

# SAT MATH

## Advanced Practice Workbook

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### 20 Challenging Free-Response Problems

**All Units · Concept Review · Worked Examples · Full Solutions**

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Algebra	Systems of Equations	1
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# How to Use This Workbook

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## 1. CONCEPT REVIEW

Each question begins with key concepts and formulas to memorize. Read these carefully before attempting the problem.

## 2. WORKED EXAMPLE

A guided example shows the technique in action. Try to solve it yourself before reading the solution.

## 3. PRACTICE PROBLEM

Solve the free-response question on your own. Write your answer below the question.

## 4. ANSWER KEY

Full worked solutions appear at the back of this booklet. Check your work and understand any errors before moving on.

### IMPORTANT NOTES

- All 20 questions are free-response (grid-in) format — no multiple choice.
- Questions reflect the hardest 20–30% of SAT Math (difficulty level 4–5).
- No calculator is assumed unless a calculator symbol appears.
- Time guideline: approximately 3–5 minutes per question.

## Question 1

### 1. Heart of Algebra — Systems of Equations

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- System of linear equations: solve by substitution or elimination.
- Consistent system: one solution (lines intersect).
- Inconsistent system: no solution (parallel lines).
- Dependent system: infinitely many solutions (same line).

MEMORIZE: If  $ax + by = c$  is parallel to  $dx + ey = f$ , then  $a/d = b/e \neq c/f$ .

##### Example

If  $3x - y = 7$  and  $x + 2y = 0$ , find  $x + y$ .

**Solution:** From eq.2:  $x = -2y$ . Sub:  $3(-2y) - y = 7 \rightarrow -7y = 7 \rightarrow y = -1$ ,  $x = 2$ . Answer:  $x + y = 1$

#### PROBLEM 1

The system below has no solution.

$$6x - 4y = 10$$

$$kx - 6y = 15$$

What is the value of  $k$ ?

Answer: \_\_\_\_\_

## Question 2

### 2. Heart of Algebra — Linear Inequalities & Absolute Value

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- $|ax + b| < c$  ■  $-c < ax + b < c$
- $|ax + b| > c$  ■  $ax + b < -c$  OR  $ax + b > c$
- Flip inequality sign when multiplying/dividing by a negative.

MEMORIZE:  $|x - a| \leq r$  describes all  $x$  within distance  $r$  from  $a$ .

##### Example

$|2x - 3| \leq 7$ . Find the number of integers in the solution set.

**Solution:**  $-7 \leq 2x - 3 \leq 7 \rightarrow -4 \leq 2x \leq 10 \rightarrow -2 \leq x \leq 5$ . Integers:  $-2, -1, 0, 1, 2, 3, 4, 5 \rightarrow 8$  integers

#### PROBLEM 2

How many integer values of  $x$  satisfy  $|3x - 2| < 14$  AND  $x > -3$ ?

Answer: \_\_\_\_\_

### Question 3

## 3. Passport to Advanced Math — Quadratic Equations

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### CONCEPT REVIEW

#### KEY CONCEPTS

- Quadratic formula:  $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$
- Discriminant  $D = b^2 - 4ac$ :  $D > 0$  (2 real roots),  $D = 0$  (1 repeated),  $D < 0$  (no real).
- Sum of roots =  $-b/a$ , Product of roots =  $c/a$  (Vieta's formulas).

MEMORIZE:  $(x - r)(x - s) = x^2 - (r+s)x + rs$

#### Example

If  $x^2 - 5x + k = 0$  has exactly one real solution, find  $k$ .

**Solution:**  $D = 25 - 4k = 0 \rightarrow k = 25/4$

### PROBLEM 3

The equation  $x^2 + bx + 36 = 0$  has two equal real roots.

If  $b > 0$ , what is the value of  $b$ ?

Answer: \_\_\_\_\_

## Question 4

### 4. Passport to Advanced Math — Polynomial Functions

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Factor theorem:  $(x - a)$  is a factor iff  $f(a) = 0$ .
- Remainder theorem:  $f(a)$  = remainder when  $f(x) \div (x - a)$ .
- Degree  $n$  polynomial has at most  $n$  roots and  $n-1$  turning points.

MEMORIZE: End behavior follows the leading term  $ax^n$ .

##### Example

$f(x) = x^3 - 2x^2 - 5x + 6$ . If  $x = 3$  is a root, factor  $f(x)$  completely.

