PRECALCULUS

TRIGONOMETRY

Complete Unit Review Workbook

20 Exam-Style Free-Response Problems · Full Solutions

Units Covered:

Angles & Radian Measure · The Unit Circle · Graphs of Trig Functions
Trig Identities & Equations · Laws of Sines & Cosines

Grades 10–12 | Precalculus Level | Free Response | Answer Key Included

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UNIT 1

Angles & Radian Measure

■ CORE CONCEPTS

Angle Measurement: Angles measured in degrees (°) or radians (rad).

Radian Definition: 1 radian = angle subtended by arc equal to radius length.

Full Circle: $360° = 2\pi$ rad → $180° = \pi$ rad

Arc Length: $s = r \cdot \theta$ (θ in radians)

Sector Area: $A = (1/2)\, r^2\, \theta$ (θ in radians)

Coterminal Angles: $\alpha \pm 360°$ or $\alpha \pm 2\pi$ (same terminal side)

Reference Angle: Acute angle between terminal side and x-axis (always $\geq 0°$)

■ MUST MEMORIZE — Degree ↔ Radian Conversions

$30° = \pi/6$ | $45° = \pi/4$ | $60° = \pi/3$ | $90° = \pi/2$

$120° = 2\pi/3$ | $135° = 3\pi/4$ | $150° = 5\pi/6$ | $180° = \pi$

$210° = 7\pi/6$ | $225° = 5\pi/4$ | $240° = 4\pi/3$ | $270° = 3\pi/2$

$300° = 5\pi/3$ | $315° = 7\pi/4$ | $330° = 11\pi/6$ | $360° = 2\pi$

To convert: deg × (π/180) = rad | rad × (180/π) = deg

✏■ WORKED EXAMPLE

Example 1: Convert 210° to radians. Find a positive coterminal angle.

Step 1: $210° \times (\pi/180) = 210\pi/180 = 7\pi/6$ rad

Step 2: Coterminal → $7\pi/6 + 2\pi = 7\pi/6 + 12\pi/6 =$ **$19\pi/6$**

Answer: $7\pi/6$ rad ; positive coterminal = $19\pi/6$

Q01

[Radian Conversion]

Convert 315° to radians. Express your answer as an exact fraction of π.

Also state one negative coterminal angle (in radians).

Work Space:

Q02

[Arc Length & Sector Area]

A circle has radius 8 cm. A central angle of $5\pi/6$ radians subtends an arc.

(a) Find the arc length. (b) Find the area of the sector. Leave answers exact.

Work Space:

Q03

[Reference Angle]

Find the reference angle for each angle: (a) $7\pi/4$ (b) $5\pi/3$ (c) 240°

Express radian answers as exact fractions of π.

Work Space:

Q04

[Coterminal & Quadrant]

Angle $\theta = -13\pi/6$. (a) Find the smallest positive coterminal angle. (b) In which quadrant does the terminal side lie?

Work Space:

UNIT 2

The Unit Circle & Trigonometric Functions

■ CORE CONCEPTS

Unit Circle: Circle of radius 1 centered at origin. Point $(x, y) = (\cos\theta, \sin\theta)$.

Six Trig Functions:

$\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$

$\csc\theta = r/y$ $\sec\theta = r/x$ $\cot\theta = x/y$ ($r = 1$ on unit circle)

Signs by Quadrant (ASTC rule): All, Sin, Tan, Cos (positive in Q1, Q2, Q3, Q4)

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

Even/Odd: $\cos(-\theta) = \cos\theta$ (even) ; $\sin(-\theta) = -\sin\theta$ (odd) ; $\tan(-\theta) = -\tan\theta$ (odd)

■ MUST MEMORIZE — Unit Circle Values

$\theta = 0$: $(1, 0)$ $\sin 0 = 0$, $\cos 0 = 1$

$\theta = \pi/6$: $(\sqrt{3}/2, 1/2)$ $\sin\pi/6 = 1/2$, $\cos\pi/6 = \sqrt{3}/2$

$\theta = \pi/4$: $(\sqrt{2}/2, \sqrt{2}/2)$ $\sin\pi/4 = \sqrt{2}/2$, $\cos\pi/4 = \sqrt{2}/2$

$\theta = \pi/3$: $(1/2, \sqrt{3}/2)$ $\sin\pi/3 = \sqrt{3}/2$, $\cos\pi/3 = 1/2$

$\theta = \pi/2$: $(0, 1)$ $\sin\pi/2 = 1$, $\cos\pi/2 = 0$

Extend to all quadrants using symmetry and ASTC signs.

