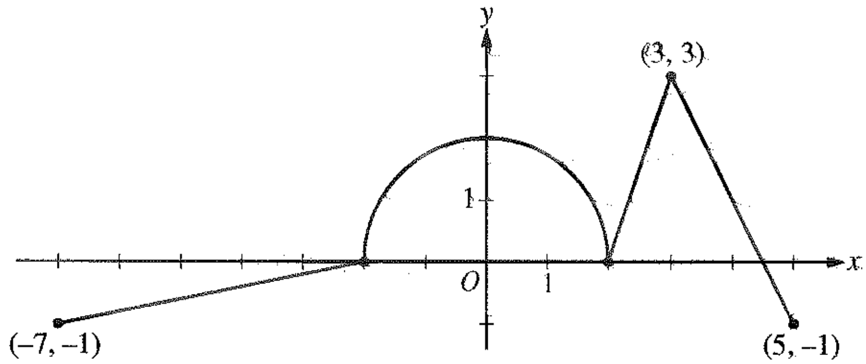
AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

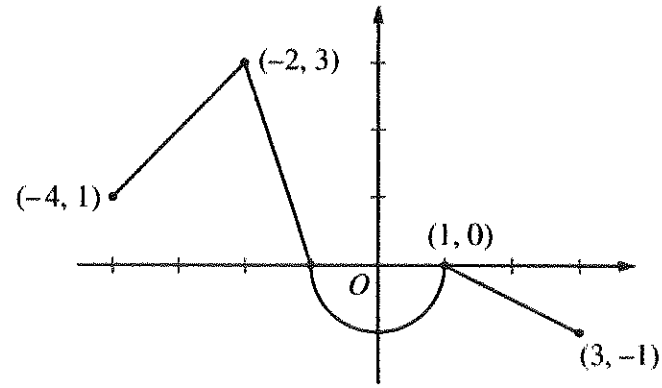
(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

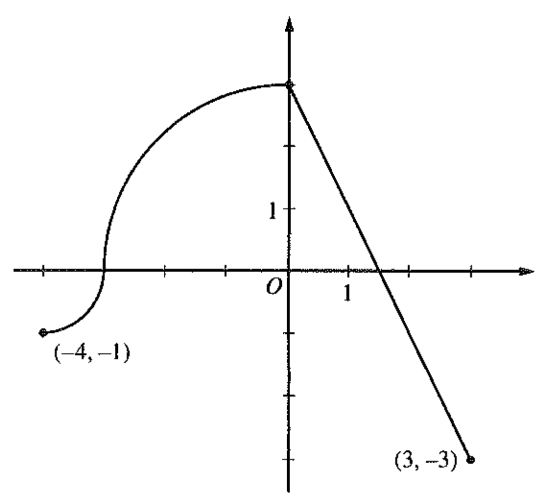
(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.



Graph of f

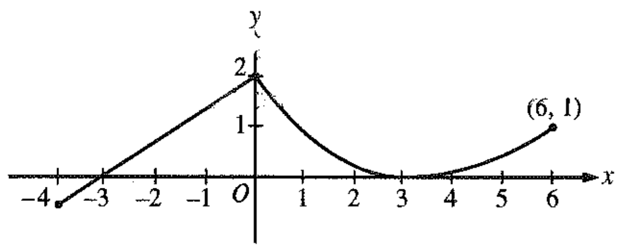
(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

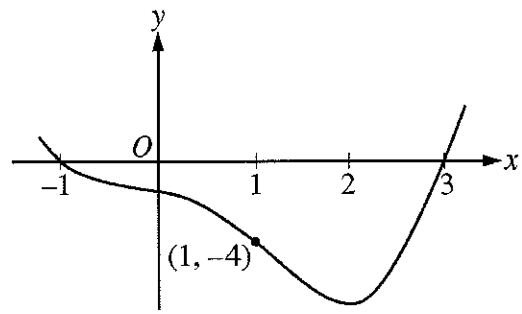
(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\, dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

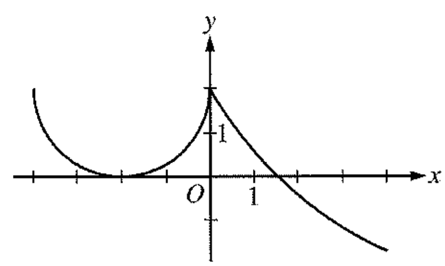
AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 6

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \le 4 \end{cases}.$$

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.



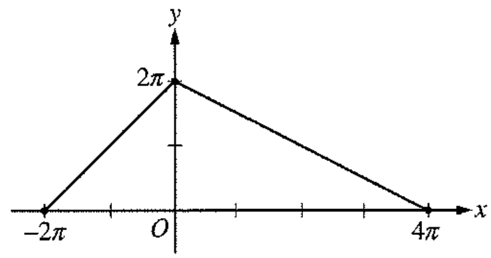
Graph of f'

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

Question 6



Graph of g

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.