

# MASTERCLASS

---

20 Core Problems · All Topics Covered

Concept Review · Key Formulas · Exam-Style Questions · Full Solutions

★ EXAM READY ★

# PART 1 — CONCEPTS & KEY FORMULAS

Study each concept card. Memorize the formulas and tips before attempting problems.

## 1 · Angles & Lines

### KEY FORMULAS

**Supplementary angles:  $a + b = 180^\circ$**

**Complementary angles:  $a + b = 90^\circ$**

**Vertical angles:  $a = b$  (opposite angles at intersection)**

**Parallel lines cut by transversal: alt. interior = co-interior supplementary**

### MEMORIZE

\* *SUPP = 180, COMP = 90*

\* *Vertical angles are ALWAYS equal*

\* *Co-interior (same-side) angles sum to 180*

### EXAMPLE

Q: Two angles are supplementary. One is  $3x + 10$ , other is  $x + 50$ . Find  $x$ .

A:  $3x+10 + x+50 = 180 \Rightarrow 4x = 120 \Rightarrow x = 30$

## 2 · Triangles — Angles & Types

### KEY FORMULAS

**Sum of interior angles =  $180^\circ$**

**Exterior angle = sum of two non-adjacent interior angles**

**Isosceles: 2 equal sides, 2 equal base angles**

**Equilateral: all sides equal, each angle =  $60^\circ$**

### MEMORIZE

\* *Triangle angle sum = 180 ALWAYS*

\* *Exterior angle SHORTCUT: skip the 180 step*

\* *Equilateral = all  $60^\circ$*

### EXAMPLE

Q: A triangle has angles  $2x$ ,  $3x$ , and  $x+30$ . Find  $x$ .

A:  $2x + 3x + x+30 = 180 \Rightarrow 6x = 150 \Rightarrow x = 25$

## 3 · Pythagorean Theorem & Distance

### KEY FORMULAS

**$a^2 + b^2 = c^2$  ( $c$  = hypotenuse)**

**Common triples: 3-4-5, 5-12-13, 8-15-17**

**Distance formula:  $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$**

**Midpoint:  $M = ((x_1+x_2)/2, (y_1+y_2)/2)$**

### MEMORIZE

- \* Hypotenuse is ALWAYS opposite the right angle
- \* Memorize 3-4-5 and 5-12-13 triples!
- \* Distance = Pythagorean theorem on coordinate grid

#### EXAMPLE

Q: Right triangle legs: 6 and 8. Find hypotenuse.

**A:  $6^2 + 8^2 = 36 + 64 = 100 \Rightarrow c = 10$**

## 4 · Area & Perimeter

### KEY FORMULAS

**Rectangle:  $A = lw$ ,  $P = 2(l+w)$**

**Triangle:  $A = (1/2)bh$**

**Parallelogram:  $A = bh$**

**Trapezoid:  $A = (1/2)(b_1+b_2)h$**

**Circle:  $A = \pi r^2$ ,  $C = 2\pi r$**

### MEMORIZE

- \* All areas use **BASE** x **HEIGHT** (perpendicular!)
- \* Trapezoid = average of parallel sides x height
- \* Circle uses **RADIUS** not diameter

#### EXAMPLE

Q: Find area of trapezoid with bases 5 and 9, height 4.

**A:  $A = (1/2)(5+9)(4) = (1/2)(14)(4) = 28$**

## 5 · Similar & Congruent Triangles

### KEY FORMULAS

**Similar (~): same shape, proportional sides, equal angles**

**Congruent (=): same shape AND size**

**SSS, SAS, ASA, AAS, HL — congruence criteria**

**AA — similarity criterion (two angles equal)**

**Scale factor k: perimeter ratio = k, area ratio =  $k^2$**

### MEMORIZE

- \* AA is **ENOUGH** for similarity
- \* Area ratio = (scale factor)<sup>2</sup>
- \* Set up proportions carefully: match **CORRESPONDING** sides

#### EXAMPLE

Q: Two similar triangles have sides 4,6,8 and 6,9,x. Find x.

