Dr. Ela Sharma's

Digital SAT Math 5 Practice Tests

- Review the required math concepts
- Take 2 full-length tests with easy module 2
- Take 3 full-length tests with hard module 2
- Learn the quickest solution to a question

Second Edition 2024-2025

Peter's Digital SAT Math

Study Guide

PREFACE

Dear Esteemed Students,

I extend a warm welcome to Peter's Digital SAT Math Prep!
This comprehensive resource is designed to be an invaluable tool for your preparation for the digital College Board SAT Math. I have meticulously reviewed all released digital College Board SAT tests, along with numerous previous examinations. It has been my privilege and honor to collaborate with students, assisting them in enhancing their SAT Math scores and achieving their academic goals over the years.

Throughout my extensive SAT Math tutoring career, I have provided personalized SAT Math point notes and crafted analogous practice problems, aligning with the rigor of College Board SAT Math assessments. Recognizing the potential benefits for students like yourself, I am pleased to publish this SAT Math prep book, sharing a wealth of techniques and knowledge.

It is crucial to acknowledge that the SAT is a timed test. The explanations provided for the problems may deviate from conventional methods taught in your math classes. My intention is to elucidate the most efficient and fastest methodologies to reach the correct answers. Techniques such as employing the process of elimination, strategic use of numeric substitutions, and visual aids like graphs or diagrams have been incorporated. All problems within this book are designed to be solvable ideally within one minute, reflecting my ultimate goal for your success. If you find the process taking longer, I encourage a thorough review of the provided explanation.

It is imperative to note that purchasing this book alone will not guarantee automatic improvement in your scores. The SAT lacks a magic formula for success. I implore you to commit dedicated time and effort to mastering SAT Math. Once you embark on this commitment, consider this book as a potent tool on your journey to success.

I recommend beginning your study with the SAT Math point notes summarized in the initial section of this book before engaging in 15 full-length practice tests. This approach will fortify your understanding of crucial concepts in digital SAT Math.

For any queries or suggestions regarding this book, I welcome you to reach out to me at peterseducation@gmail.com. I am committed to providing comprehensive answers to your inquiries.

Now armed with this exceptional SAT Math study guide, I encourage you to persevere diligently and achieve your desired SAT Math score. Your efforts are commendable, and I wish you continued success.

Thank you,
Peter So.

Digital SAT Math
5 Practice Tests
+ Refresher Course

2024 - 2025

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About This Book

This book contains two parts:

Refresher Course

The first part is the synopsis of the required math areas, arranged in 27 sections. Each section contains the fundamentals required to answer questions in that math area.

To win the SAT math sections, a student should know how to speed through the questions on the test and have enough time left for the questions that are challenging to a student. Each section contains tips on what type of questions in the respective section can be solved using the mental math, or the Desmos graphing calculator, or may not require any calculation.

Digital SAT Math Practice Tests

This part contains 5 full-length SAT math tests that are written in the format of the digital adaptive tests. These practice tests are created after meticulous analysis of the questions on the released digital SAT tests. The purpose of these tests is to provide students with an opportunity to assess their preparedness and test themselves before taking the adaptive tests in Bluebook™.

Practice tests 1 and 2 contain relatively easier questions in Module 2.

Practice tests 3, 4, and 5 contain relatively harder questions in Module 2.

The answer solutions to the questions that can be solved using the Desmos graphing calculator are marked with an asterisk (*). To learn more on the Desmos graphing calculator please refer to the book titled "Desmos Graphing Calculator: Digital SAT Math prep" by the author of this book.

After completing the five practice tests, a student will see familiar questions on the real SAT test. This should increase "the comfort level" and alleviate "the anxiety".

Disclaimer: Since the scoring of the tests in this book is not digital, having two adaptive Module 2 sections for each test is not required. Instead, the author has created two easy module 2 tests and three hard module 2 tests.

How To Use This Book

Determine the Areas of Improvement before Beginning the SAT Preparation

Students can take the practice tests in this book and determine the skills/knowledge that need improvement. It is recommended to take one practice test with easy Module 2 and one practice test with hard Module 2 to begin with. The remaining tests can be taken at varying time intervals during the SAT preparation process or after the SAT preparation.

Assess Preparedness after Completing the SAT Preparation

Students can take the practice tests in this book and determine if they are fully prepared for the SAT math section. Students are encouraged to take the 5 math practice tests in this book before taking the practice tests in the Bluebook™ testing app. Since the practice tests in the Bluebook™ testing app mimic the real SAT testing environment and scoring, students are recommended to take them when they are fully prepared.

Take 1-to-2-day Crash Course

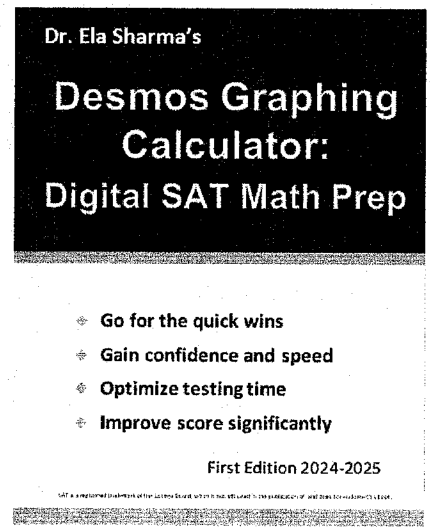
Students who are rushing to prepare for the SAT math in 1 or 2 days right before the test can familiarize themselves with the required math concepts in the Refresher Course and take the 5 practice tests to familiarize themselves with the type of questions that they will see on the SAT test. While this preparation does not guarantee an 800 score on the math section, it will boost the score.

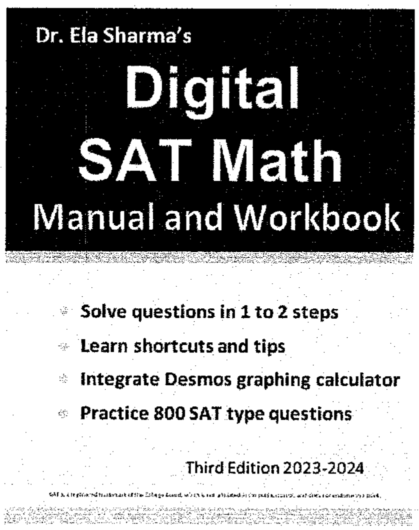
Students looking for a revision/cheat sheet right before the test can go over the 27 sections to refresh the math concepts and take the practice tests to be prepared for the types of questions that they will see on the SAT math sections.

Note From the Author

My mission is to provide students preparing for the SAT math with resources that will improve their score while maximizing study time. See the complete SAT math study plan below.
I wish good luck to all the students preparing for the SAT!!