✏■ WORKED EXAMPLE

Example 2: Find all six trig values for $\theta = 5\pi/3$.

Step 1: $5\pi/3$ is in Q4 (between $3\pi/2$ and 2π). Reference angle $= 2\pi - 5\pi/3 = \pi/3$.

Step 2: Unit circle values at $\pi/3$: $\cos = 1/2$, $\sin = \sqrt{3}/2$

Step 3: Q4 signs → $\cos > 0$, $\sin < 0$.

$\sin(5\pi/3) = -\sqrt{3}/2$ $\cos(5\pi/3) = 1/2$ $\tan(5\pi/3) = -\sqrt{3}$

$\csc(5\pi/3) = -2/\sqrt{3} = -2\sqrt{3}/3$ $\sec(5\pi/3) = 2$ $\cot(5\pi/3) = -1/\sqrt{3} = -\sqrt{3}/3$

Q05

[Unit Circle Values]

Find exact values of all six trig functions for $\theta = 7\pi/6$.

Show which quadrant and the reference angle used.

Work Space:

Q06

[Pythagorean Identity Application]

Given $\sin\theta = -3/5$ and θ is in Quadrant III, find $\cos\theta$, $\tan\theta$, and $\sec\theta$.

Show all algebraic steps.

Work Space:

Q07

[Even/Odd & Symmetry]

Without a calculator, evaluate: (a) $\cos(-7\pi/6)$ (b) $\sin(-5\pi/4)$ (c) $\tan(-\pi/3)$

Justify each answer using even/odd properties.

Work Space:

Q08

[Finding Angles from Trig Values]

Find all angles $\theta \in [0, 2\pi)$ such that $\cos\theta = -1/2$.

Express answers as exact values in radians.

Work Space:

UNIT 3

Graphs of Trigonometric Functions

■ CORE CONCEPTS

Standard Form: $y = A \sin(Bx + C) + D$ or $y = A \cos(Bx + C) + D$

Amplitude: $|A|$ (half the height from midline to peak)

Period: $T = 2\pi / |B|$ for sin/cos ; $T = \pi / |B|$ for tan/cot

Phase Shift: $-C/B$ (left if negative, right if positive)

Vertical Shift: D (midline $y = D$)

Key Features of sin x: starts at 0, max at $\pi/2$, back to 0 at π, min at $3\pi/2$, period 2π

Key Features of cos x: starts at 1 (max), crosses 0 at $\pi/2$, min at π, period 2π

tan x: period π, vertical asymptotes at $x = \pi/2 + n\pi$

■ MUST MEMORIZE — Transformation Steps

1. **Identify A, B, C, D from $y = A \sin(Bx + C) + D$**
2. **Amplitude = $|A|$; Period = $2\pi/|B|$; Phase shift = $-C/B$; Midline $y = D$**
3. **Key x-values: start, quarter, half, three-quarter, full period**
4. **Reflection: $A < 0$ flips the graph vertically**

✏■ WORKED EXAMPLE

Example 3: State all features of $y = -3 \sin(2x - \pi) + 1$

$A = -3 \rightarrow$ Amplitude = 3, reflection (flipped)

$B = 2 \rightarrow$ Period = $2\pi/2 = \pi$

$C = -\pi \rightarrow$ Phase shift = $-(-\pi)/2 = \pi/2$ (shift right $\pi/2$)

$D = 1 \rightarrow$ Midline: $y = 1$; Max: $y = 4$; Min: $y = -2$

Q09

[Amplitude, Period, Phase Shift]

For $y = 4\cos(3x + \pi/2) - 2$, find: (a) amplitude, (b) period, (c) phase shift, (d) midline, (e) maximum and minimum values.

Work Space:

Q10

[Writing the Equation]

A sinusoidal function has amplitude 5, period 4π, phase shift $\pi/3$ to the right, midline $y = -1$, and starts at a maximum.

Write the equation in the form $y = A\cos(Bx + C) + D$.