**A: Scale factor =  $6/4 = 1.5 \Rightarrow x = 8 \times 1.5 = 12$**

## 6 · Circles — Arcs, Chords, Angles

### KEY FORMULAS

**Central angle = intercepted arc**

**Inscribed angle =  $(1/2)$  x intercepted arc**

**Tangent-chord angle =  $(1/2)$  x intercepted arc**

**Two chords intersect inside: angle =  $(1/2)(\text{arc1} + \text{arc2})$**

**Chord-chord:  $AE \times EB = CE \times ED$**

**MEMORIZE**

\* Inscribed angle = HALF the arc

\* Central angle = arc (1:1 ratio)

\* Intersecting chords: multiply the segments

**EXAMPLE**

Q: Inscribed angle is  $35^\circ$ . Find the intercepted arc.

A: Arc =  $2 \times 35 = 70^\circ$

## 7 · Volume & Surface Area

**KEY FORMULAS**

**Prism:  $V = Bh$  (B = base area)**

**Cylinder:  $V = \pi r^2 h$ ,  $SA = 2\pi r^2 + 2\pi r h$**

**Pyramid:  $V = (1/3)Bh$**

**Cone:  $V = (1/3)\pi r^2 h$ ,  $SA = \pi r l + \pi r^2$**

**Sphere:  $V = (4/3)\pi r^3$ ,  $SA = 4\pi r^2$**

**MEMORIZE**

\* Pyramid/Cone =  $(1/3)$  x cylinder/prism

\* Sphere SA = 4 circles ( $4\pi r^2$ )

\* Always identify what B (base area) equals first

**EXAMPLE**

Q: Cylinder:  $r = 3$ ,  $h = 5$ . Find volume.

A:  $V = \pi(3^2)(5) = 45\pi$

## 8 · Coordinate Geometry

**KEY FORMULAS**

**Slope:  $m = (y_2 - y_1)/(x_2 - x_1)$**

**Parallel lines: equal slopes ( $m_1 = m_2$ )**

**Perpendicular lines:  $m_1 \times m_2 = -1$**

**Slope-intercept:  $y = mx + b$**

**Point-slope:  $y - y_1 = m(x - x_1)$**

**MEMORIZE**

\* Parallel = same slope

\* Perpendicular = negative reciprocal slope

\*  $m = \text{RISE over RUN}$

**EXAMPLE**

Q: Points (1,2) and (4,8). Find the slope.

**A:  $m = (8-2)/(4-1) = 6/3 = 2$**

## PART 2 — EXAM PROBLEMS

Show all working. Write your answers on the lines provided.

### Q01 ANGLES & LINES

Two parallel lines are cut by a transversal. One co-interior angle is  $(3x + 15)^\circ$  and the other is  $(2x + 25)^\circ$ . Find the value of  $x$  and both angles.

---

---

---

### Q02 ANGLES & LINES

Angle A and angle B are supplementary. Angle A is 4 times the size of angle B. Find the measure of each angle.

---

---

---

### Q03 TRIANGLES

In triangle ABC, angle A =  $(2x + 10)^\circ$ , angle B =  $(x + 20)^\circ$ , and angle C =  $(3x - 6)^\circ$ . Find  $x$  and classify the triangle by its angles.

---

---

---

### Q04 TRIANGLES

An exterior angle of a triangle is  $112^\circ$ . One of the non-adjacent interior angles is  $47^\circ$ . Find the other non-adjacent interior angle.

---

---

---

### Q05 PYTHAGOREAN THEOREM

A ladder 13 m long leans against a wall. The foot of the ladder is 5 m from the base of the wall. How high up the wall does the ladder reach?

---

---

---

## Q06 PYTHAGOREAN THEOREM

Point A is at (1, 2) and point B is at (7, 10). Find the exact distance AB and the coordinates of the midpoint M.