Step 1: Go for the quick wins
Check out the type of questions that can be quickly solved using the Desmos graphing calculator.
Learn from 50 use cases.
Practice 50 questions.





Step 2: Complete your study
Learn to maximize time and build confidence.
Cover all required topics, with 200+ examples.
Practice 800+ questions.

Step 3: Test yourself
Assess your preparedness.
Experience the type of questions you will see on the day of the test on the easy and hard modules.

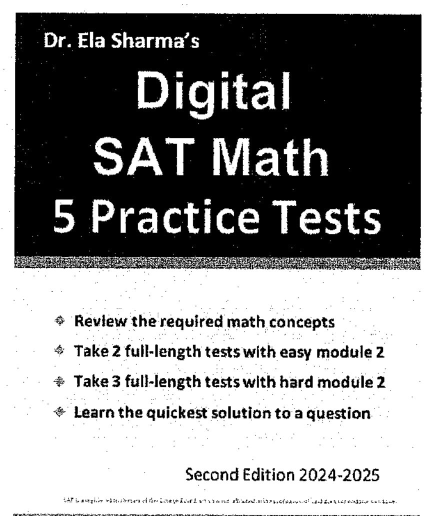


Table of Contents

Introduction to the Digital SAT

The full SAT Suite of Assessments is administered digitally to all students taking the SAT at international test centers starting March 2023. Nationally (USA) the test will be digitally administered starting spring 2024.

The digital SAT continues to measure the same skills and knowledge as the paper and pencil test and continues to be scored on the same scale as the paper and pencil test. The Reading and Writing section is scored on a scale of 200-800 and the Math section is scored on a scale of 200-800.

Each section of the digital SAT has two parts, known as modules.

Digital administration

Students will take the SAT on a laptop or tablet using a custom-built digital testing application, known as the Bluebook™. Students are responsible for downloading Bluebook™ and setting it up before the test day. Students will be provided with scratch paper and can bring a pen or pencil.

The digital testing application starts with the first module of the Reading and Writing section and moves to the second module after the time runs out on the first module. There is a 10-minute break at the end of the Reading and Writing section. After the break the testing application moves to the first module of the Math section, followed by the second module after the time runs out on the first module.

Students can move back and forth among questions in a module before time runs out.

Information on additional topics such as device requirements, device lending, accommodations, and what to bring on test day can be obtained by visiting https://satsuite.collegeboard.org/digital and selecting the submenu of interest.

Adaptive test design

The second module of each section is adaptive.

Students begin each test section by answering the questions in the first module. This module contains a mix of easy, medium, and hard questions. The questions in the second module are based on the performance of a student in the first module. The better a student performs on the first module, the harder are the questions on the second module.

Number of questions and duration

	Reading and Writing Section	**Math Section**
Number of questions per module	1st module: 25 operational questions and 2 pretest questions 2nd module: 25 operational questions and 2 pretest questions	1st module: 20 operational questions and 2 pretest questions 2nd module: 20 operational questions and 2 pretest questions
Time per module	1st module: 32 minutes 2nd module: 32 minutes	1st module: 35 minutes 2nd module: 35 minutes
Total number of questions	54 questions	44 questions
Total time allocated	64 minutes	70 minutes
Question type(s)	Four-option multiple choice	Four-option multiple choice approximately 75% and student produced responses approximately 25%

Only operational questions count towards the score. Pretest questions are included to aid the College Board with the test development process and do not count towards the score. Since it is not possible to identify pretest questions on the test, students should treat all questions equally important.

Digital SAT practice tests

Four full-length digital adaptive practice tests are available in Bluebook™. These tests mimic the actual digital adaptive test interface, format, and scoring system. They are a valuable resource to familiarize students with the digital platform interface and the adaptive nature of the test.

Additionally, pdf versions of four linear (nonadaptive) tests are available at the College Board website. Though they are not adaptive and not taken digitally, they provide students with an additional resource for the digital test format and type of questions. However, some of the content overlaps with Bluebook™

Digital SAT math section

The four math content domains for the digital SAT, along with the required skills and the question distribution for each content domain, are shown in the table below. Mapping of sections in this book to the four math content domains is included.

Content Domain	**Skill/Knowledge Required**	**Operation Question Distribution**	**Section in this Book**
Algebra	Linear equations in one variable. Linear equations in two variables. Linear functions. Systems of two linear equations in two variables. Linear inequalities in one or two variables.	Approximately 35% with 13-15 questions	1, 2, 3, 4
Advanced Math	Equivalent expressions. Nonlinear equations in one variable and systems of equations in two variables. Nonlinear functions.	Approximately 35% with 13-15 questions	5, 6, 7, 10, 11
Problem-Solving and Data Analysis	Ratios, rates, proportional relationships, and units. Percentages. One-variable data: distributions and measures of center and spread. Two-variable data: models and scatterplots. Probability and conditional probability. Inference from sample statistics and margin of error. Evaluating statistical claims: observational studies and experiments.	Approximately 15% with 5-7 questions	8, 9, 12
Geometry and Trigonometry	Area and volume. Lines, angles, and triangles. Right triangles and trigonometry. Circles.	Approximately 15% with 5-7 questions	13, 14

The four-option multiple choice questions and the student produced response questions are distributed throughout each math module.

Calculators are allowed for all the math questions. Students may use their own approved calculator on test day or utilize the Desmos graphing calculator embedded in Bluebook™.

The availability of the Desmos graphing calculator is an advantage. Several math questions on the SAT, a few seemingly hard, can be solved within seconds with the Desmos graphing calculator. Students not familiar with the graphing calculator do not need to stress. This book contains examples of several questions that can be solved using the basic features of the Desmos graphing calculator.

Disclaimers:

* *The information on digital adaptive SAT summarized above is taken from the College Board website.*

Refresher Course

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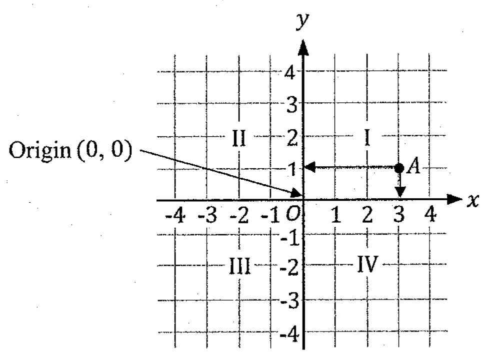
Commonly Used Terms and Concepts

xy-plane and ordered pair

The xy-plane refers to a coordinate system that has a horizontal x-axis and a vertical y-axis. See figure below. The axes are perpendicular to each other and intersect at a point known as the origin. Each point in the coordinate system has a coordinate on the x-axis and a coordinate on the y-axis, collectively known as the ordered pair (x, y). For example, in the figure below, the x-coordinate of point A is 3 and the y-coordinate is 1. The ordered pair is (3, 1).



