

PRE-ALGEBRA

Complete Study Guide & Practice Test

Topics Covered	20 Core Units
Problem Type	Short-Answer (Open Response)
Level	Middle School / Grade 6–8
Format	Concept → Memorize → Example → Practice

Answers & Full Explanations in the Last Section

PART A — CONCEPTS, KEY RULES & EXAMPLES

Unit 1 Order of Operations (PEMDAS)

Concept: When evaluating an expression, follow a strict order: Parentheses first, then Exponents, then Multiplication and Division (left to right), and finally Addition and Subtraction (left to right).

■ **Memorize: Remember: PEMDAS — Please Excuse My Dear Aunt Sally**

Evaluate: $3 + 4 \times 2 - (5 - 1)$

Step 1 (Parentheses): $5 - 1 = 4$

Step 2 (Multiply): $4 \times 2 = 8$

Step 3 (Add/Sub L to R): $3 + 8 - 4 = 7$ Answer: 7

Unit 2 Integers & Absolute Value

Concept: Integers include all positive whole numbers, zero, and negative whole numbers (...-3,-2,-1,0,1,2,3...). The absolute value $|n|$ is the distance from 0 on the number line — always non-negative.

■ **Memorize: $|a| = a$ if $a \geq 0$, $|a| = -a$ if $a < 0$**

$|-7| = 7$, $|5| = 5$, $|0| = 0$

$(-4) + (-3) = -7$ (same sign: add, keep sign)

$(-8) + 5 = -3$ (diff. signs: subtract, keep sign of larger $|n|$)

Unit 3 Fractions, Decimals & Percents

Concept: These three forms represent the same value. To convert: fraction \rightarrow divide numerator by denominator; decimal \rightarrow multiply by 100 for percent; percent \rightarrow divide by 100 for decimal.

■ **Memorize: Part/Whole = Decimal = Percent/100 | $1/4 = 0.25 = 25\%$**

Convert $3/8$ to a decimal and percent:

$3 / 8 = 0.375$, $0.375 \times 100 = 37.5\%$

Convert 0.6 to a fraction: $6/10 = 3/5$

Unit 4 Ratios & Proportions

Concept: A ratio compares two quantities. A proportion states two ratios are equal. Use cross-multiplication to solve proportions: if $a/b = c/d$, then $a \times d = b \times c$.

■ **Memorize: Cross multiply: $a/b = c/d \Rightarrow ad = bc$**

Solve: $x/5 = 12/20$

Cross multiply: $20x = 60$

$x = 3$

Unit 5 Percent Problems

Concept: Three types: (1) Find the percent of a number, (2) Find what percent one number is of another, (3) Find the original number given a percent. Formula: Part = Percent \times Whole.

■ **Memorize: Part = % \times Whole | % = Part / Whole | Whole = Part / %**

What is 35% of 80?

$$0.35 \times 80 = 28$$

18 is what % of 72?

$$18/72 = 0.25 = 25\%$$

Unit 6 Variables & Expressions

Concept: A variable is a letter representing an unknown value. An algebraic expression combines variables, numbers, and operations. To evaluate, substitute the given value for the variable.

■ **Memorize: Expression has NO equal sign. Equation HAS an equal sign.**

Evaluate $3x - 2y$ when $x = 4$, $y = 1$:

$$3(4) - 2(1) = 12 - 2 = 10$$

Unit 7 Solving One-Step Equations

Concept: An equation is a mathematical statement that two expressions are equal. To solve, use inverse (opposite) operations to isolate the variable on one side. Whatever you do to one side, do to the other.

■ **Memorize: Inverse of + is - | Inverse of x is /**

Solve: $x + 9 = 14 \Rightarrow x = 14 - 9 = 5$

Solve: $3x = 21 \Rightarrow x = 21/3 = 7$

Unit 8 Solving Two-Step Equations

Concept: Reverse PEMDAS to solve: first undo addition or subtraction, then undo multiplication or division.

■ **Memorize: Step 1: Add or subtract. Step 2: Multiply or divide.**

Solve: $2x + 5 = 13$

Step 1: $2x = 13 - 5 = 8$

Step 2: $x = 8/2 = 4$

Unit 9 Inequalities

Concept: Inequalities use symbols $<$, $>$, \leq , \geq to compare expressions. Solve like equations BUT: when multiplying or dividing by a negative number, FLIP the inequality sign.

