  $x = 16$

■ **SOLUTION — Correct Answer: (B)  $x = 22$**

$$(2x+10) + (3x-5) = 115 \rightarrow 5x+5 = 115 \rightarrow 5x = 110 \rightarrow x = 22.$$

Q20

Trigonometry

★★★★

SOH-CAH-TOA

■ CORE CONCEPT

Right triangle trig:  $\sin \theta = \text{opp/hyp}$ ;  $\cos \theta = \text{adj/hyp}$ ;  $\tan \theta = \text{opp/adj}$ . Remember: SOH-CAH-TOA.

$$\sin \theta = \text{opp/hyp} \quad | \quad \cos \theta = \text{adj/hyp} \quad | \quad \tan \theta = \text{opp/adj}$$

— ■ WORKED EXAMPLE

Angle  $A = 30^\circ$ , hypotenuse = 10. Find the side opposite  $A$ .

$$\sin 30^\circ = \text{opp}/10 \rightarrow \text{opp} = 10 \times 0.5 = 5.$$

■ Angle  $\theta = 37^\circ$ , adjacent side = 20. Find the opposite side. (Use  $\tan 37^\circ \approx 0.75$ )

- (A) 12
- (B) 15
- (C) 18
- (D) 26.67
- (E) 10

■ SOLUTION — Correct Answer: (B) 15

$$\tan 37^\circ = \text{opp/adj} \rightarrow 0.75 = \text{opp}/20 \rightarrow \text{opp} = 20 \times 0.75 = 15.$$