GEOMETRY

Essential Workbook

20 Core Problems · 5 Units · Full Solutions

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U1 · Lines & Angles

Key Definitions

Complementary angles: two angles whose measures sum to 90°.

Supplementary angles: two angles whose measures sum to 180°.

Vertical angles: opposite angles formed by two intersecting lines — always congruent.

Linear pair: adjacent supplementary angles that together form a straight line.

Parallel Lines Cut by a Transversal

Corresponding angles are congruent (F-shape).

Alternate interior angles are congruent (Z-shape).

Co-interior (same-side interior) angles are supplementary.

Key Formulas

- **Complementary: a + b = 90°**
- **Supplementary: a + b = 180°**
- **Vertical angles: angle1 = angle2**

Worked Example

Q: Two supplementary angles are in the ratio 2 : 3. Find each angle.

Solution: Let the angles be 2x and 3x. 2x + 3x = 180 => x = 36. Angles: 72° and 108°.

Practice Problems

Problem 1

Two complementary angles have measures (3x + 5)° and (2x)°. Find x and both angle measures.

Answer:

Problem 2

Lines l and m are parallel, cut by transversal t. One co-interior (same-side interior) angle is (4x + 10)°. Find x if the other co-interior angle is (6x - 30)°.

Answer:

Problem 3

Vertical angles are formed when two lines intersect. One angle measures (5y - 12)° and its vertical angle measures (3y + 20)°. Find y.

Answer:

Problem 4

A straight line is divided by a point into two adjacent angles. One angle is three times the other. Find both angles.

Answer:

U2 · Triangles

Triangle Angle Sum

The sum of interior angles of any triangle = 180°.

Exterior angle of a triangle = sum of the two non-adjacent interior angles.

Triangle Congruence & Similarity

Congruence shortcuts: SSS, SAS, ASA, AAS, HL (right triangles).

Similarity shortcuts: AA, SAS~, SSS~.

In similar triangles, corresponding sides are proportional and corresponding angles are equal.

Key Formulas

- **Angle sum: A + B + C = 180°**
- **Exterior angle: ext = A + B (where C is adjacent interior)**
- **Area: (1/2) x base x height**

Worked Example

Q: In triangle ABC, angle A = 55° and angle B = 75°. Find angle C.

Solution: C = 180 - 55 - 75 = 50°.

Practice Problems

Problem 5

In triangle PQR, the exterior angle at R is 115°. Angle P = 48°. Find angle Q.

Answer:

Problem 6

Triangle ABC ~ Triangle DEF with AB = 6, BC = 8, AC = 10 and DE = 9. Find EF and DF.

Answer:

Problem 7

Two angles of a triangle are equal. The third angle is 40°. Find each of the equal angles.

Answer:

Problem 8

In an isosceles triangle, the vertex angle is 36°. Find the base angles.

Answer:

U3 · Pythagorean Theorem

The Theorem

In a right triangle with legs a, b and hypotenuse c: a^2 + b^2 = c^2.

The hypotenuse is always the side opposite the right angle (the longest side).

Common Pythagorean Triples

3-4-5, 5-12-13, 8-15-17, 7-24-25.

Multiples also work: 6-8-10, 9-12-15, etc.

Key Formulas

- **a^2 + b^2 = c^2**
- **Leg: a = sqrt(c^2 - b^2)**
- **Distance formula: d = sqrt((x2-x1)^2 + (y2-y1)^2)**

Worked Example

Q: A right triangle has legs of length 9 and 12. Find the hypotenuse.

Solution: c = sqrt(81 + 144) = sqrt(225) = 15.

Practice Problems

Problem 9

A ladder 13 m long leans against a wall. Its foot is 5 m from the base of the wall. How high up the wall does the ladder reach?

Answer:

Problem 10

Find the distance between points A(1, 2) and B(7, 10) on the coordinate plane.

Answer:

Problem 11

Is a triangle with sides 7, 24, 25 a right triangle? Justify your answer.

Answer:

Problem 12

A rectangular room is 8 m long and 6 m wide. Find the length of the diagonal of the room.