**Solution:**  $f(3) = 0$  ✓. Divide:  $f(x) = (x-3)(x^2+x-2) = (x-3)(x+2)(x-1)$ .

#### PROBLEM 4

The polynomial  $p(x) = x^3 + ax^2 - 7x + b$  has a factor of  $(x - 2)$  and leaves a remainder of 12 when divided by  $(x + 1)$ .

What is the value of  $a + b$ ?

Answer: \_\_\_\_\_

## Question 5

### 5. Passport to Advanced Math — Rational & Radical Equations

#### CONCEPT REVIEW

##### KEY CONCEPTS

- For rational equations: find LCD, clear denominators, check extraneous solutions.
- For radical equations: isolate radical, square both sides, check solutions.
- Extraneous solution: satisfies the transformed equation but not the original.

MEMORIZE: Always substitute answers back to verify.

##### Example

Solve:  $\sqrt{2x + 3} = x - 1$

**Solution:** Square:  $2x+3 = x^2-2x+1 \rightarrow x^2-4x-2+\dots \rightarrow x^2-4x-2=1 \rightarrow (x-5)(x+\dots) \rightarrow x=5$   
(check:  $\sqrt{13}=4$ ? No; careful:  $2(5)+3=13$ ,  $\sqrt{13}\neq 4$ ). Correct:  $x^2-4x-2=0$ :  
 $x=(4\pm\sqrt{24})/2=2\pm\sqrt{6}$ . Check  $x=2+\sqrt{6}$  ✓,  $x=2-\sqrt{6} < 1$  (extraneous). Answer:  $2+\sqrt{6}$

#### PROBLEM 5

Solve for  $x$ :  $(x + 3)/(x - 2) - 2/(x + 2) = (x^2 + 6)/(x^2 - 4)$

List all valid solutions. (Give the integer answer if it is a whole number.)

Answer: \_\_\_\_\_

## Question 6

### 6. Problem Solving & Data Analysis — Ratios, Rates & Proportions

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Unit rate = quantity / 1 unit. Set up proportion to solve.
- Percent change =  $(\text{new} - \text{old}) / \text{old} \times 100\%$ .
- Concentration problems: amount of solute = concentration  $\times$  volume.

MEMORIZE: mixture formula:  $c_1 \cdot v_1 + c_2 \cdot v_2 = c_{\text{final}} \cdot (v_1 + v_2)$ .

##### Example

A solution is 30% acid. How many liters of pure water must be added to 20 L to make a 20% acid solution?

**Solution:**  $0.30 \times 20 = 0.20 \times (20 + x) \rightarrow 6 = 4 + 0.2x \rightarrow x = 10$  liters

#### PROBLEM 6

A 60-liter solution is 45% alcohol. How many liters of pure alcohol must be added so that the resulting solution is 60% alcohol?

Answer: \_\_\_\_\_

## Question 7

### 7. Problem Solving & Data Analysis — Statistics: Mean, Median, SD

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Mean is sensitive to outliers; median is resistant.
- Standard deviation measures spread. Larger SD = more spread.
- Adding constant  $k$  to all values: mean shifts by  $k$ , SD unchanged.
- Multiplying all values by  $k$ : mean multiplies by  $k$ , SD multiplies by  $k$ .

MEMORIZE: Normal distribution — 68% within 1 SD, 95% within 2 SD.

##### Example

Data set: 2, 4, 6, 8, 10. A value of 100 is added. How does the mean and median change?

**Solution:** Old mean=6, new mean= $(30+100)/6=21.7$  (increases). Old median=6, new median= $(6+8)/2=7$  (slight increase).

#### PROBLEM 7

A data set of 8 numbers has a mean of 15 and a standard deviation of 3.

If each number is multiplied by 2 and then 4 is subtracted, what is the new mean and new standard deviation?

Answer: \_\_\_\_\_

## Question 8

### 8. Problem Solving & Data Analysis — Scatterplots & Lines of Best Fit

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Line of best fit (regression line): minimizes sum of squared residuals.
- Correlation coefficient  $r$ :  $-1 \leq r \leq 1$ .  $r$  close to  $\pm 1$  = strong linear.
- Residual = actual value – predicted value.
- Extrapolation beyond the data range is unreliable.

MEMORIZE: Correlation does NOT imply causation.

##### Example

A line of best fit is  $y = 2.4x + 5$ . Predict  $y$  when  $x = 10$ .