Quadrant

Quadrants are the four sections formed by the intersecting x- and y-axes in the xy-plane. See figure above. They are numbered 1 to 4 in the counterclockwise direction starting from the upper right quadrant. The quadrant numbers are denoted by Roman numerals.

Line segment

A line segment is a part of a line with distinct and finite end points. For example, sides of a triangle or a square or any other geometric figure that has distinct end points.

Real numbers

A real number is any number that is not a negative square root. For example, $\sqrt{-4}$ or $\sqrt{-169}$ are not real numbers.

Variable

A variable is a placeholder for a numerical value. It is denoted by a letter. For example, in $5x - 7$, x is a variable. The numerical value given to x will determine the value of $5x - 7$.

Constant

A constant is a static number in an equation. For example, in the equation $2y = 5x - 7$, the numbers 2, 5, and 7 are constants. Their value will not change in the equation. A question that has a letter as a constant will specifically call out the constant. For example, in the equation $2y = 5x - a$, a is a constant.

Coefficient

A constant is a static number in an equation. For example, in the equation $y = x - 7$, 7 is a constant. Its value will not change in the equation. A constant may be represented by a letter. For example, in the equation $y = x - a$, a is a constant.

Term

A term refers to a number, variable, several variables multiplied together, or a number and several variables multiplied together. For example, in the equation $2xy + x + 7 = 3y + 2x$, the terms on the left side of the equation are $2xy$, x, and 7 and the terms on the right side of the equation are $3y$ and $2x$.

Expression vs. Equation

The terms on either side of an equation are collectively known as an expression. For example, in the equation $2xy + x + 7 = 3y - 2x$, the two expressions are $2xy + x + 7$ and $3y - 2x$.

Section 1 – Variables in Linear Equations and Expressions

Key Points

Expressions in Parentheses

- Expressions within parentheses can be simplified by multiplying the term outside the parentheses with each term within the parentheses and then adding/subtracting the multiplied terms. The entire outside term must be multiplied by each term within the parentheses. See the example below.

$$-3(x-5) = (-3 \times x) - (-3 \times 5) = -3x - (-15) = -3x + 15$$

Solving Variables in Linear Equations

- A variable in a linear equation can be solved by moving all terms with variables to one side of the equation and all the numbers to the other side of the equation, followed by simplifying the equation. See the example below.

$$5x - 7 = 2x + 5 \rightarrow 5x - 2x = 5 + 7 \rightarrow 3x = 12 \rightarrow x = \frac{12}{3} \rightarrow x = 4$$

- If an equation contains expressions within parentheses, then simply the expression before solving for the variable. See the example below.

$$5(x+1) - 3x = 7 \rightarrow 5x + 5 - 3x = 7 \rightarrow 2x = 7 - 5 \rightarrow 2x = 2 \rightarrow x = \frac{2}{2} \rightarrow x = 1$$

Solving Identical Expressions in Linear Equations

- Identical expressions in an equation can be added or subtracted as a unit and solved as a unit. For example, in the equation $5(2x+7) + 3(2x+7) = 24$, $(2x+7)$ can be considered as a unit. 5 units + 3 units add to 8 units. See below.

$$5(2x+7) + 3(2x+7) = 24 \rightarrow 8(2x+7) = 24 \rightarrow (2x+7) = 3$$

Tips

Mental Math: Simple equations without parentheses or multiple terms can be solved using mental math.

- In the equation $5x - 9 = 2x$, it is apparent that $2x$ moved to the left-side results in $3x$ and -9 moved to the right-side is 9. Hence, $3x$ is equal to 9 and x is equal to 3.

- In the equation, $5(2x+7) + 3(2x+7) = 24$, the sum of 5 and 3 in the left-side of the equation is 8. Hence, $8(2x+7)$ is equal to 24. The value of $(2x+7)$ is 24 divided by 8 which is equal to 3.

- In the equation, $\frac{x}{3} = \frac{4}{5}$, 4 multiplied by 3 is 12 and 12 divided by 5 is 2.4.

Desmos Graphing Calculator: Desmos can save time and avoid calculation errors to solve equations that have parentheses and multiple terms.

- Type the linear equation with one variable in the Desmos graphing calculator. This will graph a line. Read the point where the line intersects the axis.
  When the variable is x, for example $3(2x-5) + 8 = 4x - 3$, read the value of x where the graph intersects the x-axis.
  When the variable is y, for example $3(2y-5) + 8 = 4y - 3$, read the value of y where the graph intersects the y-axis.
  The Desmos graphing calculator only recognizes x and y variables. When an equation contains a variable that is not x or y, then substitute the variable with x or y. For example, in the equation $3(2z-5) + 8 = 4z - 3$, substitute z with x or y before tying the equation in the Desmos graphing calculator.

Section 2 – Lines and Linear Functions

Key Points

Slope

- The slope between any two points on a line is the same. For any two points (x_1, y_1) and (x_2, y_2) on a line, the slope can be determined using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, where m is the slope.
- On a graph, a line with a positive slope slant upward from left to right (Fig. 1 below). A line with a negative slope slant downward from left to right (Fig. 2 below).

  From a graph, the slope of a line can also be written as $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y\ (y_2 - y_1)}{\text{change in } x\ (x_2 - x_1)}$ as shown in Fig. 1 and Fig. 2 below.

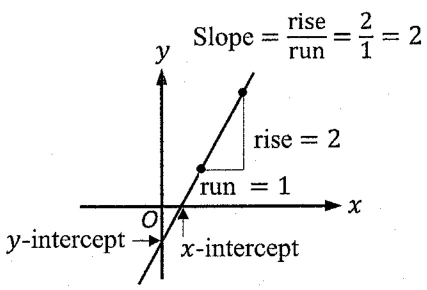


Fig. 1

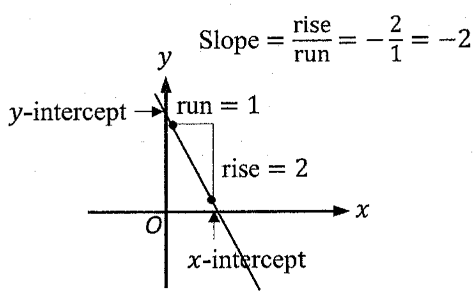


Fig. 2

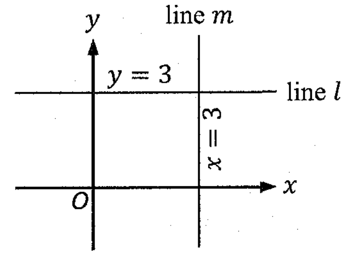


Fig. 3

x- and y-intercepts

- The x-intercept of a line is the point $(x, 0)$where the line intersects the x-axis. At the x-intercept, $y = 0$ (Fig. 1 and Fig. 2 above). For example, if a line passes through the x-axis at $x = 5$, then the x-intercept is $(5, 0)$.
- The y-intercept of a line is the point $(0, y)$where the line intersects the y-axis. At the y-intercept, $x = 0$ (Fig. 1 and Fig. 2 above). For example, if a line passes through the y-axis at $y = 5$, then the y-intercept is $(0, 5)$.

