# Core Problems Workbook

## All 10 Units | 20 Exam-Style Problems

**Unit 1 Unit 2 Unit 3 Unit 4 Unit 5 Unit 6 Unit 7 Unit 8 Unit 9 Unit 10**

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UNIT 1

# Real Numbers & Properties

## CONCEPT: NUMBER SETS

- Natural Numbers (N): 1, 2, 3, ...
- Whole Numbers (W): 0, 1, 2, 3, ...
- Integers (Z): ..., -2, -1, 0, 1, 2, ...
- Rational Numbers (Q): any number a/b where b is not 0
- Irrational Numbers: non-repeating, non-terminating decimals (e.g. sqrt(2), pi)
- Real Numbers (R): all rational + irrational numbers

## CONCEPT: KEY PROPERTIES

- Commutative: a + b = b + a | a x b = b x a
- Associative: (a+b)+c = a+(b+c)
- Distributive: a(b+c) = ab + ac
- Identity: a + 0 = a | a x 1 = a
- Inverse: a + (-a) = 0 | a x (1/a) = 1 (a not 0)

### ★ MUST MEMORIZE

• PEMDAS order: Parentheses, Exponents, Multiply/Divide, Add/Subtract

• Absolute value |x| = distance from zero — always >= 0

• Classify: sqrt(4)=2 (rational), sqrt(5) (irrational)

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Classify -7/3 | **Ans: Rational (integer over integer)** |
| **Ex 2.** | Simplify: 3 + 2 x (8 - 5)^2 | **Ans: 3 + 2 x 9 = 3 + 18 = 21** |
| **Ex 3.** | \|-13\| + \|5\| | **Ans: 13 + 5 = 18** |

UNIT 2

# Expressions & Variables

## CONCEPT: KEY TERMS

- Variable: a letter representing an unknown value (e.g. x, y, n)
- Constant: a fixed number (e.g. 5, -3, pi)
- Coefficient: the number multiplying a variable (in 4x, coefficient = 4)
- Term: a single number, variable, or product (3x, -7, 5xy)
- Expression: terms combined with + or - (3x + 5)
- Like Terms: same variable(s) and exponent(s) (3x and -7x are like terms)

## CONCEPT: SIMPLIFYING

- Combine like terms: 5x + 3y - 2x + y = 3x + 4y
- Distributive: 4(2x - 3) = 8x - 12
- Expand: -(x - 5) = -x + 5 (distribute the negative)

### ★ MUST MEMORIZE

• Only LIKE TERMS can be combined (same variable, same power)

• The term '3' means 3 x 1, the term '3x' means 3 times x

• Distributing a negative: -(a - b) = -a + b

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Simplify: 6x - 3 + 2x + 9 | **Ans: 8x + 6** |
| **Ex 2.** | Expand: -3(4x - 2) | **Ans: -12x + 6** |
| **Ex 3.** | Evaluate 2x^2 - x + 1 when x = -2 | **Ans: 2(4) - (-2) + 1 = 8 + 2 + 1 = 11** |

UNIT 3

# Solving Linear Equations

## CONCEPT: ONE-STEP & TWO-STEP

- Goal: isolate the variable on one side
- One-step: x + 7 = 12 --> x = 5
- Two-step: 2x - 3 = 11 --> 2x = 14 --> x = 7
- Division step: always divide BOTH sides by the same number

## CONCEPT: MULTI-STEP & SPECIAL CASES

- Distribute first, then combine like terms, then solve
- Variables on both sides: move all variable terms to one side
- No solution: false statement (e.g. 0 = 5)
- Infinite solutions: true statement (e.g. 0 = 0)

### ★ MUST MEMORIZE

- Whatever you do to one side, do to the OTHER side
- Move constants AWAY from the variable (use opposite operations)
- Check answer: substitute back into the original equation

