AMC MASTER

PROBLEM SET

AMC 8 • AMC 10 • AMC 12

20 Essential Problems | All Core Topics | Full Solutions

CONCEPTS • MEMORIZATION • EXAMPLES • PRACTICE

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#01 Arithmetic & Number Sense

AMC 8

CORE CONCEPT

Divisibility, LCM, GCD, Prime Factorization

MEMORIZE

★ LCM(a,b) x GCD(a,b) = a x b

★ Number of factors of n = (e1+1)(e2+1)... where n = p1^e1 * p2^e2 ...

★ Sum of factors: (p^(e+1)-1)/(p-1) for each prime power

★ Primes up to 50: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47

WORKED EXAMPLE

Find GCD(48, 180). 48 = 2^4 x 3, 180 = 2^2 x 3^2 x 5 GCD = 2^2 x 3 = 12

PRACTICE PROBLEM

How many positive integers less than 100 are divisible by both 6 and 8?

Your Answer: _____

#02 Fractions, Decimals & Percents

AMC 8

CORE CONCEPT

Converting between forms, percent change, ratio problems

MEMORIZE

★ Percent change = (New - Old)/Old x 100%

★ a% of b = b% of a = ab/100

★ To convert repeating decimal: if x = 0.abcabc..., then 1000x - x = abc

WORKED EXAMPLE

A price increases by 20% then decreases by 20%. Net change? Final = 1.2 x 0.8 = 0.96, so net change = -4%

PRACTICE PROBLEM

A jacket costs $80. After a 25% discount and then a 10% tax, what is the final price?

Your Answer: _____

#03 Ratios & Proportions

AMC 8

CORE CONCEPT

Direct/inverse proportion, mixture problems, rates

MEMORIZE

★ Direct proportion: y = kx

★ Inverse proportion: y = k/x

★ Rate x Time = Distance (or Work)

★ Combined rate: 1/T = 1/T1 + 1/T2 (for work problems)

WORKED EXAMPLE

A and B can paint a room in 4 and 6 hours. How long together? 1/T = 1/4 + 1/6 = 5/12, so T = 12/5 = 2.4 hours

PRACTICE PROBLEM

A car travels 120 miles in 2 hours. At the same rate, how many miles in 5 hours?

Your Answer: _____

#04 Geometry: Perimeter & Area

AMC 8

CORE CONCEPT

Areas of polygons, circles, composite figures

MEMORIZE

★ Rectangle: A = lw, P = 2(l+w)

★ Triangle: A = (1/2)bh

★ Circle: A = pi*r^2, C = 2*pi*r

★ Trapezoid: A = (1/2)(b1+b2)h

WORKED EXAMPLE

A circle has area 36*pi. Find its circumference. r^2 = 36, so r = 6. C = 2*pi*6 = 12*pi

PRACTICE PROBLEM

A rectangle has perimeter 36 and length twice its width. Find the area.

Your Answer: _____

#05 Statistics & Data

AMC 8

CORE CONCEPT

Mean, median, mode, range, bar/pie charts

MEMORIZE

★ Mean = (sum of values) / (number of values)

★ Median = middle value when sorted

★ Mode = most frequent value

★ Range = max - min

WORKED EXAMPLE

Data: 3, 7, 7, 9, 14. Find mean, median, mode. Mean = 40/5 = 8. Median = 7. Mode = 7.

PRACTICE PROBLEM

Five test scores are 85, 90, 78, 92, and x. If the mean is 87, find x.

Your Answer: _____

#06 Counting & Probability

AMC 8/10

CORE CONCEPT	Counting principles, permutations, combinations, probability

MEMORIZE

- ★ Permutation: P(n,r) = n!/(n-r)!
- ★ Combination: C(n,r) = n!/[r!(n-r)!]
- ★ P(A and B) = P(A) x P(B) for independent events
- ★ P(A or B) = P(A) + P(B) - P(A and B)

WORKED EXAMPLE

How many ways to choose 2 from 5 people? C(5,2) = 5!/(2!3!) = 10

PRACTICE PROBLEM

A bag has 4 red and 6 blue marbles. Two are drawn without replacement. What is the probability both are red?

Your Answer: _____

#07 Integers & Sequences

AMC 8/10

CORE CONCEPT	Arithmetic/geometric sequences, sums, patterns

MEMORIZE

- ★ Arithmetic: a_n = a_1 + (n-1)d, S_n = n(a_1+a_n)/2
- ★ Geometric: a_n = a_1 * r^(n-1), S_n = a_1(1-r^n)/(1-r)
- ★ Sum 1+2+...+n = n(n+1)/2
- ★ Sum of squares: 1^2+...+n^2 = n(n+1)(2n+1)/6

WORKED EXAMPLE

Sum of first 100 positive integers: S = 100 x 101 / 2 = 5050

PRACTICE PROBLEM

An arithmetic sequence has a1=5 and common difference 3. What is the sum of the first 20 terms?