---

---

---

## Q07 AREA & PERIMETER

A trapezoid has parallel bases of 7 cm and 13 cm and a height of 6 cm. Find its area. Then find the area of a triangle with the same base (13 cm) and same height (6 cm).

---

---

---

## Q08 AREA & PERIMETER

A circle has an area of  $36\pi \text{ cm}^2$ . Find the radius, diameter, and circumference. Leave answers in terms of pi where appropriate.

---

---

---

## Q09 SIMILAR TRIANGLES

Triangle PQR ~ Triangle XYZ. PQ = 8, QR = 12, PR = 10. XY = 12. Find YZ and XZ. Then find the ratio of their areas.

---

---

---

## Q10 SIMILAR TRIANGLES

A 6-metre pole casts a shadow of 4 m. At the same time, a tree casts a shadow of 14 m. How tall is the tree?

---

---

---

---

## Q11 CONGRUENT TRIANGLES

In triangles ABC and DEF:  $AB = DE$ , angle A = angle D, angle B = angle E. Name the congruence criterion and write the congruence statement.

---

---

---

---

## Q12 CIRCLE – ARCS & ANGLES

A central angle is  $140^\circ$ . Find (a) the major arc, (b) the inscribed angle that intercepts the minor arc, (c) the inscribed angle that intercepts the major arc.

---

---

---

---

## Q13 CIRCLE – CHORDS

Two chords AB and CD intersect inside a circle at point P.  $AP = 4$ ,  $PB = 9$ , and  $CP = 6$ . Find PD.

---

---

---

---

## Q14 VOLUME

A rectangular box has dimensions 5 cm x 4 cm x 3 cm. Find (a) its volume and (b) its total surface area.

---

---

---

## Q15 VOLUME

A cone and a cylinder share the same base radius of 4 cm and the same height of 9 cm. Find the volume of each. How many cones fill the cylinder?

---

---

---

---

## Q16 VOLUME

A sphere has a surface area of  $100\pi \text{ cm}^2$ . Find the radius and the volume of the sphere. Leave in terms of  $\pi$ .

---

---

---

---

## Q17 COORDINATE GEOMETRY

Line L1 passes through (2, 3) and (6, 7). Line L2 passes through (0, 5) and (4, 1). Show whether the lines are parallel, perpendicular, or neither.

---

---

---

---

## Q18 COORDINATE GEOMETRY

Write the equation of the line perpendicular to  $y = \frac{2}{3}x + 4$  that passes through the point (4, 1).

---

---

---

---

**Q19** MIXED — POLYGON ANGLES

Find the sum of interior angles of a hexagon. Then find each interior angle of a REGULAR hexagon. Finally, find each exterior angle of a regular hexagon.

---

---

---

---

**Q20** MIXED — COMBINED

A semicircle is placed on top of a rectangle. The rectangle is 10 cm wide and 8 cm tall. The diameter of the semicircle equals the width of the rectangle. Find the total area of the combined shape. Leave pi terms separate.

---

---

---

## PART 3 — FULL SOLUTIONS & EXPLANATIONS

Check your answers. Study each solution method carefully.

### Q01 ANGLES & LINES

Co-interior angles sum to  $180^\circ$

$$(3x+15) + (2x+25) = 180$$

$$5x + 40 = 180 \Rightarrow 5x = 140 \Rightarrow x = 28$$

$$\text{Angle 1: } 3(28)+15 = 99^\circ, \text{ Angle 2: } 2(28)+25 = 81^\circ$$

$$\text{Check: } 99 + 81 = 180 \checkmark$$

**ANSWER:  $x = 28$ ; angles are  $99^\circ$  and  $81^\circ$**

### Q02 ANGLES & LINES

Let angle B =  $x$ , then angle A =  $4x$

$$x + 4x = 180 \Rightarrow 5x = 180 \Rightarrow x = 36$$

$$\text{Angle B} = 36^\circ, \text{ Angle A} = 4(36) = 144^\circ$$

**ANSWER: Angle B =  $36^\circ$ , Angle A =  $144^\circ$**

### Q03 TRIANGLES

Sum of angles =  $180^\circ$

$$(2x+10) + (x+20) + (3x-6) = 180$$

$$6x + 24 = 180 \Rightarrow 6x = 156 \Rightarrow x = 26$$

Angles: A =  $62^\circ$ , B =  $46^\circ$ , C =  $72^\circ$  — all different  $\Rightarrow$  SCALENE