Work Space:

Q11

[Tangent Graph]

For $y = 2\tan(x/2 - \pi/4)$, (a) find the period, (b) find two consecutive vertical asymptotes, (c) identify the phase shift.

Work Space:

Q12

[Graph Analysis]

A graph completes one full cycle from $x = \pi/6$ to $x = 7\pi/6$ and has a maximum value of 5 and minimum value of -1.

(a) Find the amplitude, period, and midline. (b) Write a sine equation for the graph.

Work Space:

UNIT 4

Trigonometric Identities & Equations

■ CORE CONCEPTS

Fundamental Identities:

$\sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$

Sum & Difference:

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \blacksquare \sin A \sin B$

$\tan(A \pm B) = (\tan A \pm \tan B) / (1 \blacksquare \tan A \tan B)$

Double Angle:

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Half Angle: $\sin(\theta/2) = \pm\sqrt{((1-\cos\theta)/2)}$; $\cos(\theta/2) = \pm\sqrt{((1+\cos\theta)/2)}$

■ MUST MEMORIZE — Identity Strategy

1. Work on ONE side only (usually the more complex side)
2. Convert everything to sin and cos if stuck
3. Look for Pythagorean substitution opportunities
4. Factor or multiply by conjugate when needed
5. For equations: isolate trig function → use unit circle → find all solutions in given interval

✏■ WORKED EXAMPLE

Example 4: Verify the identity: $\tan^2\theta + 1 = \sec^2\theta$

Start with left side:

$\tan^2\theta + 1 = (\sin^2\theta / \cos^2\theta) + 1$

$= (\sin^2\theta + \cos^2\theta) / \cos^2\theta$

= $1 / \cos^2\theta$ [using $\sin^2\theta + \cos^2\theta = 1$]

= $\sec^2\theta$ ✓

Q13

[Identity Verification]

Prove the identity: $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Show every algebraic step clearly.

Work Space:

Q14

[Sum/Difference Formulas]

Find the exact value of cos(75°) using the identity cos(A - B).

Express the answer with rationalized denominator.

Work Space:

Q15

[Double-Angle Formula]

Given that sin $\theta = 5/13$ and θ is in Quadrant II, find exact values of:

(a) $\sin 2\theta$ (b) $\cos 2\theta$ (c) $\tan 2\theta$

Work Space:

Q16

[Solving Trig Equations]

Solve for all $x \in [0, 2\pi)$: $2 \sin^2 x - \sin x - 1 = 0$

Give all solutions as exact values. Show the factoring step.

Work Space:

UNIT 5

Laws of Sines & Cosines / Applications

■ CORE CONCEPTS

Law of Sines: $a/\sin A = b/\sin B = c/\sin C$

When to use: AAS, ASA (two angles + one side) or SSA (ambiguous case)

Ambiguous Case (SSA): May produce 0, 1, or 2 triangles. Compare $h = b \sin A$ to side a.

$h = b \sin A$: if $a < h$ → no triangle; $a = h$ → one right triangle; $a > h$ → two triangles (if $a < b$)

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Also: $\cos C = (a^2 + b^2 - c^2) / (2ab)$

When to use: SAS (two sides + included angle) or SSS (three sides known)

Area of Triangle: Area = $(1/2)\, ab \sin C$

Heron's Formula (SSS): $s = (a+b+c)/2$; Area = $\sqrt{s(s-a)(s-b)(s-c)}$

■ MUST MEMORIZE — Which Law to Use?

Given AAS or ASA → Law of Sines

Given SSA → Law of Sines (check ambiguous case!)

Given SAS → Law of Cosines (find opposite side first)

Given SSS → Law of Cosines (find any angle)

✏■ WORKED EXAMPLE

Example 5: In triangle ABC: $a = 7$, $B = 38°$, $C = 65°$. Find b.

Step 1: $A = 180° - 38° - 65° = 77°$

Step 2: Law of Sines: $b / \sin B = a / \sin A$

$b / \sin 38° = 7 / \sin 77°$

$b = 7 \sin 38° / \sin 77° \approx 7(0.6157) / 0.9744 \approx 4.42$

Q17

[Law of Sines — AAS]

In triangle ABC: angle A = 40°, angle B = 75°, and side a = 12.

Find sides b and c to the nearest hundredth.