Your Answer: _____

#08 Algebra: Equations & Inequalities

AMC 10

CORE CONCEPT

Linear, quadratic equations, systems, absolute value

MEMORIZE

- ★ Quadratic formula: x = (-b +/- sqrt(b^2-4ac)) / 2a
- ★ Discriminant: D=b^2-4ac. D>0: 2 real roots, D=0: 1 root, D<0: no real roots
- ★ Vieta's: x1+x2 = -b/a, x1*x2 = c/a
- ★ |x| < a means -a < x < a

WORKED EXAMPLE

Solve x^2 - 5x + 6 = 0. (x-2)(x-3)=0, so x=2 or x=3

PRACTICE PROBLEM

If x^2 - 7x + k = 0 has equal roots, find k.

Your Answer: _____

#09 Geometry: Triangles

AMC 10

CORE CONCEPT

Special triangles, similarity, Pythagorean theorem

MEMORIZE

- ★ 30-60-90: sides 1 : sqrt(3) : 2
- ★ 45-45-90: sides 1 : 1 : sqrt(2)
- ★ Pythagorean triples: (3,4,5), (5,12,13), (8,15,17), (7,24,25)
- ★ Heron's formula: A = sqrt(s(s-a)(s-b)(s-c)), s=(a+b+c)/2

WORKED EXAMPLE

Right triangle: legs 5 and 12. Find hypotenuse. 5^2+12^2 = 25+144=169 = 13^2, so c=13

PRACTICE PROBLEM

An isosceles right triangle has hypotenuse 10. Find its area.

Your Answer: _____

#10 Coordinate Geometry

AMC 10

CORE CONCEPT

Distance, midpoint, slope, lines, circles on coordinate plane

MEMORIZE

★ Distance: d = sqrt((x2-x1)^2+(y2-y1)^2)

★ Midpoint: ((x1+x2)/2, (y1+y2)/2)

★ Slope: m = (y2-y1)/(x2-x1)

★ Circle: (x-h)^2+(y-k)^2=r^2

WORKED EXAMPLE

Distance from (1,2) to (4,6): d = sqrt(9+16) = sqrt(25) = 5

PRACTICE PROBLEM

Find the midpoint of the segment joining (-3, 5) and (7, -1).

Your Answer: _____

#11 Functions & Graphs

AMC 10

CORE CONCEPT

Polynomial, absolute value, piecewise functions; transformations

MEMORIZE

★ f(x+h): shift left h units; f(x-h): shift right h

★ f(x)+k: shift up k; f(x)-k: shift down

★ -f(x): reflect over x-axis; f(-x): reflect over y-axis

★ Domain of sqrt(x): x >= 0; Domain of 1/x: x != 0

WORKED EXAMPLE

f(x) = x^2. Graph of f(x-3)+2 is f shifted right 3, up 2.

PRACTICE PROBLEM

If f(x) = 2x+1 and g(x) = x^2-1, find f(g(3)).

Your Answer: _____

#12 Circles & Polygons

AMC 10

CORE CONCEPT Arc, sector, inscribed angles, regular polygons

MEMORIZE

★ Central angle = arc measure

★ Inscribed angle = (1/2) x intercepted arc

★ Area of sector = (theta/360) x pi*r^2

★ Interior angle sum of n-gon = (n-2) x 180 degrees

WORKED EXAMPLE

Regular hexagon with side 4: area = (3*sqrt(3)/2)(4^2) = 24*sqrt(3)

PRACTICE PROBLEM

A circle has radius 6. What is the area of a sector with central angle 60 degrees?

Your Answer: _____

#13 Number Theory

AMC 10/12

CORE CONCEPT

Modular arithmetic, Euler's theorem, Chinese Remainder Theorem

MEMORIZE

- ★ a^phi(n) = 1 (mod n) for gcd(a,n)=1 [Euler's theorem]
- ★ phi(p) = p-1 for prime p [Euler's totient]
- ★ Fermat's little: a^p = a (mod p) for prime p
- ★ Last digit patterns repeat with period 4 for most bases

WORKED EXAMPLE

Find 7^100 mod 10. 7^1=7, 7^2=49, 7^3=343, 7^4=2401 -> cycle of 4. 100=4x25, so 7^100 = 1 (mod 10). Last digit = 1.

PRACTICE PROBLEM

What is the units digit of 3^2025?

Your Answer: _____

#14 Logarithms & Exponents

AMC 10/12

CORE CONCEPT

Log properties, change of base, exponential equations

MEMORIZE

- ★ log(ab) = log(a)+log(b)
- ★ log(a/b) = log(a)-log(b)
- ★ log(a^n) = n*log(a)
- ★ Change of base: log_b(a) = ln(a)/ln(b)

WORKED EXAMPLE

Solve 2^(x+1) = 32. 2^(x+1) = 2^5, so x+1=5, x=4

PRACTICE PROBLEM

Simplify: log_2(8) + log_2(4) - log_2(16).