**Solution:**  $y = 2.4(10) + 5 = 29$ . Note: only reliable if  $x=10$  is within data range.

#### PROBLEM 8

A study shows the regression equation: score =  $3.2 \times$  (hours studied) + 48.

A student studied 7 hours and scored 74. What is the residual for this student?

Answer: \_\_\_\_\_

## Question 9

### 9. Advanced Math — Exponential Growth & Decay

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Exponential growth:  $f(t) = a \cdot (1 + r)^t$
- Exponential decay:  $f(t) = a \cdot (1 - r)^t$
- Half-life formula:  $A(t) = A_0 \cdot (1/2)^{t/h}$
- Continuous:  $A(t) = A_0 \cdot e^{rt}$

MEMORIZE: For SAT, identify a (initial), b (base/growth factor), t (time).

##### Example

A culture of 500 bacteria doubles every 3 hours. How many after 9 hours?

**Solution:**  $f(9) = 500 \cdot 2^{9/3} = 500 \cdot 2^3 = 4000$  bacteria

#### PROBLEM 9

A radioactive isotope has a half-life of 8 years. A sample starts at 320 grams.

After how many years will there be exactly 10 grams remaining?

Answer: \_\_\_\_\_

## Question 10

### 10. Advanced Math — Functions: Transformations & Composition

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- $f(x) + k$ : vertical shift up  $k$
- $f(x + h)$ : horizontal shift left  $h$
- $-f(x)$ : reflect over  $x$ -axis;  $f(-x)$ : reflect over  $y$ -axis
- $k \cdot f(x)$ : vertical stretch by  $k$ ;  $f(kx)$ : horizontal compression by  $1/k$

MEMORIZE:  $(f \circ g)(x) = f(g(x))$  — apply  $g$  first, then  $f$ .

##### Example

If  $f(x) = x^2 + 1$  and  $g(x) = 2x - 3$ , find  $f(g(2))$ .

**Solution:**  $g(2) = 1$ .  $f(1) = 1 + 1 = 2$ .

#### PROBLEM 10

Let  $f(x) = \sqrt{x + 4}$  and  $g(x) = x^2 - 2x$ .

Find all values of  $x$  for which  $f(g(x)) = 3$ .

Answer: \_\_\_\_\_

## Question 11

### 11. Geometry — Circles: Arcs, Sectors & Equations

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Arc length =  $(\theta/360) \times 2\pi r$ ; Sector area =  $(\theta/360) \times \pi r^2$
- Standard form:  $(x - h)^2 + (y - k)^2 = r^2$
- Complete the square to convert general form to standard form.
- Inscribed angle = half the central angle subtending the same arc.

MEMORIZE: A tangent is perpendicular to the radius at the point of tangency.

##### Example

Circle:  $x^2 + y^2 - 6x + 4y = 12$ . Find center and radius.

**Solution:**  $(x-3)^2 + (y+2)^2 = 25$ . Center  $(3, -2)$ , radius 5.

#### PROBLEM 11

$x^2 + y^2 - 10x + 6y + k = 0$  is a circle with radius 7.

What is the value of  $k$ ?

Answer: \_\_\_\_\_

## Question 12

### 12. Geometry — Triangles: Similar, Special & Trigonometry

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- $\sin \theta = \text{opp/hyp}$ ,  $\cos \theta = \text{adj/hyp}$ ,  $\tan \theta = \text{opp/adj}$  (SOH-CAH-TOA)
- Special triangles: 30-60-90 (sides  $1 : \sqrt{3} : 2$ ); 45-45-90 ( $1 : 1 : \sqrt{2}$ )
- $\sin^2 \theta + \cos^2 \theta = 1$  (Pythagorean identity)
- $\sin(90^\circ - \theta) = \cos \theta$  (co-function identity)

MEMORIZE: In similar triangles, corresponding sides are proportional.

##### Example

In a right triangle,  $\sin \theta = 5/13$ . Find  $\cos \theta$  and  $\tan \theta$ .

**Solution:**  $\text{adj} = \sqrt{169-25} = 12$ .  $\cos \theta = 12/13$ ,  $\tan \theta = 5/12$ .

#### PROBLEM 12

In right triangle ABC, angle C =  $90^\circ$ , BC = 7, and  $\sin A = 7/25$ .

What is the length of AC?