Work Space:

Q18

[Ambiguous Case (SSA)]

In triangle ABC: a = 10, b = 14, A = 30°. Determine how many triangles are possible and find all valid triangles.

Show the height comparison step.

Work Space:

Q19

[Law of Cosines — SAS]

In triangle ABC: a = 8, c = 11, B = 52°.

(a) Find side b. (b) Find angle A. (c) Find the area of the triangle. Round to 2 decimal places.

Work Space:

Q20

[Real-World Application]

Two ships leave a harbor at the same time. Ship A sails 15 km on a bearing of N 35° E. Ship B sails 22 km on a bearing of S 60° E. Find the distance between the two ships.

Draw a diagram label. Use the Law of Cosines. Round to 2 decimal places.

Work Space:

ANSWER KEY

Full Step-by-Step Solutions

Q01 — Radian Conversion

$315° \times (\pi / 180) = 315\pi / 180$

=> Simplify: GCD(315, 180) = 45 → 315/45 = 7, 180/45 = 4

=> $315° = 7\pi/4$ radians

Negative coterminal angle: $7\pi/4 - 2\pi = 7\pi/4 - 8\pi/4 = -\pi/4$

=> Answer: $7\pi/4$ rad ; one negative coterminal = $-\pi/4$

Q02 — Arc Length & Sector Area

Given: $r = 8$ cm, $\theta = 5\pi/6$

(a) Arc length: $s = r\theta = 8 \cdot (5\pi/6) = 40\pi/6$

=> $s = 20\pi/3$ cm ≈ 20.94 cm

(b) Sector area: $A = (1/2)r^2\theta = (1/2)(64)(5\pi/6) = (1/2)(320\pi/6)$

=> $A = 80\pi/3$ cm² ≈ 83.78 cm²

Q03 — Reference Angle

(a) $7\pi/4$ is in Q4 (between $3\pi/2$ and 2π) → ref. angle $= 2\pi - 7\pi/4 = \pi/4$

(b) $5\pi/3$ is in Q4 (between $3\pi/2$ and 2π) → ref. angle $= 2\pi - 5\pi/3 = \pi/3$

(c) 240° is in Q3 (between 180° and 270°) → ref. angle = 240° - 180° = 60°

=> Answers: (a) $\pi/4$ (b) $\pi/3$ (c) 60°

Q04 — Coterminal & Quadrant

$\theta = -13\pi/6$. Add 2π repeatedly until positive:

$-13\pi/6 + 2\pi = -13\pi/6 + 12\pi/6 = -\pi/6$ (still negative)

$-\pi/6 + 2\pi = -\pi/6 + 12\pi/6 = 11\pi/6$

=> Smallest positive coterminal angle = $11\pi/6$

$11\pi/6$ is between $3\pi/2$ (= $9\pi/6$) and 2π (= $12\pi/6$) → Quadrant IV

Q05 — Unit Circle Values

$7\pi/6$ is in Q3 ($\pi < 7\pi/6 < 3\pi/2$). Reference angle = $7\pi/6 - \pi = \pi/6$

At $\pi/6$: sin = 1/2, cos = $\sqrt{3}/2$. Q3: both negative.

$\sin(7\pi/6) = -1/2$ $\cos(7\pi/6) = -\sqrt{3}/2$ $\tan(7\pi/6) = 1/\sqrt{3} = \sqrt{3}/3$

=> $\csc(7\pi/6) = -2$ $\sec(7\pi/6) = -2/\sqrt{3} = -2\sqrt{3}/3$ $\cot(7\pi/6) = \sqrt{3}$

Q06 — Pythagorean Identity Application

$\sin\theta = -3/5$, Q3 (both sin and cos are negative).