Slope-Intercept Equation

- The slope-intercept form of a line is $y = mx + b$, where m is the slope and b is the y-coordinate of the y-intercept.

Standard Form Equation

- The standard form equation of a line is $Ax + By = C$, where A, B, and C are constants.
  - Slope $= -\frac{A}{B}$.
  - y-intercept $= \frac{C}{B}$.

  For example, in the equation $3x + 2y = 5$, the slope is $-\frac{A}{B} = -\frac{3}{2}$ and the y-intercept is $\frac{C}{B} = \frac{5}{2} = 2.5$.

Horizontal and Vertical Lines

- A horizontal line passes through the same point on the y-axis. See line l in Fig. 3 above. Since the difference in the y values of any two points on a horizontal line is 0, the slope of a horizontal line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$.
- A vertical line passes through the same point on the x-axis. See line m in Fig. 3 above. Since the difference in the x values of any two points on a vertical line is 0, the slope of a vertical line is $\frac{y_2 - y_1}{0} =$ undefined.

Parallel and Perpendicular Lines

- Parallel lines have the same slopes but different y-intercepts. For example, if the slope of a line k is 3, then the slope of any line parallel to k is also 3.
- Perpendicular lines have negative reciprocal slopes. For example, if the slope of a line k is 3, then the slope of any line perpendicular to k is the negatve reciprocal of 3, which is $-\frac{1}{3}$.

Linear Functions

- A linear function graphs a straight line. It is written as $f(x) = mx + b$, where $f(x)$ is the y value for an input value of x. For any (x, y) point on a line, the equation is same as $y = mx + b$. For example, in the linear function $f(x) = 1.5x + 3$, if $x = 2$, then $y = f(2) = (1.5 \times 2) + 3 = 6$. Hence, $f(2) = 6$. The corresponding (x, y) pair is (2, 6).
  Each (x, y) pair of a linear function is a point on the line graphed by that linear function.

Linear Functions Represented in a Table

- In a table containing x and y values of a linear function, each (x, y) pair is a point on the line graphed by the function. For example, in the below table the points $(-6, 0)$, $(-2, 2)$, $(0, 3)$, and $(4, 5)$ are graphed by a linear function f where $f(x)$ is the y value for the corresponding x value.
  Note that the point $(-6, 0)$ is the x-intercept of the line and the point $(0, 3)$ is the y-intercept of the line.

x	-6	-2	0	4
$f(x)$	0	2	3	5

- Remember that if a question mentions linear relationship between two variables, it is referring to a linear function.

Tips

No Calculation Required:

- The coordinates of the x- and y-intercepts can be directly read from a graph of a line by looking for the point where the line intersects the corresponding axes. From a table, they can be read as described above.
- The slope and the y-intercept can be directly read from a slope-intercept equation.
- The slope of parallel and perpendicular lines can be directly determined from a given line equation. For example, if the equation of a line is $y = 3x + 5$, then the slope of any parallel line = 3 and the slope of any perpendicular line = $-\frac{1}{3}$.

Mental Math:

- From a graph, the slope can be directly read as rise/run. Look for two points on the gridlines of the graph and read the rise (the vertical distance between them) and the run (the horizontal distance between them). The slant of the line determines negative/positive slope.
- The slope or the y-intercept from a standard form equation can be determined by evaluating $-\frac{A}{B}$ and $\frac{C}{B}$ from the equation, respectively. For example, in the equation $3x + 2y = 5$, the slope is $-\frac{3}{2}$ and the y-intercept is $\frac{5}{2}$.

Desmos Graphing Calculator:

- When two points on a line are given, the slope, x-intercept, and y-intercept can be determined using the Desmos graphing calculator. Create a table for the two points and in the next row type the linear regression equation. This will graph the corresponding line.
- When one point on a line and the slope of the line are given, the slope, x-intercept, and y-intercept can be determined using the Desmos graphing calculator. Create a table for the given point and in the next row type the linear regression equation substituting the given slope. This will graph the corresponding line.
- The output value of a linear function can be determined by typing the definition of the function in a row and what needs to be determined in the next row. For example, to determine the value of $f(-2.6)$ for the linear function $f(x) = 5.5x + 1.5$, type the definition of f in a row and in the next row type $f(-2.6)$. This will display the value of $f(-2.6)$ to the right in the same row.

Section 3 – Systems of Linear Equations

Key Points

Systems of Linear Equations and Number of Solutions

- Two or more linear equations of a line are collectively known as a system of linear equations.
- When compared in the standard form, $Ax + By = C$, the following rules apply for two equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.
  - If $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, and $\frac{c_1}{c_2}$ are same, then the system has infinitely many solutions. The equations are for the same line.
  - If $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$ are same but $\frac{c_1}{c_2}$ is different, then the system has no solution. The equations are for parallel lines.
  - If $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$ are different, then the system has one solution. The equations are for two lines that intersect at exactly one point (x, y). The system can be solved for the value of (x, y). The value of c is irrelevant in this situation.
- The equations must be in the standard form to compare the ratios, as shown above. If one or both equations are not in the standard form, then convert to the standard form. For example, $y = 3x - 2$ can be converted as $3x - y = 2$.

Unknown Constants and Coefficients in System of Linear Equations

- When linear equations in a system contain unknown constants or coefficients, evaluate the values of constants /coefficients that make the ratios same or different.

  See example below for the equations $ax - 4y = 12$ and $2x - y = c$ in a system, where a and c are constants.

  a_1, b_1, and c_1 correspond to the equation $ax - 4y = 12$.

  a_2, b_2, and c_2 correspond to the equation $2x - y = c$.

$$\frac{a_1}{a_2}:\frac{b_1}{b_2}:\frac{c_1}{c_2} \rightarrow \frac{a}{2}:\frac{-4}{-1}:\frac{12}{c} \rightarrow \frac{a}{2}:\frac{4}{1}:\frac{12}{c}$$

  - The system will have infinitely many solutions when all the ratios are the same. Hence, a must be 8 and c must be 3.
  - The system will have no solution when the ratios of a and b are same but different than c. Hence, a must be 8 for the ratios of a and b to be same, and c must not be 3 for the ratio of c to be different than a and b.
  - The system will have one solution when the ratios of a and b are different. Hence, a must not be 8 for the ratios of a and b to be different. a can be any value other than 8.