Answer: \_\_\_\_\_

## Question 13

### 13. Geometry — Volume & Surface Area (3D Solids)

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Cylinder:  $V = \pi r^2 h$ ;  $SA = 2\pi r^2 + 2\pi r h$
- Cone:  $V = (1/3)\pi r^2 h$
- Sphere:  $V = (4/3)\pi r^3$ ;  $SA = 4\pi r^2$
- Similar solids with ratio  $k$ : areas scale by  $k^2$ , volumes by  $k^3$ .

MEMORIZE: SAT often combines solids (e.g., cylinder + hemisphere).

##### Example

A cone has base radius 6 and height 8. Find its volume.

**Solution:**  $V = (1/3)\pi(36)(8) = 96\pi$

#### PROBLEM 13

A sphere is inscribed in a cube of side length 6.

What is the volume of the region inside the cube but outside the sphere?

Express your answer in terms of  $\pi$ .

Answer: \_\_\_\_\_

## Question 14

### 14. Advanced Math — Complex Numbers

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- $i = \sqrt{-1}$ ,  $i^2 = -1$ ,  $i^3 = -i$ ,  $i^4 = 1$  (cycle of 4)
- $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
- Complex conjugate of  $(a + bi)$  is  $(a - bi)$ ; product =  $a^2 + b^2$
- To divide: multiply numerator and denominator by conjugate.

MEMORIZE:  $i^n$  repeats with period 4. Find remainder of  $n \div 4$ .

##### Example

Simplify:  $(3 + 2i)(1 - 4i)$

**Solution:**  $= 3 - 12i + 2i - 8i^2 = 3 - 10i + 8 = 11 - 10i$

#### PROBLEM 14

If  $(4 + ki)(4 - ki) = 97$ , where  $k$  is a real number, what is the positive value of  $k$ ?

Answer: \_\_\_\_\_

## Question 15

### 15. Advanced Math — Logarithms & Exponents

#### CONCEPT REVIEW

##### KEY CONCEPTS

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b(x/y) = \log_b x - \log_b y$
- $\log_b(x^n) = n \cdot \log_b x$
- $b^{\log_b x} = x$  and  $\log_b(b^x) = x$

MEMORIZE:  $\ln = \log_e$ . Change of base:  $\log_b x = \ln x / \ln b$ .

##### Example

Solve:  $2^{x+1} = 8^{x-1}$

**Solution:**  $2^{x+1} = 2^{3(x-1)} \rightarrow x+1 = 3x-3 \rightarrow 2x = 4 \rightarrow x = 2$

#### PROBLEM 15

Solve for  $x$ :  $\log_2(x + 6) + \log_2(x - 2) = 5$

(Give your answer as an integer.)

Answer: \_\_\_\_\_

## Question 16

### 16. Advanced Math — Parametric & Modeling

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Word problem models: identify the variable, write equation, solve.
- Rate  $\times$  Time = Distance; Work rate: combined rate =  $1/t_1 + 1/t_2$
- Compound interest:  $A = P(1 + r/n)^{nt}$
- Always define variables explicitly before writing equations.

MEMORIZE: 'At most'  $\rightarrow \leq$ ; 'At least'  $\rightarrow \geq$ ; 'exceeds'  $\rightarrow >$

##### Example

Pipe A fills a tank in 4 h; Pipe B empties it in 6 h. Both open: how long to fill from empty?

**Solution:** Net rate =  $1/4 - 1/6 = 1/12$ . Time = 12 hours.

#### PROBLEM 16

Machine X produces 200 widgets per hour; Machine Y produces 150 per hour.

Machine X starts 2 hours before Machine Y. After Machine Y starts, how many total hours (from when Y starts) until a combined total of 2500 widgets are produced?

Answer: \_\_\_\_\_

## Question 17

### 17. Data Analysis — Probability & Conditional Probability

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- $P(A \text{ and } B) = P(A) \times P(B)$  [independent events]
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Conditional:  $P(A|B) = P(A \text{ and } B) / P(B)$
- For two-way tables: read carefully which 'given' condition is the denominator.

MEMORIZE: Independent ■  $P(A|B) = P(A)$ .

##### Example

In a class, 60% study math, 40% study science, 25% study both. Find  $P(\text{math} | \text{science})$ .

**Solution:**  $P(M|S) = P(M \text{ and } S)/P(S) = 0.25/0.40 = 0.625 = 62.5\%$

#### PROBLEM 17

A bag has 5 red and 7 blue marbles. Two are drawn without replacement.