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | 5x + 3 = 2x - 9 | **Ans: 3x = -12 --> x = -4** |
| **Ex 2.** | 3(x - 2) = 2x + 7 | **Ans: 3x - 6 = 2x + 7 --> x = 13** |
| **Ex 3.** | 2(x + 1) = 2x + 2 | **Ans: Infinite solutions (identity)** |

UNIT 4

# Inequalities

## CONCEPT: SOLVING & GRAPHING

- Solve like equations, but: flip the inequality when multiplying or dividing by a NEGATIVE
- Solution is a range: x > 3, x <= -1, etc.
- Open circle ( o ) = strict inequality (< or >)
- Closed circle ( . ) = inclusive inequality (<= or >=)

## CONCEPT: COMPOUND INEQUALITIES

- AND: both conditions true --> -2 < x AND x < 5 --> -2 < x < 5
- OR: at least one condition true --> x < -1 OR x > 4

**★ MUST MEMORIZE**

• FLIP the sign when multiplying or dividing by a NEGATIVE

• -3x > 12 --> x < -4 (sign flipped!)

• AND = intersection (overlap), OR = union (either region)

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Solve: -4x + 1 > 13 | **Ans: -4x > 12 --> x < -3** |
| **Ex 2.** | Solve: -1 <= 2x + 3 < 9 | **Ans: -4 <= 2x < 6 --> -2 <= x < 3** |
| **Ex 3.** | Solve: \|x - 2\| < 5 | **Ans: -5 < x - 2 < 5 --> -3 < x < 7** |

UNIT 5

# Linear Functions & Slope

## CONCEPT: SLOPE & FORMS

- Slope m = (y2 - y1) / (x2 - x1) (rise over run)
- Slope-intercept form: y = mx + b (m=slope, b=y-intercept)
- Standard form: Ax + By = C
- Point-slope form: y - y1 = m(x - x1)
- Horizontal line: slope = 0 (y = k)
- Vertical line: slope = undefined (x = k)

## CONCEPT: PARALLEL & PERPENDICULAR

- Parallel lines: SAME slope (m1 = m2), different y-intercepts
- Perpendicular lines: slopes are NEGATIVE RECIPROCALS (m1 x m2 = -1)

### ★ MUST MEMORIZE

• Slope = rise/run = change in y / change in x

• Parallel: same slope | Perpendicular: slopes multiply to -1

• y-intercept is where x = 0 | x-intercept is where y = 0

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Slope through (-1, 4) and (3, -4) | **Ans: m = (-4-4)/(3-(-1)) = -8/4 = -2** |
| **Ex 2.** | Equation through (2, 1), slope 3 | **Ans: y - 1 = 3(x - 2) --> y = 3x - 5** |
| **Ex 3.** | Is y=2x+1 parallel to y=2x-7? | **Ans: Yes — same slope m = 2** |

UNIT 6

# Systems of Equations

## CONCEPT: THREE METHODS

- Graphing: find the intersection point of two lines
- Substitution: solve one equation for one variable, substitute into the other
- Elimination: add or subtract equations to eliminate one variable

## CONCEPT: TYPES OF SOLUTIONS

- One solution: lines intersect at exactly one point (consistent & independent)
- No solution: parallel lines — no intersection (inconsistent)
- Infinite solutions: same line — overlap everywhere (consistent & dependent)

**★ MUST MEMORIZE**

- Substitution works best when one variable has coefficient 1
- Elimination: multiply equations so coefficients match, then add/subtract
- Always check the solution in BOTH original equations

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Solve: x + y = 5, x - y = 1 | **Ans: Add: 2x = 6 --> x = 3, y = 2** |
| **Ex 2.** | Solve: 2x + 3y = 7, y = x - 1 | **Ans: Sub y: 2x + 3(x-1) = 7 --> x = 2, y = 1** |
| **Ex 3.** | Lines y = 3x + 1 and y = 3x - 4 | **Ans: No solution (parallel, different intercepts)** |