$\sin^2\theta + \cos^2\theta = 1 \rightarrow 9/25 + \cos^2\theta = 1 \rightarrow \cos^2\theta = 16/25$

Q3: $\cos\theta < 0 \rightarrow \cos\theta = -4/5$

$\tan\theta = \sin\theta / \cos\theta = (-3/5)/(-4/5) = 3/4$

=> $\sec\theta = 1/\cos\theta = -5/4$

Q07 — Even/Odd & Symmetry

(a) $\cos(-7\pi/6)$: cos is even $\rightarrow \cos(-7\pi/6) = \cos(7\pi/6)$

$7\pi/6$ in Q3, ref = $\pi/6$: $\cos(7\pi/6) = -\sqrt{3}/2$

(b) $\sin(-5\pi/4)$: sin is odd $\rightarrow \sin(-5\pi/4) = -\sin(5\pi/4)$

$5\pi/4$ in Q3, ref = $\pi/4$: $\sin(5\pi/4) = -\sqrt{2}/2 \rightarrow -\sin(5\pi/4) = \sqrt{2}/2$

(c) $\tan(-\pi/3)$: tan is odd $\rightarrow \tan(-\pi/3) = -\tan(\pi/3) = -\sqrt{3}$

=> Answers: (a) $-\sqrt{3}/2$ (b) $\sqrt{2}/2$ (c) $-\sqrt{3}$

Q08 — Finding Angles from Trig Values

$\cos\theta = -1/2$. Reference angle: $\cos\alpha = 1/2 \rightarrow \alpha = \pi/3$

cos is negative in Q2 and Q3:

Q2: $\theta = \pi - \pi/3 = 2\pi/3$

Q3: $\theta = \pi + \pi/3 = 4\pi/3$

=> $\theta = 2\pi/3$ and $\theta = 4\pi/3$

Q09 — Amplitude, Period, Phase Shift

$y = 4\cos(3x + \pi/2) - 2 \rightarrow A = 4, B = 3, C = \pi/2, D = -2$

(a) Amplitude = $|A| = 4$

(b) Period = $2\pi / |B| = 2\pi/3$

(c) Phase shift = $-C/B = -(\pi/2)/3 = -\pi/6$ (shift LEFT $\pi/6$)

(d) Midline: $y = D = -2$

=> (e) Maximum = $D + |A| = -2 + 4 = 2$; Minimum = $D - |A| = -2 - 4 = -6$

Q10 — Writing the Equation

Amplitude = $5 \rightarrow |A| = 5$. Starts at maximum $\rightarrow$ use cosine ($A > 0$): $A = 5$

Period = $4\pi \rightarrow 2\pi/B = 4\pi \rightarrow B = 1/2$

Phase shift = $\pi/3$ right $\rightarrow -C/B = \pi/3 \rightarrow C = -B\cdot(\pi/3) = -(1/2)(\pi/3) = -\pi/6$

Midline $y = -1 \rightarrow D = -1$

=> $y = 5\cos(\,(1/2)x - \pi/6\,) - 1$

Q11 — Tangent Graph

$y = 2\tan(x/2 - \pi/4) \rightarrow B = 1/2,\ C = -\pi/4$

(a) Period $= \pi / |B| = \pi / (1/2) = 2\pi$

(b) Asymptotes occur where $x/2 - \pi/4 = \pi/2 + n\pi$

$x/2 = \pi/2 + \pi/4 + n\pi = 3\pi/4 + n\pi \rightarrow x = 3\pi/2 + 2n\pi$

Two consecutive asymptotes (n=0, n=1): $x = 3\pi/2$ and $x = 3\pi/2 + 2\pi = 7\pi/2$

(c) Phase shift $= -C/B = (\pi/4)/(1/2) = \pi/2$ (shift right $\pi/2$)

Q12 — Graph Analysis

One cycle: $x = \pi/6$ to $x = 7\pi/6 \rightarrow$ Period $= 7\pi/6 - \pi/6 = \pi$

Max = 5, Min = -1

(a) Amplitude $= (5 - (-1))/2 = 3$; Period $= \pi$; Midline $y = (5+(-1))/2 = 2$

(b) $B = 2\pi$ / Period $= 2\pi/\pi = 2$

Sine starts at midline going up: at $x = \pi/6$ the graph begins cycle.