All angles  $< 90^\circ \Rightarrow$  ACUTE triangle

**ANSWER:  $x = 29.4$ ; triangle is scalene acute (angles  $68.8^\circ$ ,  $49.4^\circ$ ,  $81.8^\circ$ )**

### Q04 TRIANGLES

Exterior angle theorem: ext. angle = sum of two non-adjacent interior angles

$$112 = 47 + \text{other angle}$$

$$\text{Other angle} = 112 - 47 = 65^\circ$$

**ANSWER: Other interior angle =  $65^\circ$**

### Q05 PYTHAGOREAN THEOREM

Hypotenuse (ladder) = 13, one leg (base) = 5

$$a^2 + 5^2 = 13^2$$

$$a^2 = 169 - 25 = 144$$

$$a = \sqrt{144} = 12 \text{ m}$$

Recognise the 5-12-13 Pythagorean triple!

**ANSWER: Height = 12 m**

### Q06 DISTANCE & MIDPOINT

$$d = \sqrt{(7-1)^2 + (10-2)^2}$$

$$d = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$M = ((1+7)/2, (2+10)/2) = (4, 6)$$

**ANSWER: AB = 10, M = (4, 6)**

### Q07 AREA & PERIMETER

$$\text{Trapezoid: } A = (1/2)(b_1+b_2)h = (1/2)(7+13)(6) = (1/2)(20)(6) = 60 \text{ cm}^2$$

$$\text{Triangle: } A = (1/2)(13)(6) = 39 \text{ cm}^2$$

**ANSWER: Trapezoid area = 60 cm<sup>2</sup>; Triangle area = 39 cm<sup>2</sup>**

### Q08 CIRCLE

$$\pi r^2 = 36\pi \Rightarrow r^2 = 36 \Rightarrow r = 6 \text{ cm}$$

$$\text{Diameter} = 2r = 12 \text{ cm}$$

$$\text{Circumference} = 2\pi r = 2\pi \cdot 6 = 12\pi \text{ cm}$$

**ANSWER: r = 6 cm, d = 12 cm, C = 12π cm**

### Q09 SIMILAR TRIANGLES

$$\text{Scale factor} = XY/PQ = 12/8 = 3/2$$

$$YZ = QR \times (3/2) = 12 \times 1.5 = 18$$

$$XZ = PR \times (3/2) = 10 \times 1.5 = 15$$

$$\text{Area ratio} = (3/2)^2 = 9/4, \text{ so } XYZ : PQR = 9 : 4$$

**ANSWER: YZ = 18, XZ = 15; Area ratio = 9:4**

### Q10 SIMILAR TRIANGLES

Using similar triangles (same sun angle):

$$6/4 = h/14$$

$$h = (6 \times 14)/4 = 84/4 = 21 \text{ m}$$

**ANSWER: Tree height = 21 m**

### **Q11** CONGRUENT TRIANGLES

Given:  $AB = DE$  (side), angle  $A =$  angle  $D$  (angle), angle  $B =$  angle  $E$  (angle)

The side is BETWEEN the two angles  $\Rightarrow$  ASA criterion

Write corresponding vertices in order:  $ABC \cong DEF$

**ANSWER: ASA congruence; Triangle  $ABC \cong$  Triangle  $DEF$**

### **Q12** CIRCLE ANGLES

Minor arc = central angle =  $140^\circ$

Major arc =  $360 - 140 = 220^\circ$

Inscribed angle (minor arc) =  $140/2 = 70^\circ$

Inscribed angle (major arc) =  $220/2 = 110^\circ$

**ANSWER: (a) Major arc =  $220^\circ$ , (b) inscribed angle for minor arc =  $70^\circ$ , (c) inscribed angle for major arc =  $110^\circ$**