UNIT 7

# Exponents & Polynomials

## CONCEPT: EXPONENT RULES

- Product rule: x^a x x^b = x^(a+b)
- Quotient rule: x^a / x^b = x^(a-b)
- Power rule: (x^a)^b = x^(ab)
- Zero exponent: x^0 = 1 (x not 0)
- Negative exponent: x^(-n) = 1/x^n

## CONCEPT: POLYNOMIAL OPERATIONS

- Add/Subtract: combine like terms only
- Multiply: use Distributive Property (FOIL for two binomials)
- FOIL: (a+b)(c+d) = ac + ad + bc + bd
- Special products: (a+b)^2 = a^2 + 2ab + b^2
- Difference of squares: (a+b)(a-b) = a^2 - b^2

**★ MUST MEMORIZE**

• Adding exponents = multiplying bases (x^3 x x^4 = x^7)

• FOIL: First, Outer, Inner, Last

• (a+b)^2 ≠ a^2 + b^2 (don't forget the 2ab middle term!)

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Simplify: (3x^2)(4x^3) | **Ans: 12x^5** |
| **Ex 2.** | Expand: (x + 5)(x - 3) | **Ans: x^2 + 2x - 15** |
| **Ex 3.** | Expand: (2x - 3)^2 | **Ans: 4x^2 - 12x + 9** |

UNIT 8

# Factoring

## CONCEPT: FACTORING METHODS

- GCF (Greatest Common Factor): always check first! 6x^2 + 9x = 3x(2x + 3)
- Trinomial (a=1): x^2 + bx + c = (x + p)(x + q) where p+q=b, pxq=c
- Trinomial (a>1): use AC method or guess-and-check
- Difference of Squares: a^2 - b^2 = (a+b)(a-b)
- Perfect Square Trinomial: a^2 + 2ab + b^2 = (a+b)^2

## CONCEPT: SOLVING BY FACTORING

- Set the equation equal to zero: ax^2 + bx + c = 0
- Factor completely
- Zero Product Property: if AB = 0, then A = 0 OR B = 0
- Solve each factor for the variable

### ★ MUST MEMORIZE

• Always factor out GCF FIRST before other methods

• For x^2 + bx + c: find two numbers that ADD to b and MULTIPLY to c

• Zero Product Property: if (x-3)(x+5) = 0, then x = 3 or x = -5

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Factor: x^2 - 5x - 14 | **Ans: (x - 7)(x + 2)** |
| **Ex 2.** | Factor: 4x^2 - 25 | **Ans: (2x + 5)(2x - 5) [difference of squares]** |
| **Ex 3.** | Solve: x^2 + x - 12 = 0 | **Ans: (x+4)(x-3) = 0 --> x = -4 or x = 3** |

UNIT 9

# Quadratic Functions

## CONCEPT: STANDARD & VERTEX FORMS

- Standard form: y = ax^2 + bx + c
- Vertex form: y = a(x - h)^2 + k (vertex at (h, k))
- Vertex x-coordinate: x = -b / (2a) from standard form
- Axis of symmetry: x = -b / (2a) (vertical line through vertex)
- Parabola opens UP if a > 0, opens DOWN if a < 0

## CONCEPT: SOLVING QUADRATICS

- By factoring: set = 0, factor, use Zero Product Property
- By square root: x^2 = k --> x = +/- sqrt(k)
- Completing the square: rewrite as (x + d)^2 = e
- Quadratic Formula: x = (-b +/- sqrt(b^2 - 4ac)) / (2a)
- Discriminant: b^2 - 4ac > 0: 2 real roots | = 0: 1 root | < 0: no real roots

### ★ MUST MEMORIZE

• Vertex x = -b/(2a), then plug in to find y

• Discriminant tells you HOW MANY solutions without solving

• Quadratic Formula works for ALL quadratics

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Vertex of y = 2x^2 - 8x + 3 | **Ans: x = 4/2 = 2, y = 8 - 16 + 3 = -5 --> (2, -5)** |
| **Ex 2.** | Solve: x^2 = 49 | **Ans: x = +/- 7** |
| **Ex 3.** | Discriminant of x^2 - 4x + 5 | **Ans: 16 - 20 = -4 --> no real solutions** |

