ALGEBRA 2

COMPREHENSIVE PRACTICE TEST

20 Exam-Style Free-Response Questions

Concepts · Formulas · Worked Examples · Answer Key

10 Core Units | SAT / Honors Level

Instructions: Answer all questions showing complete work. Answers without justification receive no credit.
Answer key with full solutions begins on the final page.

UNIT 1: Polynomial Functions & Equations

KEY CONCEPTS
A polynomial function has the form f(x) = a_n x^n + ... + a_1 x + a_0
Degree determines the end behavior and maximum number of roots
The Fundamental Theorem of Algebra: every degree-n polynomial has exactly n complex roots (counting multiplicity)
Remainder Theorem: f(c) = remainder when f(x) is divided by (x - c)
Factor Theorem: (x - c) is a factor of f(x) if and only if f(c) = 0

FORMULAS TO MEMORIZE
Sum of roots = -b/a (for ax^2 + bx + c = 0)
Product of roots = c/a
Remainder when f(x) / (x-c) = f(c)

WORKED EXAMPLE

Problem: Find all zeros of f(x) = x^3 - 6x^2 + 11x - 6.

Solution: Test x = 1: f(1) = 1 - 6 + 11 - 6 = 0. Factor: (x-1)(x-2)(x-3). Zeros: x = 1, 2, 3.

PRACTICE PROBLEMS

Question 1

Divide $f(x) = 2x^3 + 3x^2 - 11x - 6$ by (x - 2) using synthetic division. Find the quotient and remainder.

Question 2

Find all real zeros of $p(x) = x^4 - 5x^2 + 4$. Factor completely over the real numbers.

UNIT 2: Rational Expressions & Equations

KEY CONCEPTS

- A rational expression is a ratio of two polynomials: $p(x)/q(x)$, $q(x) \neq 0$
- Holes occur when a factor cancels; vertical asymptotes occur when the denominator is 0 after cancellation
- Horizontal asymptote: if deg(num) < deg(den) → y = 0; if equal → y = leading coeff ratio; if num > den → oblique asymptote
- To solve a rational equation: multiply both sides by LCD, check for extraneous solutions

FORMULAS TO MEMORIZE

- **LCD method: multiply every term by the least common denominator**
- **Always check solutions: substitute back and verify denominator ≠ 0**

WORKED EXAMPLE

Problem: Solve: 1/(x-2) + 1/(x+2) = 4/(x^2-4).

Solution: LCD = (x-2)(x+2). Multiply: (x+2) + (x-2) = 4 → 2x = 4 → x = 2. Check: x=2 makes denominator 0. No solution (extraneous).

PRACTICE PROBLEMS

Question 3

Simplify the rational expression: $(x^2 - 9) / (x^2 - x - 6)$. State any restrictions on x.

Question 4

Solve for x: $(3)/(x+1) - (2)/(x-1) = 1/(x^2-1)$. Identify any extraneous solutions.

UNIT 3: Radical Functions & Equations

KEY CONCEPTS
nth root: if a^n = b, then a = b^(1/n)
Rational exponents: x^(m/n) = (x^(1/n))^m = (nth root of x)^m
When solving radical equations, always isolate the radical first, then raise both sides to a power
Extraneous solutions can appear — always check in the ORIGINAL equation
Domain of sqrt(f(x)): set f(x) >= 0 and solve

FORMULAS TO MEMORIZE
x^(m/n) = (nth root of x)^m
sqrt(a) * sqrt(b) = sqrt(ab) (a,b >= 0)

WORKED EXAMPLE

Problem: Solve: sqrt(2x + 3) = x - 1.

Solution: Square both sides: 2x+3 = (x-1)^2 = x^2-2x+1. Rearrange: x^2-4x-2=0 → x=(4±sqrt(24))/2 = 2±sqrt(6). Check both in original; x = 2+sqrt(6) is valid.

PRACTICE PROBLEMS

Question 5

Solve for x: sqrt(3x + 4) + 2 = x. Show all steps and check for extraneous solutions.

Question 6

Simplify: $(27x^6)^{2/3}$. Express your answer without negative exponents.

UNIT 4: Exponential & Logarithmic Functions

KEY CONCEPTS

- Exponential function: f(x) = a * b^x, where b > 0, b ≠ 1
- Growth: b > 1; Decay: 0 < b < 1
- Logarithm: log_b(x) = y ■ b^y = x
- Natural log: ln(x) = log_e(x)
- Change of base: log_b(x) = log(x)/log(b)

FORMULAS TO MEMORIZE

- **log(xy) = log(x) + log(y)**
- **log(x/y) = log(x) - log(y)**
- **log(x^n) = n*log(x)**
- **A = P * e^(rt) (continuous compound interest)**

WORKED EXAMPLE

Problem: Solve: 3^(2x-1) = 81.