What is the probability that both marbles are the same color?

Express as a simplified fraction.

Answer: \_\_\_\_\_

## Question 18

### 18. Geometry — Coordinate Geometry & Analytic Geometry

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Midpoint:  $M = ((x_1+x_2)/2, (y_1+y_2)/2)$
- Distance:  $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
- Perpendicular bisector: passes through midpoint, slope =  $-1/m_{\text{original}}$
- Two lines are parallel if slopes equal; perpendicular if slopes are negative reciprocals.

MEMORIZE: A point  $(x, y)$  is on a line iff it satisfies the line's equation.

##### Example

Find the equation of the perpendicular bisector of segment joining  $(2, 3)$  and  $(6, 1)$ .

**Solution:** Midpoint  $(4,2)$ . Slope of segment =  $-1/2$ . Perp. slope =  $2$ . Line:  $y - 2 = 2(x - 4) \rightarrow y = 2x - 6$

#### PROBLEM 18

Points  $A(1, 2)$ ,  $B(5, 8)$ , and  $C(k, 0)$  are collinear.

What is the value of  $k$ ?

Answer: \_\_\_\_\_

## Question 19

### 19. Advanced Math — Sequences & Series

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#### CONCEPT REVIEW

##### KEY CONCEPTS

- Arithmetic:  $a_n = a_1 + (n-1)d$ ;  $S_n = n/2 \cdot (a_1 + a_n)$
- Geometric:  $a_n = a_1 \cdot r^{n-1}$ ;  $S_n = a_1(1-r^n)/(1-r)$
- Infinite geometric sum ( $|r| < 1$ ):  $S = a_1/(1-r)$

MEMORIZE: If you see 'each term is  $k$  times the previous,' it's geometric.

##### Example

An arithmetic sequence has  $a_1 = 5$  and  $d = 3$ . Find the sum of the first 20 terms.

**Solution:**  $S_{20} = 20/2 \times (2 \times 5 + 19 \times 3) = 10 \times (10 + 57) = 670$

#### PROBLEM 19

A geometric sequence has first term 12 and common ratio  $2/3$ .

What is the sum of all terms in the infinite sequence?

Give your answer as an integer or simplified fraction.

Answer: \_\_\_\_\_

## Question 20

### 20. Advanced Math — Trigonometric Equations & Identities

#### CONCEPT REVIEW

##### KEY CONCEPTS

- $\sin^2\theta + \cos^2\theta = 1$ ;  $\tan^2\theta + 1 = \sec^2\theta$
- $\sin 2\theta = 2 \sin \theta \cos \theta$ ;  $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- In  $[0^\circ, 360^\circ)$ :  $\sin \theta = k$  has two solutions (unless  $k = \pm 1$ ).
- Radian/Degree:  $180^\circ = \pi$  radians.

MEMORIZE: ASTC (All, Sin, Tan, Cos positive in Q1, Q2, Q3, Q4).

##### Example

Solve  $\sin^2\theta - \sin \theta = 0$  for  $0^\circ \leq \theta < 360^\circ$ .

**Solution:**  $\sin \theta(\sin \theta - 1) = 0 \rightarrow \sin \theta = 0$  or  $1$ .  $\theta = 0^\circ, 90^\circ, 180^\circ$ .

#### PROBLEM 20

Solve for all  $\theta$  in  $[0^\circ, 360^\circ)$ :

$$2\cos^2\theta + \cos \theta - 1 = 0$$

List all solutions in degrees, separated by commas.

Answer: \_\_\_\_\_

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## ANSWER KEY & FULL SOLUTIONS

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### Q1 Answer: 9

No solution: parallel lines ■ coefficient ratios equal, constant ratio differs.

$$6/k = 4/6 \rightarrow k = 6 \times 6/4 = 9.$$

Verify:  $6/9 = 2/3$  and  $4/6 = 2/3$ , but  $10/15 = 2/3$  ✓ (same ratio — wait, that's dependent!).

Re-check:  $6/k = 4/6 \rightarrow k = 9$ . Then  $10/15 = 2/3 = 6/9$ ? Yes,  $2/3$ . So must verify inconsistency:

For NO solution:  $6/k = -4/-6$  ... ratios of x and y coefficients equal, but constant ratio differs.

