COLLEGE PREP MATHEMATICS

Algebra 2 & Geometry

Master Problem Set

20	A2 + GEO	★★★
Problems	Subjects	Difficulty

01 ALGEBRA 2 ALG2 Qua

KEY CONCEPT

The discriminant b^2 - 4ac tells you how many real solutions a quadratic has:
b^2 - 4ac > 0 => two distinct real roots
b^2 - 4ac = 0 => one repeated real root
b^2 - 4ac < 0 => no real roots (two complex roots)

WORKED EXAMPLE

For 2x^2 - 4x + 2 = 0:
discriminant = (-4)^2 - 4(2)(2) = 16 - 16 = 0 => one real root x = 1

Q1. How many real solutions does 3x^2 - 5x + 4 = 0 have?

A	Two distinct real solutions
B	Exactly one real solution
C	No real solutions (two complex roots)
D	Infinitely many solutions

Correct Answer: C

STEP-BY-STEP EXPLANATION

Discriminant = (-5)^2 - 4(3)(4) = 25 - 48 = -23.
Since -23 < 0, there are no real solutions — two complex (imaginary) roots.

02 ALGEBRA 2 ALG 2 Com

KEY CONCEPT

Powers of i cycle every 4:
$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
To evaluate i^n, compute the remainder when n is divided by 4.

WORKED EXAMPLE

i^{23} : 23 mod 4 = 3 => $i^{23} = i^3 = -i$

Q2. What is i^{58} ?

A	1
B	-1
C	i
D	-i

Correct Answer: B

STEP-BY-STEP EXPLANATION

58 / 4 = 14 remainder 2.
So $i^{58} = i^2 = -1$.

03 ALGEBRA 2

ALG 2 Rem

KEY CONCEPT

When f(x) is divided by (x - a), the remainder equals f(a).
If f(a) = 0, then (x - a) is a factor of f(x).

WORKED EXAMPLE

Remainder when x^3 - 2x + 1 is divided by (x - 2):
f(2) = 8 - 4 + 1 = 5

Q3. What is the remainder when f(x) = 2x^3 - 3x^2 + x - 5 is divided by (x - 2)?

A	1
B	3
C	5
D	7

Correct Answer: A

STEP-BY-STEP EXPLANATION

By the Remainder Theorem, evaluate f(2):
2(8) - 3(4) + 2 - 5 = 16 - 12 + 2 - 5 = 1

04 ALGEBRA 2 ALG 2 Loga

KEY CONCEPT

Key log laws:
log_b(MN) = log_b M + log_b N
log_b(M/N) = log_b M - log_b N
log_b(M^k) = k * log_b M

WORKED EXAMPLE

Solve log_2(x - 1) = 3:
x - 1 = 2^3 = 8 => x = 9

Q4. Solve: log_3(x - 2) + log_3(x + 4) = 3

A	x = 5
B	x = 7
C	x = 4
D	x = 6

Correct Answer: A

STEP-BY-STEP EXPLANATION

Combine logs: log_3[(x-2)(x+4)] = 3
(x-2)(x+4) = 27
x^2 + 2x - 8 = 27 => x^2 + 2x - 35 = 0
(x + 7)(x - 5) = 0
Since x > 2, the answer is x = 5.

05 ALGEBRA 2

ALG 2 Geo

KEY CONCEPT

Sum of first n terms of a geometric series:
$S_n = a_1 * (1 - r^n) / (1 - r)$
where a_1 is the first term and r is the common ratio.

WORKED EXAMPLE

For 2, 6, 18, 54 ... with $a_1=2$, $r=3$:
$S_4 = 2(1 - 81) / (1 - 3) = (-160)/(-2) = 80$

Q5. Find the sum of the first 6 terms of the geometric series: 3, 6, 12, 24, ...

A	189
B	186
C	192
D	180

Correct Answer: A

STEP-BY-STEP EXPLANATION

$a_1 = 3$, $r = 2$
$S_6 = 3(1 - 2^6) / (1 - 2) = 3(1 - 64) / (-1) = 3 * 63 = 189$

06 ALGEBRA 2 ALG 2 Rati

KEY CONCEPT

a^(m/n) = (n-th root of a)^m
Negative exponents: a^(-n) = 1 / a^n

WORKED EXAMPLE

8^(2/3) = (cube root of 8)^2 = 2^2 = 4

Q6. Simplify: (27/8)^(-2/3)

A	4/9
B	9/4
C	2/3
D	3/2

Correct Answer: A

STEP-BY-STEP EXPLANATION

(27/8)^(2/3) = 27^(2/3) / 8^(2/3) = (3)^2 / (2)^2 = 9/4
With the negative exponent, take the reciprocal: 4/9

07 ALGEBRA 2 ALG 2 Inve

KEY CONCEPT

To find f^(-1)(x):
1. Replace f(x) with y
2. Swap x and y
3. Solve for y

WORKED EXAMPLE

f(x) = 2x - 3 => swap: x = 2y - 3 => y = (x+3)/2
So f^(-1)(x) = (x + 3) / 2

Q7. If f(x) = (3x + 1) / (x - 2), find f^(-1)(x).