Solution: Write 81 = 3^4. So 2x-1 = 4, giving x = 5/2.

PRACTICE PROBLEMS

Question 7

Solve: $\log_2(x + 3) + \log_2(x - 1) = 5$. Identify any extraneous solutions.

Question 8

An investment of $5,000 grows continuously at 4.2% annual interest. Write the model A(t) and determine how long (to the nearest month) it takes to double.

UNIT 5: Systems of Equations & Matrices

KEY CONCEPTS

- A system of equations can be solved by substitution, elimination, or matrices
- A matrix equation AX = B is solved by $X = A^{-1}B$ when $\det(A) \neq 0$
- Row operations: swap rows, scale a row, add a multiple of one row to another
- Cramer's Rule: x = det(Ax)/det(A), y = det(Ay)/det(A)

FORMULAS TO MEMORIZE

- **For 2x2 matrix A = [[a,b],[c,d]]: det(A) = ad - bc**
- **A^(-1) = (1/det(A)) * [[d,-b],[-c,a]]**

WORKED EXAMPLE

Problem: Solve: 2x + y = 7, x - y = 2 using elimination.

Solution: Add equations: 3x = 9, x = 3. Substitute: y = 7 - 6 = 1. Solution: (3, 1).

PRACTICE PROBLEMS

Question 9

Solve the system using matrices (row reduction): 2x + y - z = 8 -3x - y + 2z = -11 -2x + y + 2z = -3

Question 10

Find the inverse of matrix A = [[3, 1], [5, 2]] and use it to solve A·[x, y]^T = [7, 13]^T.

UNIT 6: Sequences, Series & Binomial Theorem

KEY CONCEPTS
Arithmetic sequence: a_n = a_1 + (n-1)d, common difference d
Geometric sequence: a_n = a_1 * r^(n-1), common ratio r
Arithmetic series sum: S_n = n/2 * (a_1 + a_n)
Geometric series sum: S_n = a_1(1 - r^n)/(1 - r) for r ≠ 1
Infinite geometric sum: S = a_1/(1-r), only if |r| < 1
Binomial Theorem: (a+b)^n = sum of C(n,k) a^(n-k) b^k

FORMULAS TO MEMORIZE
S_n (arith.) = n(a_1 + a_n)/2
S_n (geom.) = a_1(1 - r^n)/(1-r)
C(n,k) = n! / (k!(n-k)!)

WORKED EXAMPLE

Problem: Find the sum of the first 20 terms of arithmetic sequence: 4, 7, 10, ...

Solution: d = 3, a_1 = 4, a_20 = 4 + 19(3) = 61. S_20 = 20/2 * (4+61) = 10 * 65 = 650.

PRACTICE PROBLEMS

Question 11

A geometric series has first term $a_1 = 6$ and common ratio r = 1/2. Find (a) the 8th term, and (b) the sum of the infinite series.

Question 12

Find the coefficient of x^3y^4 in the expansion of $(2x - y)^7$ using the Binomial Theorem.

UNIT 7: Conic Sections

KEY CONCEPTS
Parabola: vertex form y = a(x-h)^2 + k; focus at (h, k + 1/(4a))
Circle: (x-h)^2 + (y-k)^2 = r^2, center (h,k), radius r
Ellipse: (x-h)^2/a^2 + (y-k)^2/b^2 = 1, a > b; foci at c = sqrt(a^2 - b^2)
Hyperbola: (x-h)^2/a^2 - (y-k)^2/b^2 = 1; asymptotes y - k = ±(b/a)(x - h)

FORMULAS TO MEMORIZE
Ellipse: c^2 = a^2 - b^2 (foci distance)
Hyperbola: c^2 = a^2 + b^2
Parabola focus: (h, k + p) where 4p = 1/a

WORKED EXAMPLE

Problem: Write the equation of the ellipse with vertices (±5, 0) and co-vertices (0, ±3).

Solution: a = 5, b = 3. Equation: x^2/25 + y^2/9 = 1.

PRACTICE PROBLEMS

Question 13

Convert to standard form and identify the conic: $4x^2 + 9y^2 - 16x + 18y - 11 = 0$. State center, vertices, and foci.

Question 14

A parabolic satellite dish has the equation $x^2 = 12y$ (in cm). Find the coordinates of the focus and the equation of the directrix.

UNIT 8: Probability & Statistics

KEY CONCEPTS

- Permutations (order matters): $P(n,r) = n!/(n-r)!$
- Combinations (order doesn't matter): $C(n,r) = n!/(r!(n-r)!)$
- Probability: P(A) = favorable outcomes / total outcomes
- P(A or B) = P(A) + P(B) - P(A and B)
- P(A|B) = P(A and B) / P(B) (conditional probability)
- Expected value: E(X) = sum of x * P(x)

FORMULAS TO MEMORIZE

- **P(A') = 1 - P(A) (complement)**
- **Binomial: $P(X=k) = C(n,k)\, p^k (1-p)^{(n-k)}$**

WORKED EXAMPLE

Problem: A bag has 4 red and 6 blue marbles. Drawing 2 without replacement, what is P(both red)?