(x, y) pair in System of Linear Equations with One Solution

The system can be solved for (x, y) by making one of the variable same in both the equations and then adding/subtracting the equations to eliminate that variable. See example below for two equations $6x + 4y = 72$ and $2x + 2y = 28$.

$$\begin{array}{c} 6x + 4y = 72 \\ -2(2x + 2y = 28) \end{array} \longrightarrow \begin{array}{r} 6x + \cancel{4y} = 72 \\ -4x - \cancel{4y} = -56 \\ \hline 2x \quad = 16 \end{array} \rightarrow x = 8$$

The value of y can be determined by plugging the value of x in any of the two equations.

Tips

Mental Math:

- In the system of equations $3x - 4y = 2$ and $9x - 12y = 6$, it can be observed that the values of a, b, and c in the equation $9x - 12 = 6$ are triple the values in the equation $3x - 4y = 2$. Hence, the ratios of a, b, and c are the same and the system has infinitely many solutions.

Desmos Graphing Calculator:

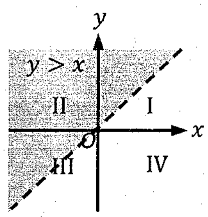
- The number of solutions to a system of equations can be determined by typing the two equations in individual rows and determining from the graph whether the two lines are parallel lines, same lines, or intersect at one point.
- The intersection point (x, y) of a system of equations with one solution can be determined by typing the two equations in individual rows and reading the point where the graph of the two lines intersect.

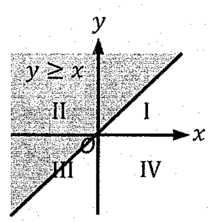
Section 4 – Linear Inequalities and Systems of Linear Inequalities

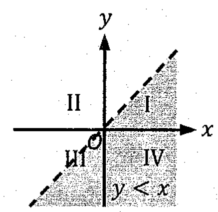
Key Points

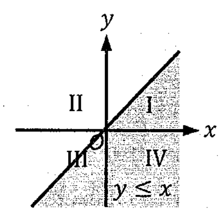
Linear Inequalities

- The equality symbols are less than (<), less than or equal to (≤), greater than (>), and greater than or equal to (≥).
- The inequality symbols determine whether the solution set of an inequality, shown as the shaded region in graphs below, is above or below a line in the xy-plane. When $y > x$, the solution set is above the line. When $y \geq x$, the solution set is on and above the line. When $y < x$, the solution set is below the line. When $y \leq x$, the solution set is on and below the line.
- The inequalities with ≥ and ≤ symbols have a solid line and the inequalities with > and < symbols have a dashed line.





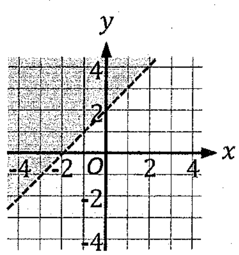




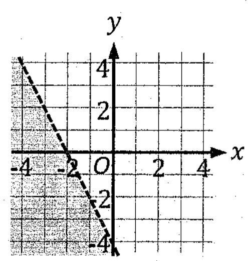
- The solution set of an inequality on a graph depends on the slope and the y-intercept of the inequality. In the examples above, the inequalities have slope $= 1$ and y-intercept $= 0$. The slant of the line determines whether the slope is negative or positive. In the above graphs, the lines slant upwards from left to right. Hence, the slope is positive.
- Any point within the solution set is a solution to the linear inequality.

Systems of Linear Inequalities

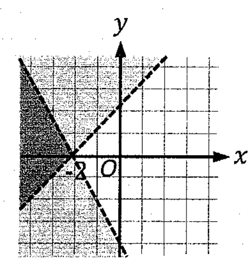
- A system of two linear inequalities is comprised of two linear inequalities. The solution set of a system of two linear inequalities is the region on the graph where the graphs of the two inequalities overlap. It may span one or more quadrants. See example below.
- Any point within the overlapping region, the solution set, is a solution to the system of linear inequalities.



$y > x + 2$



$y < -2x - 4$



Overlapping Region (darker shaded region)

Tips

Mental Math:

- A graph can be matched to a linear inequality by looking at the slant of the line, the inequality operator, and the y-intercept.

Desmos Graphing Calculator:

- A graph of an inequality can be matched to the inequality in the answer choices by typing the inequality from each answer choice in the Desmos graphing calculator, one at a time, and matching the corresponding graph to the given graph.
- The graph of a system of inequalities can be matched to the given system of inequalities in the answer choices by tying the two inequalities from each answer choice in the Desmos graphing calculator, one at a time, and matching the corresponding graph to the given graph.
- The points within the solution set of an inequality or a system of inequalities can be determined by typing the inequalities and looking for the points that are within the solution set. Same can be applied to determine the quadrants that contain the solution set of an inequality or a system of inequalities.

Section 5 – Word Problems on Linear Equations and Inequalities

Key Points

Constant and an Unknown Number in Linear Equations

- A word problem may be given that contains a constant number and an unknown number. The value of the constant number does not change in the equation. The unknown number is comprised of a fixed number and an associated variable.
  For example, if the annual membership to a gym is $90 and participation in each yoga class offered at the gym is $5, then the total annual cost of gym membership plus participating in yoga classes is
  total annual cost = annual membership fee of $90 + ($5 × number of yoga classes in a year)
  If x is the number of yoga classes in a year, then the total annual cost is $= 90 + (5 \times x) = 90 + 5x$.
  $5x$ is the unknown number and x is the associated variable. The value given to x will determine the total annual cost. For example, the total annual cost for 3 yoga classes in a year is $90 + (5 \times 3) = 105$ and so on.

Two Unknown Numbers in Linear Equations

- A word problem may be given that contains two unknown numbers. For example, the cost of one bottle of water is $2 and the cost of one bag of chips is $1.50. If the total cost of a bottles of water and b packets of chips is $22, then the equation is $2a + 1.5b = 22$. If the value of a is given, then b can be calculated. If the value of b is given, then a can be calculated.

Systems of Linear Equations

- A word problem may be given that contains two unknown variables and the value of neither variable is given. The word problem will contain sufficient information to create a system of equations. For example, Sara bought a total of 8 shirts and hats. The cost of each shirt is $6 and the cost of each hat is $4. Sara spent a total of $42 on x shirts and y hats.
  - One equation can be created for total number of shirts and hats as $x + y = 8$.
  - Second equation can be created for the total cost as $6x + 4y = 42$.
  - The system of two equations can now be solved for x and y.