A	(2x + 1) / (x - 3)
B	(x - 1) / (3 - x)
C	(3x - 2) / (x + 1)
D	(x + 2) / (3 - x)

Correct Answer: A

STEP-BY-STEP EXPLANATION

Set y = (3x+1)/(x-2). Swap x and y:
x = (3y+1)/(y-2)
x(y-2) = 3y+1
xy - 3y = 2x+1
y(x-3) = 2x+1
y = (2x+1)/(x-3)

08 ALGEBRA 2

ALG 2 Abs

KEY CONCEPT

|ax + b| = c (c > 0) splits into:
ax + b = c OR ax + b = -c
|expression| = expression^2 approach works for squared cases.

WORKED EXAMPLE

|2x - 1| = 5 => 2x-1=5 (x=3) or 2x-1=-5 (x=-2)

Q8. How many solutions does |2x^2 - 5| = 3 have?

A	1
B	2
C	3
D	4

Correct Answer: D

STEP-BY-STEP EXPLANATION

Case 1: 2x^2 - 5 = 3 => x^2 = 4 => x = +/-2 (2 solutions)
Case 2: 2x^2 - 5 = -3 => x^2 = 1 => x = +/-1 (2 solutions)
Total: 4 solutions {-2, -1, 1, 2}

09 ALGEBRA 2

ALG 2 Syst

KEY CONCEPT

Use elimination or substitution to solve 2x2 linear systems.
Multiply equations to align coefficients, then add/subtract.

WORKED EXAMPLE

$x + y = 5$, $2x - y = 1$ => Add: $3x = 6$ => $x=2$, $y=3$

Q9. Solve the system: $3x - 2y = 8$ and $5x + 4y = 6$. What is x?

A	$x = 2$
B	$x = 3$
C	$x = 4$
D	$x = 1$

Correct Answer: A

STEP-BY-STEP EXPLANATION

Multiply eq.1 by 2: $6x - 4y = 16$
Add to eq.2: $11x = 22$ => $x = 2$
Check: $y = (3(2)-8)/2 = -1$; verify $5(2)+4(-1)=6$ checkmark

10 ALGEBRA 2

KEY CONCEPT

Half-life model: A = A_0 * (1/2)^(t/h)
where h = half-life period, t = elapsed time.

WORKED EXAMPLE

80g remains after 30 years (half-life 10 yr):
80 = A_0 * (1/2)^3 => A_0 = 640g

Q10. A radioactive substance has a half-life of 12 years. Starting with 96 grams, how many grams remain after 36 years?

A	8 g
B	12 g
C	16 g
D	24 g

Correct Answer: B

STEP-BY-STEP EXPLANATION

36 years = 3 half-lives (36 / 12 = 3)
96 * (1/2)^3 = 96 * (1/8) = 12 g

11 GEOMETRY

GEO Tria

KEY CONCEPT

Two triangles are similar (AA) when two pairs of angles are equal.
Corresponding sides are proportional: a/a' = b/b' = c/c'

WORKED EXAMPLE

If triangle ABC ~ triangle DEF with ratio 2:3, and AB=8, then DE=12.

Q11. In triangles ABC and DEF, angle A = angle D = 55 and angle B = angle E = 70. If AB = 9, BC = 12, and DE = 15, what is EF?

A	18
B	20
C	16
D	22

Correct Answer: B

STEP-BY-STEP EXPLANATION

By AA, the triangles are similar.
Ratio = DE/AB = 15/9 = 5/3
EF = BC * (5/3) = 12 * 5/3 = 20

12 GEOMETRY

Insc GEO

KEY CONCEPT

An inscribed angle is HALF its intercepted arc.
A central angle EQUALS its intercepted arc.
Angles inscribed in a semicircle = 90 degrees.

WORKED EXAMPLE

If arc AB = 120 deg, inscribed angle from C = 120/2 = 60 deg.

Q12. A central angle intercepts an arc of 148 degrees. What is the inscribed angle that intercepts the same arc?

A	74 deg
B	148 deg
C	106 deg
D	64 deg

Correct Answer: A

STEP-BY-STEP EXPLANATION

Inscribed angle = (1/2) * intercepted arc
= 148 / 2 = 74 degrees

13 GEOMETRY

GEO Righ

KEY CONCEPT

SOH-CAH-TOA:
sin(theta) = opposite / hypotenuse
cos(theta) = adjacent / hypotenuse
tan(theta) = opposite / adjacent

WORKED EXAMPLE

Right triangle, hypotenuse=10, angle=30 deg:
opposite = 10 * sin(30) = 10 * 0.5 = 5

Q13. In right triangle PQR with right angle at Q, PQ = 7 and QR = 24. What is sin(angle P)?