UNIT 10

# Statistics & Data Analysis

## CONCEPT: MEASURES OF CENTER & SPREAD

- Mean: sum of all values divided by count
- Median: middle value when ordered (average of two middle if even count)
- Mode: most frequently occurring value
- Range: maximum - minimum
- Interquartile Range (IQR): Q3 - Q1
- Outlier: value below Q1 - 1.5(IQR) or above Q3 + 1.5(IQR)

## CONCEPT: SCATTER PLOTS & CORRELATION

- Positive correlation: as x increases, y increases
- Negative correlation: as x increases, y decreases
- No correlation: no visible pattern
- Line of best fit (trend line): drawn to minimize distance to all points
- Correlation does NOT imply causation

### ★ MUST MEMORIZE

• Mean is affected by outliers; median is more RESISTANT

• IQR = Q3 - Q1 (middle 50% spread)

• r close to 1 or -1 = strong correlation | r close to 0 = weak

## WORKED EXAMPLES

| | | |
|---|---|---|
| **Ex 1.** | Mean of: 4, 7, 7, 9, 3 | **Ans: Sum=30, n=5 --> Mean = 6** |
| **Ex 2.** | Median of: 2, 5, 8, 11, 14 | **Ans: Middle value = 8** |
| **Ex 3.** | IQR of: 2, 4, 6, 8, 10 | **Ans: Q1=4, Q3=8 --> IQR = 4** |

EXAM

# 20 Exam-Style Problems

Write your full solution for each problem. Show all work to receive full credit. Answer key with explanations is on the final pages.

**Q01** Unit 1: Real Numbers

Which of the following is an irrational number?
(A) -5/3 (B) sqrt(16) (C) sqrt(7) (D) 0.333...

**Answer:** _____

**Q02** Unit 1: Real Numbers

Evaluate: 18 / 3 + 2^3 - (4 - 1) x 2

**Answer:** _____

**Q03** Unit 2: Expressions

Simplify: 4(3x - 1) - 2(x + 5)

**Answer:** _____

**Q04** Unit 2: Expressions

Evaluate 3x^2 - 2x + 4 when x = -3

**Answer:** _____

**Q05** Unit 3: Equations

Solve for x: 3(2x - 4) = 2(x + 6)

**Answer:** _____

**Q06** Unit 3: Equations

Solve: (x/3) + 5 = 2x - 1

**Answer:** _____

**Q07** Unit 4: Inequalities

Solve and write in interval notation: 3 - 2x >= 9

**Answer:** _____

**Q08** Unit 4: Inequalities

Solve: -3 < 4x - 1 <= 11

**Answer:** _____

**Q09** Unit 5: Linear Functions

Find the equation of the line through (3, -2) and (-1, 6).

**Answer:**

## Q10

Unit 5: Linear Functions

Are the lines 4x - 2y = 8 and y = 2x + 5 parallel, perpendicular, or neither?

**Answer:**

**Q11** Unit 6: Systems

Solve the system: 2x + 3y = 12 and x - y = 1

**Answer:** _____

**Q12** Unit 6: Systems

Solve using elimination: 3x - 2y = 7 and -3x + 5y = -1

**Answer:** _____

**Q13** Unit 7: Exponents

Simplify: (2x^2 y^3)^3 / (4x^3 y)

**Answer:** _____

**Q14** Unit 7: Polynomials

Multiply: (3x - 2)(2x + 5)

**Answer:** _____

**Q15** Unit 8: Factoring

Factor completely: 2x^3 - 8x^2 - 24x

**Answer:** _____

**Q16** Unit 8: Factoring

Solve by factoring: 6x^2 - 7x - 3 = 0

**Answer:** _____

**Q17** Unit 9: Quadratics

Find the vertex and axis of symmetry: y = -3x^2 + 12x - 5

**Answer:** _____

**Q18** Unit 9: Quadratics

Solve using the Quadratic Formula: 2x^2 - 3x - 5 = 0

**Answer:** _____

**Q19** Unit 10: Statistics

Data set: 12, 15, 11, 14, 18, 11, 20. Find the mean, median, and mode.