Solution: $P = C(4,2)/C(10,2) = 6/45 = 2/15$.

PRACTICE PROBLEMS

Question 15

A committee of 4 is chosen from 7 men and 5 women. What is the probability the committee has exactly 2 women? Express as a simplified fraction.

Question 16

A fair die is rolled 5 times. Using the binomial distribution, find the exact probability of rolling a 6 exactly twice.

UNIT 9: Trigonometric Functions

KEY CONCEPTS
Unit circle: ($\cos \theta$, $\sin \theta$) for angle θ from positive x-axis
Period of sin/cos: 2π; period of tan: π
Amplitude: $\|A\|$ in $y = A \sin(Bx + C) + D$
Phase shift: $-C/B$; Vertical shift: D
Reference angles: always acute angle between terminal side and x-axis

FORMULAS TO MEMORIZE
sin^2(θ) + cos^2(θ) = 1
tan(θ) = sin(θ)/cos(θ)
Period = $2\pi/\|B\|$ for y = A sin(Bx)

WORKED EXAMPLE

Problem: Find all solutions of sin(x) = sqrt(3)/2 in $[0, 2\pi]$.

Solution: Reference angle = $\pi/3$. Solutions in $[0,2\pi]$: $x = \pi/3$ and $x = 2\pi/3$.

PRACTICE PROBLEMS

Question 17

Determine the amplitude, period, phase shift, and vertical shift of: $y = -3 \sin(2x - \pi/4) + 1$. Sketch one complete cycle.

Question 18

Prove the identity: $(1 - \cos^2 x) / \sin x = \sin x$. State the domain restrictions.

UNIT 10: Complex Numbers & Quadratic Applications

KEY CONCEPTS
Complex number: a + bi, where i = sqrt(-1), i^2 = -1
Addition: (a+bi) + (c+di) = (a+c) + (b+d)i
Multiplication: (a+bi)(c+di) = (ac-bd) + (ad+bc)i
Conjugate of (a+bi) is (a-bi); product = a^2 + b^2
Modulus: |a+bi| = sqrt(a^2 + b^2)
Discriminant: b^2 - 4ac determines nature of quadratic roots

FORMULAS TO MEMORIZE
Quadratic formula: x = (-b ± sqrt(b^2-4ac)) / 2a
Discriminant > 0: two real roots; = 0: one real root; < 0: two complex roots

WORKED EXAMPLE

Problem: Find the roots of 2x^2 - 4x + 5 = 0.

Solution: Discriminant = 16 - 40 = -24 < 0. x = (4 ± sqrt(-24))/4 = 1 ± (sqrt(6)/2)i.

PRACTICE PROBLEMS

Question 19

Simplify: $(3 + 2i)^2 - (1 - 4i)(2 + i)$. Write the result in standard form a + bi.

Question 20

A rectangular garden has perimeter 40 m and area 84 m^2. Set up and solve a quadratic equation to find the dimensions. Verify your answer makes physical sense.

ANSWER KEY & FULL SOLUTIONS

Answer 1

Synthetic division with root 2:

Coefficients: 2 | 3 | -11 | -6. Bring down 2. Multiply: $2\times2=4$. Add: $3+4=7$. Continue: $7\times2=14$, $-11+14=3$; $3\times2=6$, $-6+6=0$. Quotient: $2x^2 + 7x + 3$. Remainder: 0. Factor: $(x-2)(2x+1)(x+3)$.

Answer 2

Treat as quadratic in x^2: let $u = x^2$.

$u^2 - 5u + 4 = 0 \rightarrow (u-1)(u-4) = 0 \rightarrow u = 1$ or $u = 4$. So $x^2 = 1 \rightarrow x = \pm1$; $x^2 = 4 \rightarrow x = \pm2$. All four are real zeros. Factored: $(x-1)(x+1)(x-2)(x+2)$.

Answer 3

Factor numerator and denominator:

Numerator: $(x-3)(x+3)$. Denominator: $(x-3)(x+2)$. Cancel $(x-3)$: simplified = $(x+3)/(x+2)$. Restrictions: $x \neq 3$, $x \neq -2$.

Answer 4

Multiply both sides by $(x+1)(x-1) = x^2-1$:

$3(x-1) - 2(x+1) = 1 \rightarrow 3x-3-2x-2 = 1 \rightarrow x = 6$. Check: denominators $\neq 0$ at $x=6$. Solution: $x = 6$.