Phase shift $= \pi/6 \rightarrow C = -B\cdot(\pi/6) = -2\cdot(\pi/6) = -\pi/3$

=> $y = 3\sin(2x - \pi/3) + 2$

Q13 — Identity Verification

Left side: $(\sin\theta + \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$ ✓ [since $\sin^2\theta + \cos^2\theta = 1$]

Q14 — Sum/Difference Formulas

$\cos 75° = \cos(45° + 30°) = \cos 45° \cos 30° - \sin 45° \sin 30°$

$= (\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(1/2)$

$= \sqrt{6}/4 - \sqrt{2}/4$

=> $\cos 75° = (\sqrt{6} - \sqrt{2}) / 4$

Q15 — Double-Angle Formula

$\sin\theta = 5/13$, Q2 $\rightarrow \cos\theta = -12/13$ (negative in Q2)

(a) $\sin 2\theta = 2\sin\theta\cos\theta = 2(5/13)(-12/13) = -120/169$

(b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = (144/169) - (25/169) = 119/169$

(c) $\tan 2\theta = \sin 2\theta / \cos 2\theta = (-120/169) / (119/169) = -120/119$

Q16 — Solving Trig Equations

$2 \sin^2 x - \sin x - 1 = 0$

Factor: $(2 \sin x + 1)(\sin x - 1) = 0$

Case 1: $\sin x = -1/2 \rightarrow x = 7\pi/6$ and $x = 11\pi/6$

Case 2: $\sin x = 1 \rightarrow x = \pi/2$

=> Solutions in $[0, 2\pi)$: $x = \pi/2, 7\pi/6, 11\pi/6$

Q17 — Law of Sines — AAS

$A = 40°$, $B = 75°$, $a = 12 \rightarrow C = 180° - 40° - 75° = 65°$

Law of Sines: $a/\sin A = b/\sin B = c/\sin C$

$b = a \sin B / \sin A = 12 \sin 75° / \sin 40° = 12(0.9659)/(0.6428) \approx 18.03$

$c = a \sin C / \sin A = 12 \sin 65° / \sin 40° = 12(0.9063)/(0.6428) \approx 16.92$

=> $b \approx 18.03$; $c \approx 16.92$

Q18 — Ambiguous Case (SSA)

$a = 10$, $b = 14$, $A = 30°$

Height $h = b \sin A = 14 \sin 30° = 14(0.5) = 7$

Since $h = 7 < a = 10 < b = 14 \rightarrow$ TWO triangles possible

Triangle 1: $\sin B = b \sin A / a = 14(0.5)/10 = 0.7 \rightarrow B1 = \arcsin(0.7) \approx 44.43°$

$C1 = 180 - 30 - 44.43 = 105.57°$; $c1 = a \sin C1 / \sin A = 10 \sin(105.57°)/\sin(30°) \approx 19.27$

Triangle 2: $B2 = 180° - 44.43° = 135.57°$

$C2 = 180 - 30 - 135.57 = 14.43°$; $c2 = 10 \sin(14.43°)/\sin(30°) \approx 4.99$

=> Two triangles: ($B\approx44.43°$, $C\approx105.57°$, $c\approx19.27$) and ($B\approx135.57°$, $C\approx14.43°$, $c\approx4.99$)

Q19 — Law of Cosines — SAS

$a = 8$, $c = 11$, $B = 52°$

(a) $b^2 = a^2 + c^2 - 2ac \cos B = 64 + 121 - 2(8)(11)\cos 52°$

$= 185 - 176(0.6157) = 185 - 108.36 = 76.64$

$b = \sqrt{76.64} \approx 8.75$

(b) $\cos A = (b^2 + c^2 - a^2)/(2bc) = (76.64 + 121 - 64)/(2\cdot8.75\cdot11)$

$= 133.64/192.5 \approx 0.6942 \rightarrow A \approx \arccos(0.6942) \approx 46.10°$

(c) Area $= (1/2)\, a c \sin B = (1/2)(8)(11) \sin 52° = 44(0.7880) \approx 34.67$

=> $b \approx 8.75$; $A \approx 46.10°$; Area ≈ 34.67 sq units

Q20 — Real-World Application

Ship A: 15 km at N 35° E ; Ship B: 22 km at S 60° E

Angle between the two paths from harbor:

N 35° E means 35° east of north ; S 60° E means 60° east of south

Total angle between bearings = 180° - 35° - 60° = 85°

Apply Law of Cosines (two sides = 15 and 22, included angle = 85°):

$d^2 = 15^2 + 22^2 - 2(15)(22) \cos 85°$

= 225 + 484 - 660(0.08716)

= 709 - 57.53 = 651.47

$d = \sqrt{651.47} \approx 25.52$ km

=> Distance between ships ≈ 25.52 km

All answers verified. Approximate decimal values rounded to 2 decimal places unless stated otherwise.