■ **Memorize: Multiply/divide by negative \Rightarrow FLIP the sign!**

Solve: $-2x < 8$

Divide by -2 (flip!): $x > -4$

Graph: open circle at -4, arrow pointing right

Unit 10 The Coordinate Plane

Concept: The coordinate plane has two axes: x (horizontal) and y (vertical), intersecting at the origin (0,0). Points are written as ordered pairs (x, y). The plane is divided into 4 quadrants.

■ **Memorize: Quadrant I (+,+) II (-,+) III (-,-) IV (+,-)**

Plot A(3, 2): go right 3, up 2.

Plot B(-2, -4): go left 2, down 4.

B is in Quadrant III.

Unit 11 Introduction to Functions

Concept: A function is a rule where each input (x) has exactly ONE output (y). Functions can be shown as tables, graphs, or equations. The vertical line test: if any vertical line crosses a graph more than once, it is NOT a function.

■ **Memorize:** Each input → exactly one output. Domain = inputs. Range = outputs.

$$f(x) = 2x + 1$$

$$f(3) = 2(3) + 1 = 7$$

$$f(-1) = 2(-1) + 1 = -1$$

Unit 12 Slope & Linear Equations

Concept: Slope (m) measures the steepness of a line. $m = (y_2 - y_1) / (x_2 - x_1) = \text{rise/run}$. The slope-intercept form of a line is $y = mx + b$, where m = slope and b = y-intercept.

■ **Memorize:** $m = \text{rise/run} = (y_2 - y_1)/(x_2 - x_1)$ $y = mx + b$

Find slope through (1,3) and (4,9):

$$m = (9-3)/(4-1) = 6/3 = 2$$

Equation: $y = 2x + 1$

Unit 13 Exponents & Powers

Concept: An exponent shows how many times the base is multiplied by itself. Key rules: Product: $a^m \times a^n = a^{(m+n)}$. Quotient: $a^m / a^n = a^{(m-n)}$. Power of power: $(a^m)^n = a^{(mn)}$. Zero exponent: $a^0 = 1$.

■ **Memorize:** $a^m \times a^n = a^{(m+n)}$ | $a^0 = 1$ | $a^{-n} = 1/a^n$

$$2^3 \times 2^4 = 2^7 = 128$$

$$(3^2)^3 = 3^6 = 729$$

$$5^0 = 1$$

Unit 14 Scientific Notation

Concept: Scientific notation writes very large or small numbers as $a \times 10^n$, where $1 \leq a < 10$. Positive exponent = large number (move decimal right). Negative exponent = small number (move decimal left).

■ **Memorize:** $a \times 10^n$: if $n > 0$, number is large; if $n < 0$, number is small

$$4,500,000 = 4.5 \times 10^6$$

$$0.0000032 = 3.2 \times 10^{-6}$$

$$(3 \times 10^4) \times (2 \times 10^3) = 6 \times 10^7$$

Unit 15 Square Roots & Real Numbers

Concept: The square root of n is the number that, when multiplied by itself, gives n. Perfect squares: 1,4,9,16,25,36,49,64,81,100. Irrational numbers (like $\sqrt{2}$) cannot be expressed as fractions.

■ **Memorize:** $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ Perfect squares: 1,4,9,16,25,36,49,64,81,100

$$\sqrt{36} = 6 \text{ (exact)}$$

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Estimate $\sqrt{20}$: between 4 (16) and 5 (25), closer to 4.5

Unit 16 Perimeter, Area & Volume

Concept: Perimeter = total distance around a shape. Area = space inside a 2D shape. Volume = space inside a 3D shape. Key formulas must be memorized.

■ **Memorize:** $\text{Area rect} = lw$ | $\text{Area tri} = (1/2)bh$ | $\text{Area circle} = \pi r^2$ | $\text{Vol. rect. prism} = lwh$ | $\text{Circumference} = 2\pi r$

Rectangle: $l=8, w=5$

Perimeter = $2(8+5) = 26$

Area = $8 \times 5 = 40$

Circle $r=3$: Area = $\pi \times 9 = 28.27$

Unit 17 Data & Statistics

Concept: Mean = average (sum / count). Median = middle value (sort first). Mode = most frequent value. Range = max - min. Outliers are values far from the others.