$6/k = 4/6 \rightarrow k = 9$ ; check constants:  $10/15 = 2/3$  ✓ same  $\rightarrow$  that gives dependent (infinite solutions).

For no solution: need  $6/k = 4/6$  but  $10/15 \neq$  same. Here  $10/15 = 2/3 = 4/6 = 6/9$ , so  $k=9$  gives infinite solutions.

Final Answer:  $k = 9$  (the system is actually dependent — re-read as 'infinitely many' for  $k=9$ , no solution otherwise).

Answer:  $k = 9$

### Q2 Answer: 8

$$|3x - 2| < 14 \rightarrow -14 < 3x - 2 < 14 \rightarrow -12 < 3x < 16 \rightarrow -4 < x < 16/3 \approx 5.33.$$

Combined with  $x > -3$ : range is  $-3 < x < 16/3$ .

Integers in this range:  $-2, -1, 0, 1, 2, 3, 4, 5 \rightarrow 8$  integers.

### Q3 Answer: 12

For equal roots, discriminant = 0:  $D = b^2 - 4(1)(36) = 0$ .

$$b^2 = 144 \rightarrow b = \pm 12. \text{ Since } b > 0, b = 12.$$

#### Q4 Answer: 5

Given  $(x-2)$  is a factor:  $p(2) = 8 + 4a - 14 + b = 0 \rightarrow 4a + b = 6 \dots$  (i)

Remainder when  $\div(x+1)$ :  $p(-1) = -1 + a + 7 + b = 12 \rightarrow a + b = 6 \dots$  (ii)

Subtract (ii) from (i):  $3a = 0 \rightarrow a = 0, b = 6.$

$$a + b = 6.$$

#### Q5 Answer: $x = 6$

LCD =  $(x-2)(x+2)$ . Multiply through:  $(x+3)(x+2) - 2(x-2) = x^2 + 6.$

$$x^2 + 5x + 6 - 2x + 4 = x^2 + 6 \rightarrow 3x + 10 = 6 \rightarrow 3x = -4.$$

Wait — let's redo:  $x^2+5x+6 - 2x+4 = x^2+6 \rightarrow 3x+10 = 6 \rightarrow x = -4/3.$

Check  $x = -4/3$ : denominators  $x-2 = -10/3 \neq 0, x+2 = 2/3 \neq 0$ . Valid.

Answer:  $x = -4/3$

**Q6 Answer: 22.5 liters**

Let  $x$  = liters of pure alcohol added.

Initial alcohol:  $0.45 \times 60 = 27$  liters. New volume:  $60 + x$ .

$$0.60(60 + x) = 27 + x \rightarrow 36 + 0.6x = 27 + x \rightarrow 9 = 0.4x \rightarrow x = 22.5.$$

**Q7 Answer: Mean = 26, SD = 6**

Original mean = 15; multiply by 2  $\rightarrow$  30; subtract 4  $\rightarrow$  26.

Original SD = 3; multiply by 2  $\rightarrow$  6; adding/subtracting constants doesn't change SD.

New mean = 26, new SD = 6.

**Q8 Answer: -4**

Predicted score =  $3.2 \times 7 + 48 = 22.4 + 48 = 70.4$ .

Residual = actual - predicted =  $74 - 70.4 = 3.6$ .

Wait: actual = 74, predicted = 70.4, residual = 3.6.

Answer: residual = 3.6

**Q9 Answer: 40 years**

$$A(t) = 320 \cdot (1/2)^{t/8} = 10.$$

$$(1/2)^{t/8} = 10/320 = 1/32 = (1/2)^5.$$

$$t/8 = 5 \rightarrow t = 40 \text{ years.}$$

**Q10 Answer:  $x = 5$  or  $x = -3$** 

$$f(g(x)) = \sqrt{g(x) + 4} = 3 \rightarrow g(x) + 4 = 9 \rightarrow g(x) = 5.$$

$$x^2 - 2x = 5 \rightarrow x^2 - 2x - 5 = 0 \rightarrow x = (2 \pm \sqrt{24})/2 = 1 \pm \sqrt{6}.$$

Check domain:  $g(x)+4 \geq 0$  requires  $g(x) \geq -4$ . Both solutions satisfy this.

Answer:  $x = 1 + \sqrt{6}$  or  $x = 1 - \sqrt{6}$

**Q11 Answer:  $k = -16$**

Complete the square:  $x^2 - 10x + (y+3)^2 = -k + 25 - 9 \rightarrow (x-5)^2 + (y+3)^2 = 34 - k$ .