Linear Inequalities

- A word problem may be given that evaluates a conditional relationship between one or more numbers, where at least one number is unknown. The unknown number contains a variable that evaluates the condition.
  - One unknown number: For example, if John cannot spend more than $720 to buy a number of books at $35 each, then $35a$ is the unknown number and the condition is $35a$ is less than or equal to 720. The inequality is $35a \leq 720$.
    Based on this condition, John can buy $a \leq \frac{720}{35} \rightarrow a \leq 20.5$ books. Hence, John can buy no more than 20 books.
  - Two unknown numbers: For example, if John cannot spend more than $720 to buy a number of books at $35 each and b number of magazines at $42 each, then the condition is the sum of $35a$ and $42b$ is less than or equal to 720. The inequality is $35a + 42b \leq 720$. If the value of a is given, then b can be evaluated and vice versa.

Systems of Linear Inequalities

- A word problem on linear inequality may be given that contains two unknown variables and the value of neither variable is given. The word problem will contain sufficient information to create a system of inequalities. For example, Sara wants to buy at least 8 shirts and hats. The cost of each shirt is $6 and the cost of each hat is $4. Sara can spend a maximum of $42 on x shirts and y hats.
  - One inequality can be created for total number of shirts and hats as $x + y \geq 8$.
  - Second inequality can be created for the total cost as $6x + 4y \leq 42$.
  - The values of x and y can now be evaluated.

Tips

Mental Math:

- In some questions, matching the given answer choices to the information in a word problem should be apparent as shown in the examples in the Key Points.

Desmos Graphing Calculator:

- For the system of equations word problems, the Desmos graphing calculator can be utilized to solve for the two variables after the two equations have been formed.

Section 6 – Polynomial Functions

Key Points

Equations

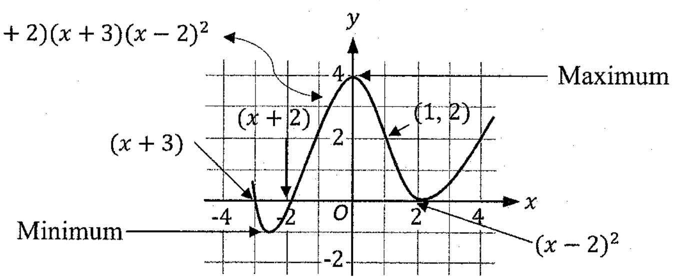
- A polynomial function contains terms in the form of ax^n, where n is a positive integer. For example, $f(x) = ax^4 + ax^2 + x$. $f(x)$ is the y value for a given value of x. If $x = 2$, then $y = f(2) = a(2)^4 + a(2)^2 + (2)$.

Graphs

- On the graph of a function f, $f(x)$ is the y-value corresponding to a particular value of x on the x-axis. For example, on the graph of a function f shown below, for $x = 1$, $y = f(1) = 2$.
- The highest value of y on the graph of a polynomial function is known as the maximum and the lowest value of y is known as the minimum. See the graph below.

Zeros and Factors

- The zeros (also known as the roots and the x-intercepts) of a polynomial function are the values of x where the graph of the polynomial function intersects the x-axis. The graph of $y = f(x)$ below has three distinct zeros -3, -2 and 2.
- The relationship between a factor and a zero is important to remember. If n is a zero of a function, then the factor is $(x - n)$ and if $-n$ is a zero of a function, then the factor is $(x + n)$. See the graph below for the zeros -2 and -3. The corresponding factors are $(x + 2)$ and $(x + 3)$.
- When the graph touches the x-axis and curves back, then there are two identical factors at that point. If n is a zero of a function, then the factors are $(x - n)^2$ and if $-n$ is a zero of a function, then the factors are $(x + n)^2$. See the graph below for the zero 2. The corresponding factors are $(x - 2)^2$.
- When the graph passes through the origin, the zero at that point is 0. Hence, the factor is just x or a multiple of x.



Remainder Theorem

- When a polynomial function is divided by a linear expression such as $x - 2$, the remainder can be determined by substituting $x = 2$ in the function. For example, when $p(x) = x^4 - 4x^3 + x^2 + 6x + 3$ is divided by $x - 2$, the remainder is $p(2) = (2)^4 - 4(2)^3 + (2)^2 + 6(2) + 3 = 16 - 32 + 4 + 12 + 3 = 3$.

Factor Theorem

- When a polynomial function is divided by a linear expression such as $x - 2$ and the remainder obtained by substituting $x = 2$ is 0, then 2 is a zero and $x - 2$ is a factor of the function. For example, when $p(x) = x^4 - 4x^3 + x^2 + 6x$ is divided by $x - 2$, the remainder is $p(2) = (2)^4 - 4(2)^3 + (2)^2 + 6(2) = 0$. $x - 2$ is a factor and 2 is a zero of $p(x)$.

Tips

No Calculation Required:

- Number of zeros can be directly read from an equation. For example, in the function $f(x) = x(x - 1)(x + 3)^2$, the three distinct zeros are 0, 1, and -3. Note that although $(x + 3)^2$ has two identical zeros, there is only one distinct zero.
- Zeros and factors can be directly read from a graph, as shown in the Key Points above.

Desmos Graphing Calculator:

- The output value of a linear function can be determined by typing the definition of the function in a row and what needs to be determined in the next row. For example, to determine the value of $f(8)$ for the function $f(x) = x^4 + 8x^2 + x$, type the definition of f in a row and in the next row type $f(8)$. The value of $f(8)$ will be displayed to the right in the same row. The remainder of a polynomial function can be determined similarly.

Section 7 – Quadratic Equations and Parabola

Key Points

Quadratic Function and Parabola

- A quadratic function graphs a parabola in the xy-plane. The tip of the parabola is known as the vertex.
- The highest term of x in a quadratic equation is x^2. For example, $f(x) = ax^2 + bx + c$, where $f(x)$ is the y value for an input value of x.
  - When the value of a is positive, the parabola opens upwards, and the vertex is the lowest point also known as the minimum of the parabola (Fig. 1 below).
  - When the value of a is negative, the parabola opens downwards, and the vertex is the highest point also known as the maximum of the parabola (Fig. 2 below).
- A straight vertical line passing through the vertex is known as the axis of symmetry.

Roots, x-intercepts, Zeros, and Solutions

- A parabola may intersect the x-axis at two distinct points (Fig. 1 and Fig. 2 below), one distinct point or may not intersect (Fig. 3 below). The points where a parabola intersects the x-axis are known as the x-intercepts, the roots, the solutions, or the zeros of the quadratic function.