Your Answer: _____

#15 Combinatorics: Advanced

AMC 10/12

CORE CONCEPT

Inclusion-exclusion, Pigeonhole, Binomial theorem

MEMORIZE

★ |A U B| = |A| + |B| - |A^B|

★ Pigeonhole: n+1 objects in n boxes => some box has >= 2

★ Binomial: (x+y)^n = sum C(n,k) x^k y^(n-k)

★ Pascal's identity: C(n,k) = C(n-1,k-1) + C(n-1,k)

WORKED EXAMPLE

How many integers 1-100 are divisible by 2 or 3? |2|+|3|-|6| = 50+33-16 = 67

PRACTICE PROBLEM

In how many ways can 5 people be seated in a row if 2 specific people must NOT sit next to each other?

Your Answer: _____

#16 Trigonometry

AMC 12

CORE CONCEPT

Unit circle, trig identities, law of sines/cosines

MEMORIZE

★ sin^2(x)+cos^2(x)=1

★ sin(2x) = 2sin(x)cos(x)

★ cos(2x) = cos^2(x)-sin^2(x) = 1-2sin^2(x)

★ Law of Cosines: c^2 = a^2+b^2-2ab*cos(C)

WORKED EXAMPLE

Find sin(75 degrees). sin(45+30) = sin45*cos30+cos45*sin30 = (sqrt(6)+sqrt(2))/4

PRACTICE PROBLEM

In triangle ABC, a=7, b=8, c=5. Find cos(C).

Your Answer: _____

#17 Complex Numbers

AMC 12

CORE CONCEPT

Operations, modulus, argument, De Moivre's theorem

MEMORIZE

★ |a+bi| = sqrt(a^2+b^2)

★ (a+bi)(a-bi) = a^2+b^2

★ De Moivre: (r*cis(theta))^n = r^n * cis(n*theta)

★ i^1=i, i^2=-1, i^3=-i, i^4=1 (period 4)

WORKED EXAMPLE

Compute (1+i)^4. (1+i)^2 = 2i. (2i)^2 = -4.

PRACTICE PROBLEM

Find the modulus of (3+4i)(1-i).

Your Answer: _____

#18 Polynomial & Rational Functions

AMC 12

CORE CONCEPT	Factor theorem, Remainder theorem, rational roots

MEMORIZE

- ★ Remainder theorem: f(a) = remainder when f(x)/(x-a)
- ★ Factor theorem: (x-a) is factor iff f(a)=0
- ★ Rational Root Theorem: rational roots are +/- (factor of const)/(factor of leading coeff)
- ★ Sum of roots = -b/a, Product = e/a (for degree n polynomial)

WORKED EXAMPLE

Find remainder when x^3-2x+1 divided by (x-2). f(2) = 8-4+1 = 5

PRACTICE PROBLEM

If (x-3) is a factor of x^3-2x^2-5x+k, find k.

Your Answer: _____

#19 3D Geometry

AMC 12

CORE CONCEPT	Volume and surface area of solids, cross-sections

MEMORIZE

- ★ Sphere: V = (4/3)*pi*r^3, SA = 4*pi*r^2
- ★ Cylinder: V = pi*r^2*h, Lateral SA = 2*pi*r*h
- ★ Cone: V = (1/3)*pi*r^2*h, Lateral SA = pi*r*l (l=slant height)
- ★ Pyramid: V = (1/3)*base*h

WORKED EXAMPLE

Cone with r=3, h=4. Find volume. l = sqrt(9+16)=5. V = (1/3)*pi*9*4 = 12*pi

PRACTICE PROBLEM

A sphere has surface area 100*pi. Find its volume.

Your Answer: _____

#20 Probability & Expected Value

AMC 12

CORE CONCEPT

Conditional probability, Bayes' theorem, expected value

MEMORIZE

★ P(A|B) = P(A and B)/P(B)

★ Bayes: P(A|B) = P(B|A)*P(A) / P(B)

★ E(X) = sum x*P(x)

★ Var(X) = E(X^2) - [E(X)]^2

WORKED EXAMPLE

Fair die. Expected value = (1+2+3+4+5+6)/6 = 3.5

PRACTICE PROBLEM

A fair coin is tossed until Heads appears. What is the expected number of tosses?