Answer:

U4 · Quadrilaterals & Polygons

Key Quadrilateral Properties

Parallelogram: opposite sides parallel & equal; opposite angles equal; diagonals bisect each other.

Rectangle: parallelogram + 4 right angles; diagonals equal.

Rhombus: parallelogram + 4 equal sides; diagonals perpendicular bisectors of each other.

Square: rectangle + rhombus; all properties of both.

Trapezoid: exactly one pair of parallel sides (bases).

Interior Angle Sum of Polygons

Sum of interior angles of an n-gon = (n - 2) x 180°.

Each interior angle of a regular n-gon = (n - 2) x 180 / n.

Key Formulas

- **Interior angle sum: S = (n - 2) x 180°**
- **Regular polygon interior angle: I = (n-2) x 180 / n**
- **Exterior angle sum (any convex polygon) = 360°**

Worked Example

Q: Find the sum of interior angles of a hexagon.

Solution: S = (6 - 2) x 180 = 4 x 180 = 720°.

Practice Problems

Problem 13

The interior angle sum of a polygon is 1440°. How many sides does the polygon have?

Answer:

Problem 14

In a parallelogram ABCD, angle A = (2x + 15)° and angle B = (3x - 5)°. Find x and all four angles.

Answer:

Problem 15

Each interior angle of a regular polygon measures 150°. How many sides does it have?

Answer:

Problem 16

In a trapezoid, the two parallel sides (bases) measure 10 cm and 16 cm, and the height is 8 cm. Find the area.

Answer:

U5 · Circles

Circle Vocabulary

Radius (r): distance from center to any point on the circle.

Diameter (d): longest chord; d = 2r.

Chord: segment whose endpoints are on the circle.

Arc: portion of the circle between two points.

Central angle = arc it intercepts.

Inscribed angle = (1/2) x intercepted arc.

Tangent & Secant Rules

A tangent is perpendicular to the radius at the point of tangency.

Two tangent segments from the same external point are equal in length.

Key Formulas

- **Circumference: C = 2 pi r = pi d**
- **Area: A = pi r^2**
- **Arc length: L = (theta / 360) x 2 pi r**
- **Sector area: A = (theta / 360) x pi r^2**

Worked Example

Q: A circle has radius 7. Find its circumference and area. (Use pi approx 3.14)

Solution: C = 2 x 3.14 x 7 = 43.96. A = 3.14 x 49 = 153.86.

Practice Problems

Problem 17

A central angle of a circle measures 80°. The radius is 9 cm. Find the arc length. (Leave answer in terms of pi.)

Answer:

Problem 18

An inscribed angle intercepts an arc of 140°. What is the measure of the inscribed angle?

Answer:

Problem 19

Two tangent segments are drawn from an external point P to a circle. One tangent has length (2x + 3) and the other has length (5x - 9). Find x and the length of each tangent.

Answer:

Problem 20

Find the area of a sector with a central angle of 90° in a circle with radius 6 cm. (Leave answer in terms of pi.)

Answer:

Answer Key & Solutions

Detailed solutions for all 20 problems are provided below.

U1 · Lines & Angles

Problem 1

Q: Two complementary angles have measures (3x + 5)° and (2x)°. Find x and both angle measures.

Solution: 3x + 5 + 2x = 90 => 5x = 85 => x = 17. Angles: 56° and 34°.

Problem 2

Q: Lines l and m are parallel, cut by transversal t. One co-interior (same-side interior) angle is (4x + 10)°. Find x if the other co-interior angle is (6x - 30)°.

Solution: Co-interior angles are supplementary: (4x+10)+(6x-30)=180 => 10x-20=180 => x=20.

Problem 3

Q: Vertical angles are formed when two lines intersect. One angle measures (5y - 12)° and its vertical angle measures (3y + 20)°. Find y.

Solution: Vertical angles are equal: 5y - 12 = 3y + 20 => 2y = 32 => y = 16.

Problem 4

Q: A straight line is divided by a point into two adjacent angles. One angle is three times the other. Find both angles.

Solution: Let angles be x and 3x. x + 3x = 180 => x = 45°. Angles: 45° and 135°.