Answer 5

Isolate the radical: $\sqrt{3x+4} = x-2$.

Condition: $x \geq 2$. Square: $3x+4 = x^2-4x+4 \rightarrow x^2-7x = 0 \rightarrow x(x-7) = 0$. $x=0$ (rejected, < 2); $x=7$: check $\sqrt{25}+2 = 5+2 = 7$ ✓. Solution: $x = 7$.

Answer 6

Apply rational exponent rules:

$(27x^6)^{2/3} = 27^{2/3} \cdot (x^6)^{2/3} = (3^3)^{2/3} \cdot x^4 = 3^2 \cdot x^4 = 9x^4$.

Answer 7

Use log product rule: $\log_2((x+3)(x-1)) = 5$.

$(x+3)(x-1) = 2^5 = 32 \rightarrow x^2+2x-3 = 32 \rightarrow x^2+2x-35 = 0 \rightarrow (x+7)(x-5) = 0$. $x = -7$ (rejected, makes log undefined); $x = 5$. Solution: $x = 5$.

Answer 8

Continuous growth model: $A(t) = 5000 e^{0.042t}$.

To double: $10000 = 5000 e^{0.042t} \rightarrow 2 = e^{0.042t} \rightarrow t = \ln(2)/0.042 \approx 0.6931/0.042 \approx 16.5$ years $\approx$ 16 years 6 months.

Answer 9

Augmented matrix row reduction:

Matrix: [2 1 -1|8], [-3 -1 2|-11], [-2 1 2|-3]. After row operations (R2←2R2+3R1, R3←R1+R3), solve triangular system. Solution: $x = 2$, $y = 3$, $z = -1$.

Answer 10

det(A) = 3(2) - 1(5) = 1. A^-1 = [[2,-1],[-5,3]].

Solve: [x;y] = A^-1 [7;13] = [[2,-1],[-5,3]] [7;13] = [14-13; -35+39] = [1; 4]. Solution: $x = 1$, $y = 4$.

Answer 11

a_8 = 6 · (1/2)^7 = 6/128 = 3/64.

Infinite sum: S = a_1/(1-r) = 6/(1-1/2) = 6/(1/2) = 12.

Answer 12

General term: C(7,k) (2x)^(7-k) (-y)^k.

For x^3 y^4: 7-k=3 → k=4. Term = C(7,4)(2x)^3(-y)^4 = 35 · 8x^3 · y^4 = 280x^3y^4. Coefficient: 280.

Answer 13

Complete the square: 4(x^2-4x+4) + 9(y^2+2y+1) = 11+16+9 = 36.

Divide by 36: (x-2)^2/9 + (y+1)^2/4 = 1. Ellipse. Center: (2,-1). Vertices: (5,-1) and (-1,-1). c = sqrt(9-4) = sqrt(5). Foci: (2±sqrt(5), -1).

Answer 14

Standard form x^2 = 12y means 4p = 12, so p = 3.

Focus: (0, 3). Directrix: y = -3.

Answer 15

Select 2 women from 5 and 2 men from 7: C(5,2)·C(7,2) = 10·21 = 210.

Total ways: C(12,4) = 495. Probability = 210/495 = 14/33.

Answer 16

X ~ B(5, 1/6). P(X=2) = C(5,2)(1/6)^2(5/6)^3.

= 10 · (1/36) · (125/216) = 1250/7776 = 625/3888 ≈ 0.1608.

Answer 17

Identify parameters in y = -3 sin(2x - π/4) + 1:

Amplitude = |-3| = 3. Period = 2π/2 = π. Phase shift = (π/4)/2 = π/8 to the right. Vertical shift = 1 up. Range: [-2, 4]. The negative reflects over midline.

Answer 18

Left side: (1 - cos^2 x)/sin x.

Use Pythagorean identity: $1 - \cos^2 x = \sin^2 x$. So: $\sin^2 x / \sin x = \sin x$ = right side. ■ Domain: $x \neq n\pi$ for any integer n ($\sin x \neq 0$).

Answer 19

$(3+2i)^2 = 9 + 12i + 4i^2 = 9 + 12i - 4 = 5 + 12i$.

$(1-4i)(2+i) = 2 + i - 8i - 4i^2 = 2 - 7i + 4 = 6 - 7i$. Result: $(5+12i) - (6-7i) = -1 + 19i$.

Answer 20

Let length = l, width = w. System: $2(l+w) = 40 \rightarrow l+w = 20$; $lw = 84$.

So l and w are roots of $t^2 - 20t + 84 = 0$. Discriminant = 400 - 336 = 64. $t = (20\pm8)/2 \rightarrow t = 14$ or $t = 6$. Dimensions: 14 m × 6 m. Check: $2(14+6) = 40$ ✓, $14\times6 = 84$ ✓.

END OF ANSWER KEY