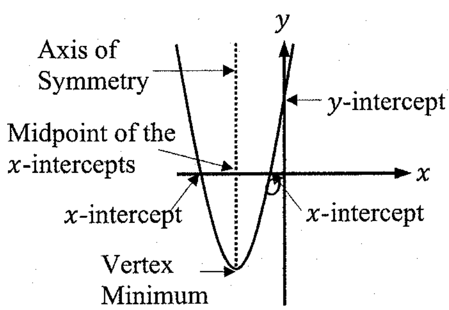


Fig. 1

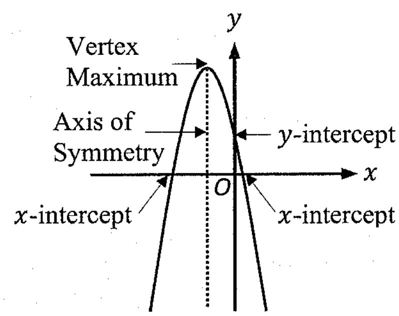


Fig. 2

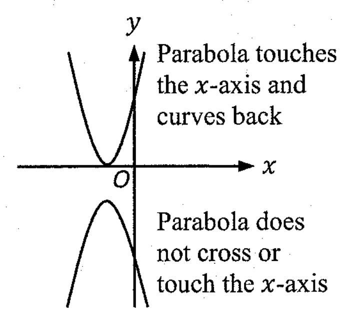


Fig. 3

Standard Form Quadratic Function, Factors, Quadratic Formula, and Discriminant

- The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where a, b, and c are constants.
  - c is the value of the y-coordinate of the y-intercept of the parabola. For example, if the parabola intersects the y-axis at $(0, 4)$, then $c = 4$.
  - The x-coordinate of the vertex can be determined as $-\frac{b}{2a}$. The y-coordinate of the vertex can be determined by plugging the above obtained value of the x-coordinate in the given equation and solving it for y.
- Factorization: The roots and factors of a quadratic function can either be determined by factoring the standard form equation or using the quadratic formula $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Use the formula when factorization is not easy, or the answer choices have square root.
- Discriminant: The expression $b^2 - 4ac$ of the quadratic formula is known as the discriminant.
  - $b^2 - 4ac > 0$ indicates two real solutions for the quadratic function. This happens when the graph of the parabola intersects the x-axis at two distinct points (Fig. 1 and Fig. 2 above).
  - $b^2 - 4ac = 0$ indicates one real solution for the quadratic function. This happens when the graph of the parabola touches the x-axis and curves back without passing through it (top parabola in Fig. 3 above). There are two identical factors at this point but one distinct root. For example, for two identical factors $(x - 2)^2$, the distinct root is 2.
  - $b^2 - 4ac < 0$ indicates no real solution for the quadratic function. This happens when the graph of the parabola does not intersect the x-axis (bottom parabola in Fig. 3). The solutions are negative square root values that are not real numbers.
- Sum and product: The sum of the roots can be determined as $-\frac{b}{a}$ and the product of the roots can be determined as $\frac{c}{a}$. For example, in the equation $2x^2 - 11x + 12 = 0$, the sum of roots $= -\frac{-11}{2} = 5.5$ and the product of roots $= \frac{12}{2} = 6$.

Vertex Form Quadratic Function

- The vertex form of a quadratic function is $f(x) = a(x-h)^2 + k$, where $y = f(x)$ and the constants h and k are the x- and y-coordinates of the vertex, respectively. For example, if the coordinates of the vertex are $(2, 3)$, then the equation is $f(x) = a(x-2)^2 + 3$ and if the coordinates of the vertex are $(-2, -3)$, then the equation is $f(x) = a(x-(-2))^2 - 3 \rightarrow f(x) = a(x+2)^2 - 3$.

Factored Form Quadratic Function

- The factored form of a quadratic function is $f(x) = a(x-r)(x-s)$, where $y = f(x)$ and the constants r and s are the x-intercepts of the parabola. For example, in the equation, $f(x) = a(x-2)(x+5)$, the x-intercepts are 2 and -5.
- The x-coordinate of the vertex is the midpoint of the two x-intercepts. It can be determined using the midpoint formula $\frac{x_2+x_1}{2}$, where x_1 and x_2 are the two x-intercepts. The y-coordinate of the vertex can be determined by plugging the above value of the x-coordinate in the given equation and solving it for y.

Intersections of Parabola and Lines

- The graphs of one vertical parabola and two or more linear lines intersect at one point, hence, have one solution.
- The graphs of one vertical parabola and one linear line may intersect at one point or two points or may not intersect at all. The two corresponding equations comprise a system of equations. When the graphs intersect at one point, then there is one solution. When the graphs intersect at two points, then there are two solutions.

  The number of solutions or the intersection points (x, y) of the system can be determined by equating the two equations to form one quadratic equation in the form $ax^2 + bx + c = 0$. For example, if the two equations are $y = x^2 + 7x + 7$ and $y = 2x + 3$, then $x^2 + 7x + 7 = 2x + 3 \rightarrow x^2 + 5x + 4 = 0$. The number of solutions can be determined by evaluating the discriminant. The points of intersection can be determined by solving the quadratic equation for x and y. Note that the quadratic equation must be in the $y = ax^2 + bx + c$ form and the linear equation must be in the $y = mx + b$ form before equating. If not, then rearrange the equations into these forms.

Equivalent Equations of a Parabola

- The standard form, the vertex form, and the factored form equations of a parabola are interchangeable. The interchangeable forms of an equation are known as equivalent equations. They graph the same parabola and have the same vertex. For example, $2(x-3)^2 + 5$ and $2x^2 - 12 + 23$ are equivalent equations. See below.

$$2(x-3)^2 + 5 \rightarrow 2(x^2 - 6x + 9) + 5 \rightarrow 2x^2 - 12x + 18 + 5 \rightarrow 2x^2 - 12x + 23$$

Movement of a Ball (or a similar object) in Air

- When a ball is thrown up in the air from the ground or from a platform, the movement of the ball in the air and back to the ground is a parabola. The time is along the x-axis and the height is along the y-axis.
  - The x-coordinate of the vertex is the time taken by the ball to reach the maximum height in the air.
  - The y-coordinate of the vertex is the maximum height of the ball in the air.
  - The positive x-intercept greater then 0 is the time taken by the ball to reach the ground.

Tips

No Calculation Required:

- From a standard form equation, the y-coordinate of the y-intercept can be read from the equation, $c = 3$.
- From a vertex form equation, the vertex can be directly read from the equation and matched to a graph and vice versa.
- From a factored form equation, the x-intercepts can be directly read as the value of x.

Mental Math:

- The x-coordinate of the vertex can be determined as $= -\frac{b}{2a}$ when the values of a and b are relatively small.

Desmos Graphing calculator:

- The solutions (x-intercepts), vertex, and y-intercept of a parabola can be determined by typing the quadratic equation or function in the Desmos graphing calculator. The corresponding parabola will be displayed. The x-intercepts, vertex, and y-intercept can be read from the parabola.