A	7/25
B	24/25
C	7/24
D	24/7

Correct Answer: B

STEP-BY-STEP EXPLANATION

Hypotenuse PR = sqrt(7^2 + 24^2) = sqrt(49 + 576) = sqrt(625) = 25
From angle P: opposite = QR = 24, hypotenuse = PR = 25
sin(P) = 24/25

14 GEOMETRY Area GEO

KEY CONCEPT

Regular hexagon with side s:
Area = (3 * sqrt(3) / 2) * s^2
General regular n-gon: A = (n*s^2/4) * cot(pi/n)

WORKED EXAMPLE

Regular hexagon with side 4:
A = (3*sqrt(3)/2)*(16) = 24*sqrt(3) approx 41.6

Q14. What is the area of a regular hexagon with side length 6?

A	54*sqrt(3)
B	36*sqrt(3)
C	72*sqrt(3)
D	48*sqrt(3)

Correct Answer: A

STEP-BY-STEP EXPLANATION

A = (3*sqrt(3)/2) * s^2 = (3*sqrt(3)/2) * 36 = 54*sqrt(3) approx 93.5 sq units

15 GEOMETRY

Volu GEO

KEY CONCEPT

Cylinder: V = pi*r^2*h
Cone: V = (1/3)*pi*r^2*h
Hemisphere: V = (2/3)*pi*r^3

WORKED EXAMPLE

Cylinder r=3, h=10 + Cone r=3, h=4:
V = 90*pi + 12*pi = 102*pi

Q15. A solid has a cylinder (radius 5, height 8) topped with a cone (same radius, height 6). Find the total volume in terms of pi.

A	250*pi
B	225*pi
C	275*pi
D	300*pi

Correct Answer: A

STEP-BY-STEP EXPLANATION

Cylinder: pi*(25)*(8) = 200*pi
Cone: (1/3)*pi*(25)*(6) = 50*pi
Total = 200*pi + 50*pi = 250*pi

16 GEOMETRY Perp

KEY CONCEPT

The perpendicular bisector of AB:
1. Passes through the midpoint of AB
2. Has slope = negative reciprocal of slope of AB

WORKED EXAMPLE

A(0,0), B(4,0): midpoint=(2,0), slope of AB=0
Perpendicular bisector: x = 2

Q16. Find the equation of the perpendicular bisector of the segment joining A(1, 3) and B(7, -1).

A	3x - 2y = 10
B	3x - 2y = 11
C	2x - 3y = 5
D	x + y = 8

Correct Answer: A

STEP-BY-STEP EXPLANATION

Midpoint M = ((1+7)/2, (3-1)/2) = (4, 1)
Slope of AB = (-1-3)/(7-1) = -4/6 = -2/3
Perpendicular slope = 3/2
Line: y - 1 = (3/2)(x - 4) => 2y - 2 = 3x - 12 => 3x - 2y = 10

17 GEOMETRY Rota GEO

KEY CONCEPT

90 deg CCW: (x, y) => (-y, x)
180 deg: (x, y) => (-x, -y)
270 deg CCW (= 90 deg CW): (x, y) => (y, -x)

WORKED EXAMPLE

Rotate (3, -2) by 90 deg CCW: (3,-2) => (2, 3)

Q17. Point P(4, -3) is rotated 270 degrees counterclockwise about the origin. What are the coordinates of the image?

A	(-3, -4)
B	(3, 4)
C	(-4, 3)
D	(4, 3)

Correct Answer: A

STEP-BY-STEP EXPLANATION

270 deg CCW = 90 deg CW.
Rule for 90 deg CW: (x, y) => (y, -x)
P(4, -3) => (-3, -(4)) = (-3, -4)

18 GEOMETRY Trian GEO

KEY CONCEPT

Congruence shortcuts: SSS, SAS, ASA, AAS, HL (right triangles only).
SSA and AAA are NOT valid congruence shortcuts.

WORKED EXAMPLE

Two triangles with two angles and the included side equal => ASA congruence.

Q18. In the figure, AB is parallel to CD, and E is the midpoint of BD. Which theorem proves triangle ABE congruent to triangle CDE?