**Answer:** _____

**Q20** Unit 10: Statistics

The five-number summary for a data set is: Min=4, Q1=9, Median=15, Q3=22, Max=31. Identify any outliers.

**Answer:** _____

ANSWER KEY

# Full Solutions & Explanations

**Q01** **Answer: C — sqrt(7)**

sqrt(7) cannot be written as a ratio of integers (approx. 2.6457..., non-terminating, non-repeating). sqrt(16)=4 is rational; -5/3 and 0.333...=1/3 are also rational.

**Q02** **Answer: 8**

Follow PEMDAS: 18/3=6, 2^3=8, (4-1)x2=6. Then: 6 + 8 - 6 = 8.

**Q03** **Answer: 10x - 14**

Distribute: 12x - 4 - 2x - 10 = 10x - 14.

**Q04** **Answer: 37**

3(9) - 2(-3) + 4 = 27 + 6 + 4 = 37.

**Q05** **Answer: x = 6**

6x - 12 = 2x + 12 --> 4x = 24 --> x = 6.

**Q06** **Answer: x = 18/5**

x/3 + 5 = 2x - 1 --> x + 15 = 6x - 3 --> 5x = 18 --> x = 18/5.

**Q07** **Answer: x <= -3 or (-inf, -3]**

3 - 2x >= 9 --> -2x >= 6 --> x <= -3 (flip sign!).

**Q08** **Answer: -1/2 < x <= 3**

Add 1: -2 < 4x <= 12. Divide by 4: -1/2 < x <= 3.

**Q09** **Answer: y = -2x + 4**

Slope = (6-(-2))/(-1-3) = 8/(-4) = -2. Use point: y+2 = -2(x-3) --> y = -2x+4.

**Q10** **Answer: Parallel**

Rewrite 4x-2y=8 as y=2x-4. Both have slope m=2 but different y-intercepts --> parallel.

**Q11** **Answer: x = 3, y = 2**

From eq2: x = y+1. Sub: 2(y+1)+3y=12 --> 5y=10 --> y=2, x=3.

**Q12** **Answer: x = 11/3, y = 2**

Add equations: 3y = 6 --> y = 2. Sub into eq1: 3x - 4 = 7 --> x = 11/3.

**Q13** **Answer: 2x^3 y^8**

Numerator: 8x^6 y^9. Divide by 4x^3 y: 2x^3 y^8.

**Q14** **Answer: 6x^2 + 11x - 10**

FOIL: 6x^2 + 15x - 4x - 10 = 6x^2 + 11x - 10.

**Q15** **Answer: 2x(x - 6)(x + 2)**

GCF=2x: 2x(x^2-4x-12). Factor trinomial: 2x(x-6)(x+2).

**Q16** **Answer: x = 3/2 or x = -1/3**

AC method: factors of -18 summing to -7 are -9,+2. 6x^2-9x+2x-3 = 3x(2x-3)+1(2x-3) = (3x+1)(2x-3)=0. x=3/2 or x=-1/3.

**Q17** **Answer: Vertex (2, 7); Axis x = 2**

x = -12/(2x(-3)) = 2. y = -3(4)+24-5 = 7. Vertex (2,7).

**Q18** **Answer: x = 5/2 or x = -1**

Discriminant = 9+40=49. x=(3+/-7)/4. x=10/4=5/2 or x=-4/4=-1.

**Q19** **Answer: Mean=101/7≈14.4; Median=14; Mode=11**

Ordered: 11,11,12,14,15,18,20. Sum=101. Mean=101/7. Middle=14. Mode=11 (appears twice).

**Q20** **Answer: No outliers**

IQR=13. Lower fence=9-19.5=-10.5; Upper fence=22+19.5=41.5. All data between -10.5 and 41.5, so no outliers.