U2 · Triangles

Problem 5

Q: In triangle PQR, the exterior angle at R is 115°. Angle P = 48°. Find angle Q.

Solution: Exterior angle = P + Q => 115 = 48 + Q => Q = 67°.

Problem 6

Q: Triangle ABC ~ Triangle DEF with AB = 6, BC = 8, AC = 10 and DE = 9. Find EF and DF.

Solution: Scale factor = 9/6 = 1.5. EF = 8 x 1.5 = 12. DF = 10 x 1.5 = 15.

Problem 7

Q: Two angles of a triangle are equal. The third angle is 40°. Find each of the equal angles.

> **Solution:** Let equal angles each = x. 2x + 40 = 180 => 2x = 140 => x = 70°.

Problem 8

Q: In an isosceles triangle, the vertex angle is 36°. Find the base angles.

> **Solution:** Base angles = (180 - 36)/2 = 144/2 = 72° each.

U3 · Pythagorean Theorem

Problem 9

Q: A ladder 13 m long leans against a wall. Its foot is 5 m from the base of the wall. How high up the wall does the ladder reach?

> **Solution:** h = sqrt(13^2 - 5^2) = sqrt(169 - 25) = sqrt(144) = 12 m.

Problem 10

Q: Find the distance between points A(1, 2) and B(7, 10) on the coordinate plane.

> **Solution:** d = sqrt((7-1)^2 + (10-2)^2) = sqrt(36 + 64) = sqrt(100) = 10.

Problem 11

Q: Is a triangle with sides 7, 24, 25 a right triangle? Justify your answer.

> **Solution:** 7^2 + 24^2 = 49 + 576 = 625 = 25^2. Yes, it is a right triangle by the converse of the Pythagorean Theorem.

Problem 12

Q: A rectangular room is 8 m long and 6 m wide. Find the length of the diagonal of the room.

> **Solution:** d = sqrt(8^2 + 6^2) = sqrt(64 + 36) = sqrt(100) = 10 m.

U4 · Quadrilaterals & Polygons

Problem 13

Q: The interior angle sum of a polygon is 1440°. How many sides does the polygon have?

> **Solution:** (n-2) x 180 = 1440 => n-2 = 8 => n = 10 sides (decagon).

Problem 14

Q: In a parallelogram ABCD, angle A = (2x + 15)° and angle B = (3x - 5)°. Find x and all four angles.

Solution: Consecutive angles supplementary: (2x+15)+(3x-5)=180 => 5x+10=180 => x=34. A=C=83°, B=D=97°.

Problem 15

Q: Each interior angle of a regular polygon measures 150°. How many sides does it have?

Solution: (n-2) x 180 / n = 150 => 180n - 360 = 150n => 30n = 360 => n = 12 sides.

Problem 16

Q: In a trapezoid, the two parallel sides (bases) measure 10 cm and 16 cm, and the height is 8 cm. Find the area.

Solution: Area = (1/2)(b1 + b2)(h) = (1/2)(10 + 16)(8) = (1/2)(26)(8) = 104 cm^2.

U5 · Circles

Problem 17

Q: A central angle of a circle measures 80°. The radius is 9 cm. Find the arc length. (Leave answer in terms of pi.)

Solution: L = (80/360) x 2 pi x 9 = (2/9) x 18 pi = 4 pi cm.

Problem 18

Q: An inscribed angle intercepts an arc of 140°. What is the measure of the inscribed angle?

Solution: Inscribed angle = (1/2) x 140 = 70°.

Problem 19

Q: Two tangent segments are drawn from an external point P to a circle. One tangent has length (2x + 3) and the other has length (5x - 9). Find x and the length of each tangent.

Solution: Tangents from same external point are equal: 2x+3 = 5x-9 => 12 = 3x => x = 4. Length = 11.

Problem 20

Q: Find the area of a sector with a central angle of 90° in a circle with radius 6 cm. (Leave answer in terms of pi.)

Solution: A = (90/360) x pi x 6^2 = (1/4) x 36 pi = 9 pi cm^2.