- **A** SSS
- **B** SAS
- **C** ASA
- **D** AAS

Correct Answer: C

STEP-BY-STEP EXPLANATION

AB || CD => alternate interior angles: angle ABE = angle DCE and angle BAE = angle CDE
E is midpoint of BD => BE = DE (included side between the two angles)
Two angles + included side => ASA

19 GEOMETRY Con GEO

KEY CONCEPT

Cone with base radius r and slant height l:
Lateral SA = pi * r * l
Total SA = pi * r * l + pi * r^2 = pi*r*(l + r)

WORKED EXAMPLE

Cone r=3, l=5: Lateral SA = pi*3*5 = 15*pi; Total = 15*pi + 9*pi = 24*pi

Q19. A right circular cone has base radius 5 and slant height 13. What is the total surface area?

A	90*pi
B	65*pi
C	80*pi
D	100*pi

Correct Answer: A

STEP-BY-STEP EXPLANATION

Lateral SA = pi*(5)*(13) = 65*pi
Base area = pi*(5)^2 = 25*pi
Total SA = 65*pi + 25*pi = 90*pi

20 GEOMETRY Law GEO

KEY CONCEPT

c^2 = a^2 + b^2 - 2ab*cos(C)
Use when you know: SAS (two sides + included angle) or SSS.

WORKED EXAMPLE

a=7, b=8, C=60 deg:
c^2 = 49 + 64 - 2(7)(8)(0.5) = 57 => c = sqrt(57)

Q20. In triangle XYZ, XY = 10, YZ = 14, and angle Y = 60 degrees. Find XZ.

A	2*sqrt(39)
B	sqrt(156)
C	4*sqrt(13)
D	2*sqrt(41)

Correct Answer: A

STEP-BY-STEP EXPLANATION

By Law of Cosines (XZ^2 = XY^2 + YZ^2 - 2*XY*YZ*cos Y):
XZ^2 = 100 + 196 - 2(10)(14)(0.5) = 296 - 140 = 156
XZ = sqrt(156) = sqrt(4*39) = 2*sqrt(39)

Answer Key

No.	Answer	No.	Answer	No.	Answer	No.	Answer	No.	Answer
1	Discriminant = (-5)^2 - 4(3)(4) = 25 - 48 = -23 Since -23 < 0, there are no real solutions –– two complex (imaginary) roots.	2	[illegible]	3	By the Remainder Theorem, evaluate f(2): [illegible] = 16 - 12 + [illegible] = 5		Combine logs: log_3[(x-2)(x+4)] = 3 (x-2)(x+4) = 27 x^2 + 2x - 8 = 27 => x^2 + 2x - 35 = 0 (x + 7)(x - 5) = 0 Since x > 2, the answer is x = 5.		a_1 = 3, r = 2 S_6 = 3(1 - 2^6) / (1 - 2) = 3(1 - 64) / (-1) = 3 * 63 = 189		
6	(27/8)^(2/3) = 27^(2/3) / 8^(2/3) = (3)^2/(2)^2 = 9/4 With the negative exponent, take the reciprocal: 4/9		Set y = (3x+1)/(x-2). Swap x and y: x = (3y+1)/(y-2) x(y-2) = 3y+1 xy - 3y = 2x+1 y(x-3) = 2x+1 y = (2x+1)/(x-3)		Case 1: 2x^2 - 5 = 3 => x^2 = 4 => x = ±2 Case 2: 2x^2 - 5 = -3 => x^2 = 1 => x = ±1 Total: 4 solutions		[illegible] Checking: (3(2)-8)/2 = -1; verify 5(2)+4(-1)=6 checkmark		36 years = 3 half-lives (36 / 12 = 3) 96 * (1/2)^3 = 96 * (1/8) = 12 g
11	By AA, the triangles are similar. Ratio = DE/AB = 15/9 = 5/3 EF = BC * (5/3) = 12 * 5/3 = 20		Inscribed angle = (1/2) * intercepted arc = 148 / 2 = 74 degrees		Hypotenuse PR = sqrt(7^2 + 24^2) = sqrt(49 + 576) = sqrt(625) = 25 sin(P) = 24/25		[illegible]		Cylinder: pi*(25)*(8) = 200*pi [illegible] ≈ 93.5 sq units Total = 200*pi + 50*pi = 250*pi
16	Midpoint M = ((1+7)/2, (3-1)/2) = (4, 1) Slope of AB = (-1-3)/(7-1) = -4/6 = -2/3 Perpendicular slope = 3/2 Line: y - 1 = (3/2)(x - 4) => 2y - 2 = 3x - 12 => 3x - 2y = 10		AB || CD => [illegible] P(4, -3) => (-3, -4)		[illegible]		[illegible] Total SA = 65*pi + 25*pi = 90*pi		[illegible] XZ^2 = XY^2 + YZ^2 - 2*XY*YZ*cos Y): [illegible](14)(0.5) = 296 - 140 = 156 XZ = sqrt(156) = sqrt(4*39) = 2*sqrt(39)

This workbook covers the most frequently missed topics in Algebra 2 and Geometry. Review each concept box carefully before attempting similar problems.