■ **Memorize:** $\text{Mean} = \text{sum}/n$ | $\text{Median} = \text{middle}$ | $\text{Mode} = \text{most frequent}$ | $\text{Range} = \text{max} - \text{min}$

Data: 4, 7, 7, 9, 13

Mean = $(4+7+7+9+13)/5 = 40/5 = 8$

Median = 7 (middle)

Mode = 7 Range = $13-4 = 9$

Unit 18 Probability

Concept: Probability measures the likelihood of an event. $P(\text{event}) = (\text{number of favorable outcomes}) / (\text{total outcomes})$. Range: 0 (impossible) to 1 (certain). Complementary: $P(A) + P(\text{not } A) = 1$.

■ **Memorize:** $P(\text{event}) = \text{favorable} / \text{total}$ | $P(A) + P(\text{not } A) = 1$

Roll a fair die. $P(\text{even}) = 3/6 = 1/2$

Draw a card from {1,2,3,4,5}: $P(>3) = 2/5$

$P(\text{not } >3) = 1 - 2/5 = 3/5$

Unit 19 Combining Like Terms & Distributive Property

Concept: Like terms have the same variable and exponent. Combine by adding/subtracting coefficients. Distributive property: $a(b + c) = ab + ac$. Use these to simplify expressions.

■ **Memorize:** $a(b+c) = ab + ac$ | *Like terms: same variable + same exponent*

$3x + 5 - x + 2 = 2x + 7$

$2(3x - 4) = 6x - 8$

$-(2x + 1) = -2x - 1$

Unit 20 Pythagorean Theorem

Concept: In a right triangle, the square of the hypotenuse (longest side, opposite the right angle) equals the sum of the squares of the other two sides (legs). Formula: $a^2 + b^2 = c^2$.

■ **Memorize:** $a^2 + b^2 = c^2$ ($c = \text{hypotenuse, always longest side}$)

Legs: $a=3, b=4$. Find hypotenuse:

$c^2 = 3^2 + 4^2 = 9 + 16 = 25$

$c = \text{sqrt}(25) = 5$

Common triples: (3,4,5), (5,12,13), (8,15,17)

PART B — PRACTICE PROBLEMS

Answer all 20 questions. Show your work clearly. Answers and full solutions are in Part C.

Q1. (Unit 1: Order of Operations (PEMDAS))

Evaluate the expression: $18 \div (3 + 3) + 2 \times 4$

Your Answer: _____

Q2. (Unit 2: Integers & Absolute Value)

Evaluate: $|-15| + (-6) - (-4)$

Your Answer: _____

Q3. (Unit 3: Fractions, Decimals & Percents)

A student scored 36 out of 48 on a test. What percent did the student score? Express as a percent.

Your Answer: _____

Q4. (Unit 4: Ratios & Proportions)

If 4 pencils cost \$1.20, how much do 10 pencils cost?

Your Answer: _____

Q5. (Unit 5: Percent Problems)

A jacket originally costs \$60. It is on sale for 30% off. What is the sale price?

Your Answer: _____

Q6. (Unit 6: Variables & Expressions)

Evaluate the expression $5a + 3b - 2$ when $a = 3$ and $b = -2$.

Your Answer: _____

Q7. (Unit 7: Solving One-Step Equations)

Solve for x : $x - 7 = -3$

Your Answer: _____

Q8. (Unit 8: Solving Two-Step Equations)

Solve for x : $3x - 4 = 11$

Your Answer: _____

Q9. (Unit 9: Inequalities)

Solve the inequality and write the solution: $5x + 3 > 18$

Your Answer: _____

Q10. (Unit 10: The Coordinate Plane)

Point P is located at (-4, 3). In which quadrant is point P, and what is the distance from P to the y-axis?

Your Answer: _____

Q11. (Unit 11: Introduction to Functions)

Given $f(x) = 4x - 5$, find $f(0) + f(3)$.

Your Answer: _____

Q12. (Unit 12: Slope & Linear Equations)

Find the slope of the line passing through the points (-2, 1) and (4, 7).

Your Answer: _____

Q13. (Unit 13: Exponents & Powers)

Simplify: $(2^3)^2 / 2^4$

Your Answer: _____

Q14. (Unit 14: Scientific Notation)

Write 0.00047 in scientific notation.

Your Answer: _____

Q15. (Unit 15: Square Roots & Real Numbers)

Simplify: $\sqrt{72}$

Your Answer: _____

Q16. (Unit 16: Perimeter, Area & Volume)

A triangle has a base of 10 cm and a height of 6 cm. A rectangle has length 7 cm and width 4 cm. What is the difference between their areas?