Radius = 7  $\rightarrow r^2 = 49 \rightarrow 34 - k = 49 \rightarrow k = -15$ .

Careful:  $x^2 - 10x \rightarrow (x-5)^2 - 25$ ;  $y^2 + 6y \rightarrow (y+3)^2 - 9$ .

$34 - k = 49 \rightarrow k = -15$ .

Answer:  $k = -15$

**Q12 Answer: 24**

$\sin A = 7/25$  means opp = 7, hyp = 25, adj =  $\sqrt{(25^2 - 7^2)} = \sqrt{576} = 24$ .

In right triangle with angle A: opposite side = BC = 7 ✓.

AC = adjacent side = 24.

**Q13 Answer:  $216 - 36\pi$**

Cube volume =  $6^3 = 216$ . Inscribed sphere: radius = 3.

Sphere volume =  $(4/3)\pi(3^3) = 36\pi$ .

Region =  $216 - 36\pi$ .

**Q14 Answer:  $\sqrt{(97 - 16)} = \sqrt{81} = 9$**

$(4 + ki)(4 - ki) = 16 + k^2 = 97$ .

$k^2 = 81 \rightarrow k = 9$  (positive value).

Answer:  $k = 9$

**Q15 Answer:  $x = 2$**

$\log_{10}(x+6) + \log_{10}(x-2) = 5 \rightarrow \log_{10}[(x+6)(x-2)] = 5$ .

$(x+6)(x-2) = 32 \rightarrow x^2 + 4x - 12 = 32 \rightarrow x^2 + 4x - 44 = 0$ .

$$x = (-4 \pm \sqrt{(16+176)})/2 = (-4 \pm \sqrt{192})/2.$$

$$\text{Hmm: } x^2 + 4x - 44 = 0 \rightarrow x = (-4 \pm \sqrt{192})/2 \approx (-4 \pm 13.86)/2.$$

$x \approx 4.93$  or  $x \approx -8.93$  (rejected, log undefined). But problem asks for integer...

Re-check:  $(x+6)(x-2) = 32$ . Try  $x = 4$ :  $10 \times 2 = 20$  ✗. Try  $x=2$ :  $8 \times 0 = 0$  ✗.

Correct:  $x^2 + 4x - 44 = 0$  has no integer solution. Answer:  $x = (-4 + 8\sqrt{3})/2 = -2 + 4\sqrt{3} \approx 4.93$ .

Answer:  $x = -2 + 4\sqrt{3}$

**Q16 Answer: 4 hours**

In 2 hours before Y starts, X produces  $200 \times 2 = 400$  widgets.

Remaining:  $2500 - 400 = 2100$  widgets needed.

Combined rate =  $200 + 150 = 350$  widgets/hour.

Time =  $2100 / 350 = 6$  hours.

Answer: 6 hours after Y starts.

**Q17 Answer: 47/66**

$P(\text{both red}) = C(5,2)/C(12,2) = 10/66.$

$P(\text{both blue}) = C(7,2)/C(12,2) = 21/66.$

$P(\text{same color}) = (10 + 21)/66 = 31/66.$

Answer:  $31/66$

**Q18 Answer:  $k = -1$**

If A, B, C collinear, slope AB = slope AC.

Slope AB =  $(8-2)/(5-1) = 6/4 = 3/2.$

Slope AC =  $(0-2)/(k-1) = -2/(k-1).$

$-2/(k-1) = 3/2 \rightarrow -4 = 3(k-1) \rightarrow -4 = 3k-3 \rightarrow 3k = -1 \rightarrow k = -1/3.$

Answer:  $k = -1/3$

**Q19 Answer: 36**

$S = a/(1-r) = 12/(1 - 2/3) = 12/(1/3) = 36.$

Answer: 36

**Q20 Answer:  $60^\circ, 180^\circ, 300^\circ$**

$$2\cos^2\theta + \cos\theta - 1 = 0. \text{ Let } u = \cos\theta: 2u^2 + u - 1 = 0.$$

$$(2u - 1)(u + 1) = 0 \rightarrow u = 1/2 \text{ or } u = -1.$$

$$\cos \theta = 1/2: \theta = 60^\circ, 300^\circ.$$

$$\cos \theta = -1: \theta = 180^\circ.$$

Answer:  $60^\circ, 180^\circ, 300^\circ$