### **Q13** CIRCLE CHORDS

Intersecting chords theorem:  $AP \times PB = CP \times PD$

$$4 \times 9 = 6 \times PD$$

$$36 = 6 \times PD \Rightarrow PD = 6$$

**ANSWER:  $PD = 6$**

### **Q14** VOLUME & SURFACE AREA

$$V = l \times w \times h = 5 \times 4 \times 3 = 60 \text{ cm}^3$$

$$SA = 2(lw + lh + wh) = 2(20 + 15 + 12) = 2(47) = 94 \text{ cm}^2$$

**ANSWER: Volume =  $60 \text{ cm}^3$ ; SA =  $94 \text{ cm}^2$**

### **Q15** VOLUME

$$\text{Cylinder: } V = \pi r^2 h = \pi(16)(9) = 144\pi \text{ cm}^3$$

$$\text{Cone: } V = (1/3)\pi r^2 h = (1/3)(144\pi) = 48\pi \text{ cm}^3$$

$144\pi / 48\pi = 3 \Rightarrow$  exactly 3 cones fill the cylinder

**ANSWER: Cylinder:  $144\pi \text{ cm}^3$ ; Cone:  $48\pi \text{ cm}^3$ ; 3 cones fill the cylinder**

### **Q16** SPHERE

$$SA = 4\pi r^2 = 100\pi \Rightarrow r^2 = 25 \Rightarrow r = 5 \text{ cm}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi \text{ cm}^3$$

**ANSWER:  $r = 5 \text{ cm}$ ;  $V = \frac{500}{3}\pi \text{ cm}^3$**

### Q17 COORDINATE GEOMETRY

$$\text{Slope of L1: } m_1 = \frac{7-3}{6-2} = \frac{4}{4} = 1$$

$$\text{Slope of L2: } m_2 = \frac{1-5}{4-0} = \frac{-4}{4} = -1$$

$$m_1 \times m_2 = 1 \times (-1) = -1 \Rightarrow \text{PERPENDICULAR}$$

**ANSWER: The lines are PERPENDICULAR**

### Q18 COORDINATE GEOMETRY

$$\text{Given line slope: } m = \frac{2}{3}$$

$$\text{Perpendicular slope} = \text{negative reciprocal} = -\frac{3}{2}$$

$$\text{Using point } (4, 1): y - 1 = -\frac{3}{2}(x - 4)$$

$$y - 1 = -\frac{3}{2}x + 6 \Rightarrow y = -\frac{3}{2}x + 7$$

**ANSWER:  $y = -\frac{3}{2}x + 7$**

### Q19 POLYGON ANGLES

$$\text{Interior angle sum} = (n-2) \times 180 = (6-2) \times 180 = 4 \times 180 = 720^\circ$$

$$\text{Each interior angle (regular)} = \frac{720}{6} = 120^\circ$$

$$\text{Each exterior angle} = \frac{360}{6} = 60^\circ$$

$$\text{Check: interior} + \text{exterior} = 120 + 60 = 180^\circ \checkmark$$

**ANSWER: Sum = 720°; each interior angle = 120°; each exterior angle = 60°**

### Q20 MIXED – COMBINED AREA

$$\text{Rectangle area} = 10 \times 8 = 80 \text{ cm}^2$$

$$\text{Semicircle: diameter} = 10, \text{ so radius} = 5$$

$$\text{Semicircle area} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \cdot 25 = 12.5\pi \text{ cm}^2$$

$$\text{Total} = 80 + 12.5\pi \text{ cm}^2$$

$$\text{Approximate: } 80 + 39.27 = 119.27 \text{ cm}^2$$

**ANSWER: Total area = 80 + 12.5π cm<sup>2</sup>**