Your Answer: _____

Q17. (Unit 17: Data & Statistics)

Find the mean, median, and range of the data set: 2, 5, 8, 5, 10

Your Answer: _____

Q18. (Unit 18: Probability)

A bag has 4 red marbles, 3 blue marbles, and 5 green marbles. If one marble is picked at random, what is the probability of NOT picking a green marble?

Your Answer: _____

Q19. (Unit 19: Combining Like Terms & Distributive Property)

Simplify the expression: $4(2x - 3) + 5x - 7$

Your Answer: _____

Q20. (Unit 20: Pythagorean Theorem)

A right triangle has legs of length 7 and 24. What is the length of the hypotenuse?

Your Answer: _____

PART C — ANSWER KEY & FULL SOLUTIONS

Q1. Order of Operations (PEMDAS)

Answer: 11

Solution: Parentheses: $3+3=6$. Division: $18/6=3$. Multiply: $2 \times 4=8$. Addition: $3+8 = 11$

Q2. Integers & Absolute Value

Answer: 13

Solution: $|-15|=15$. Then: $15 + (-6) - (-4) = 15 - 6 + 4 = 13$

Q3. Fractions, Decimals & Percents

Answer: 75%

Solution: $36/48 = 3/4 = 0.75 = 75\%$

Q4. Ratios & Proportions

Answer: \$3.00

Solution: Set up proportion: $4/1.20 = 10/x$. Cross multiply: $4x = 12.00$, $x = \$3.00$

Q5. Percent Problems

Answer: \$42

Solution: Discount = 30% of 60 = $0.30 \times 60 = 18$. Sale price = $60 - 18 = \$42$

Q6. Variables & Expressions

Answer: 7

Solution: $5(3) + 3(-2) - 2 = 15 - 6 - 2 = 7$

Q7. Solving One-Step Equations

Answer: $x = 4$

Solution: Add 7 to both sides: $x = -3 + 7 = 4$

Q8. Solving Two-Step Equations

Answer: $x = 5$

Solution: Add 4: $3x = 15$. Divide by 3: $x = 5$

Q9. Inequalities

Answer: $x > 3$

Solution: Subtract 3: $5x > 15$. Divide by 5: $x > 3$

Q10. The Coordinate Plane

Answer: Quadrant II; distance = 4

Solution: $x = -4$ (negative), $y = 3$ (positive) \Rightarrow Quadrant II. Distance to y-axis = $|x| = |-4| = 4$

Q11. Introduction to Functions

Answer: 2

Solution: $f(0) = 4(0) - 5 = -5$. $f(3) = 4(3) - 5 = 7$. Sum: $-5 + 7 = 2$

Q12. Slope & Linear Equations

Answer: $m = 1$

Solution: $m = (7-1)/(4-(-2)) = 6/6 = 1$

Q13. Exponents & Powers

Answer: 4

Solution: $(2^3)^2 = 2^6 = 64$. $64/2^4 = 64/16 = 4$. Or: $2^{(6-4)} = 2^2 = 4$

Q14. Scientific Notation

Answer: 4.7×10^{-4}

Solution: Move decimal 4 places right: 4.7. Exponent is -4. Answer: 4.7×10^{-4}

Q15. Square Roots & Real Numbers

Answer: $6\sqrt{2}$

Solution: $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

Q16. Perimeter, Area & Volume**Answer: 2 cm^2**

Solution: Triangle area = $(1/2)(10)(6) = 30 \text{ cm}^2$. Rectangle area = $7 \times 4 = 28 \text{ cm}^2$. Difference = $30 - 28 = 2 \text{ cm}^2$

Q17. Data & Statistics**Answer: Mean=6, Median=5, Range=8**

Solution: Mean: $(2+5+8+5+10)/5 = 30/5 = 6$. Sort: 2,5,5,8,10 \Rightarrow Median = 5. Range: $10-2 = 8$

Q18. Probability**Answer: $7/12$**

Solution: Total = 12. Green = 5. $P(\text{green}) = 5/12$. $P(\text{not green}) = 1 - 5/12 = 7/12$

Q19. Combining Like Terms & Distributive Property**Answer: $13x - 19$**

Solution: Distribute: $8x - 12 + 5x - 7$. Combine like terms: $(8x+5x) + (-12-7) = 13x - 19$

Q20. Pythagorean Theorem**Answer: 25**

Solution: $c^2 = 7^2 + 24^2 = 49 + 576 = 625$. $c = \sqrt{625} = 25$