Your Answer: _____

ANSWER KEY & FULL SOLUTIONS

QUICK ANSWERS

#01 4	**#02** $66	**#03** 300 miles	**#04** 72
#05 90	**#06** 2/15	**#07** 670	**#08** 49/4
#09 25	**#10** (2, 2)	**#11** 17	**#12** 6*pi
#13 3	**#14** 1	**#15** 72	**#16** 11/14
#17 5*sqrt(2)	**#18** 6	**#19** (500/3)*pi	**#20** 2

DETAILED SOLUTIONS

#01 Arithmetic & Number Sense — Answer: 4

Q: How many positive integers less than 100 are divisible by both 6 and 8?

LCM(6,8)=24. Multiples of 24 less than 100: 24,48,72,96. Count = 4.

#02 Fractions, Decimals & Percents — Answer: $66

Q: A jacket costs $80. After a 25% discount and then a 10% tax, what is the final price?

After discount: 80 x 0.75 = $60. After tax: 60 x 1.10 = $66.

#03 Ratios & Proportions — Answer: 300 miles

Q: A car travels 120 miles in 2 hours. At the same rate, how many miles in 5 hours?

Rate = 120/2 = 60 mph. Distance = 60 x 5 = 300 miles.

#04 Geometry: Perimeter & Area — Answer: 72

Q: A rectangle has perimeter 36 and length twice its width. Find the area.

Let w = width. 2(2w+w) = 36, so w = 6, l = 12. Area = 6 x 12 = 72.

#05 Statistics & Data — Answer: 90

Q: Five test scores are 85, 90, 78, 92, and x. If the mean is 87, find x.

(85+90+78+92+x)/5 = 87. Sum = 435. 345+x=435. x=90.

#06 **Counting & Probability** **Answer: 2/15**

Q: A bag has 4 red and 6 blue marbles. Two are drawn without replacement. What is the probability both are red?

P = (4/10) x (3/9) = 12/90 = 2/15.

#07 **Integers & Sequences** **Answer: 670**

Q: An arithmetic sequence has a1=5 and common difference 3. What is the sum of the first 20 terms?

a20 = 5 + 19(3) = 62. S20 = 20(5+62)/2 = 670.

#08 **Algebra: Equations & Inequalities** **Answer: 49/4**

Q: If x^2 - 7x + k = 0 has equal roots, find k.

Equal roots when D=0: 49-4k=0, k=49/4.

#09 **Geometry: Triangles** **Answer: 25**

Q: An isosceles right triangle has hypotenuse 10. Find its area.

Legs = 10/sqrt(2) = 5*sqrt(2). Area = (1/2)(5*sqrt(2))^2 = 25.

#10 **Coordinate Geometry** **Answer: (2, 2)**

Q: Find the midpoint of the segment joining (-3, 5) and (7, -1).

Midpoint = ((-3+7)/2, (5+(-1))/2) = (2, 2).

#11 **Functions & Graphs** **Answer: 17**

Q: If f(x) = 2x+1 and g(x) = x^2-1, find f(g(3)).

g(3)=9-1=8. f(8)=2(8)+1=17.

#12 **Circles & Polygons** **Answer: 6*pi**

Q: A circle has radius 6. What is the area of a sector with central angle 60 degrees?

Area = (60/360)*pi*36 = (1/6)*36*pi = 6*pi.

#13 **Number Theory** **Answer: 3**

Q: What is the units digit of 3^2025?

Cycle of 3: 3,9,7,1 (period 4). 2025=4x506+1. Units digit = 3.

#14 Logarithms & Exponents — Answer: 1

Q: Simplify: log_2(8) + log_2(4) - log_2(16).

= 3 + 2 - 4 = 1.

#15 Combinatorics: Advanced — Answer: 72

Q: In how many ways can 5 people be seated in a row if 2 specific people must NOT sit next to each other?

Total=5!=120. Bad (together): treat them as 1 unit: 4!x2=48. Answer=120-48=72.

#16 Trigonometry — Answer: 11/14

Q: In triangle ABC, a=7, b=8, c=5. Find cos(C).

c^2=a^2+b^2-2ab*cos(C): 25=49+64-112cos(C). cos(C)=88/112=11/14.

#17 Complex Numbers — Answer: 5*sqrt(2)

Q: Find the modulus of (3+4i)(1-i).

Product = 3-3i+4i-4i^2 = 7+i. Modulus = sqrt(49+1) = sqrt(50) = 5*sqrt(2).

#18 Polynomial & Rational Functions — Answer: 6

Q: If (x-3) is a factor of x^3-2x^2-5x+k, find k.

f(3) = 27-18-15+k = 0. k=6.

#19 3D Geometry — Answer: (500/3)*pi

*Q: A sphere has surface area 100*pi. Find its volume.*

4*pi*r^2=100*pi, r=5. V=(4/3)*pi*125=(500/3)*pi.

#20 Probability & Expected Value — Answer: 2

Q: A fair coin is tossed until Heads appears. What is the expected number of tosses?

Geometric distribution with p=1/2. E(X)=1/p=1/(1/2)=2.