Section 8 – Number of Solutions of Polynomial Expressions

Key Points

- Two polynomial expressions (linear expressions, quadratic expressions, or other expressions containing ax^n terms) may or may not be equivalent to each other depending on the values given to the coefficients or constants in the expressions.
- An equation has infinitely many solutions when the expressions on both sides are equal. The constants and coefficients must be given values that make both the expressions same. For example, in the equation $3x + k = ax + 2$, expressions on both sides of the equation will be equal when $a = 3$ and $k = 2$. Similarly, in the equation $4x^2 + 8x + k = mx^2 + 8x + 5$, the expressions on both sides will be equal when $m = 4$ and $k = 5$.
- An equation has no solution when the expressions on both sides are unequal. Either the constant or the coefficient must be unequal in the two expressions. For example, in the linear equation $3x + 2 = ax + 2$, since the constants on both sides are equal the coefficient of x must not be equal. Hence, for the equation to have no solution, a cannot be 3.
- If an expression on one side of the equation does not have a variable or a constant, then its value on the other side is 0. See examples below.
  - In the equation $3x = ax + 2$, the constant on the left side is 0. Since the constants are unequal, the coefficient of x on both sides must be equal for the equation to have no solution. Hence, for the equation to have no solution, a must be 3.
  - In the equation $ax + 12 = 12$, the right-side expression does not have the variable x. Hence, for the equation to have infinitely many solutions, a must be 0.

Tips

Mental Math:

- In some linear expressions, the values of the coefficients/constants that will result in infinitely many solutions or no solution are apparent by looking at the expressions on both sides of an equation, as shown above in the Key Points.

Section 9 – Absolute Value

Key Points

Absolute Value Linear Equations

- The absolute value of a real number is denoted within two bars and is always a non-negative real number. For example, both $|x|$ and $|-x|$ are equal to x. Hence, the absolute value of a positive and a negative real number is always non-negative.
- The above is also true for absolute value expressions. For example, in the equation $|2x-1|=7$ it is equally possible that the value of $2x-1$ is -7 or 7 since both $|7|$ and $|-7|$ are equal to 7.

  Hence, an absolute value linear equation is solved by giving the expression within the absolute value bars a negative value and a positive value. In the example $|2x-1|=7$, the expression $2x-1$ is equated to 7 and -7 and solved for x. See below.

  $$2x-1=7 \rightarrow 2x=7+1 \rightarrow x=4$$
  $$2x-1=-7 \rightarrow 2x=-7+1 \rightarrow x=-3$$

  Hence, the solutions to $|2x-1|=7$ are -3 and 4.

  It is important to check that a solution is not an extraneous solution.
- All terms not within the absolute value sign should be on one side. For example, $|2x-1|+3=7 \rightarrow |2x-1|=7-3$.
- If the absolute value equates to an expression, then the entire expression should be given a positive and a negative value. For example, $|2x-1|=7-x \rightarrow 2x-1=(7-x)$ AND $2x-1=-(7-x)$.
- Since the absolute value is always a non-negative, an absolute value expression cannot equate to a negative number. For example, the equation $|2x-5|=-6$ has no solution.

Tips

Desmos Graphing Calculator:

- A solution of an absolute value equation can be determined by typing the equation in the Desmos graphing calculator and reading the value of x where the line corresponding to the solution intersects the x-axis.

  If the variable in the equation is y, then read the value of y where the corresponding line intersects the y-axis.

  If the variable in the equation is a letter other than x or y in the equation, then substitute it with x or y before typing in the Desmos graphing calculator.

  Note that the Desmos graphing calculator does not graph an extraneous solution. Hence, no need to check for an extraneous solution.

Section 10 – Ratios, Fractions, Proportions, and Rates

Key Points

Ratio and Fraction

- A ratio may compare two or more parts within a total. For example, if there are total of 7 muffins, of which 2 are blueberry muffins and 5 are banana muffins, then the ratio of the number of blueberry muffins to the number of banana muffins is $2:5$.
- A ratio may compare a part with the total. From the above example, the ratio of the number of blueberry muffins to the total number of muffins is $2:7$ and the ratio of the number of banana muffins to the total number of muffins is $5:7$.
- A part out of the total can also be written as a fraction. Of the 7 muffins in the above example, the fraction of blueberry muffins is $\frac{2}{7}$ and the fraction of banana muffins is $\frac{5}{7}$.

Proportion

- A proportion equates ratios that are equal. For example, if the number of blueberry muffins and the number of banana muffins from the above example are proportionally tripled, then the ratio would be $(2:5) \times 3 = 6:15$. The proportion in $6:15$ is triple that of $2:5$ but the ratios $2:5$ and $6:15$ are equivalent.
- Since proportions are similar ratios, they can be equated as fractions. The numerator and the denominator of a proportion must represent the same thing. In the below example, the numerators represent the blueberry muffins, and the denominators represent the total muffins.

$$\frac{\text{blueberry muffins}}{\text{total muffins}} = \frac{2}{7} = \frac{4}{14} = \frac{8}{28}$$

Rate

- A rate is a ratio of two units of measurement. For example, 25 miles per hour refers to 25 miles in 1 hour. The two units of measurement are miles and hour. The ratio can be written as 25 miles:1 hour or $\frac{25 \text{ miles}}{1 \text{ hour}}$.
- When two rates are compared, both the units in the two rates must be compared, respectively. For example, when comparing a speed in miles per hour to a speed in kilometers per second, miles must be converted to kilometers and hour to seconds.

Tips

Mental Math:

- If $\frac{1}{4}$ th of a cup is 1.5 ounces, then $\frac{3}{4}$ th of the cup is 3 times more $= 4.5$ ounces.
- If a car is traveling at a speed of 30 miles per hour, then in 20 minutes (one-third of an hour) the car will travel one-third the distance $= 10$ miles.

Section 11 – Percentages

Key Points

Percent of a Number

- Percent (%) refers to parts of a number per 100. For example, 32 parts per 100 is 32%. It can also be written as $\frac{32}{100}$ or 0.32. Similarly, 260 parts per 100 is 260% or $\frac{260}{100}$ or 2.6.

  Note that converting a percent to a decimal is as simple as removing the percent sign and putting a decimal two digits to the left of the number. For example, 20% is 0.2, 710% is 7.1, 8% is 0.08, 0.5% is 0.005, and so on.
- Easiest method to determine the percent of a number is to convert the percent to a decimal and multiply it with the number. For example, 20% of 90 is $0.2 \times 90 = 18$ and 4% of 90 is $0.04 \times 90 = 3.6$.
- Multiple percents of a number can be combined. For example, 5% of 80% of 70 $= 0.05 \times 0.8 \times 70 = 2.8$.

Percent Increase/Decrease of a Number

- A percent decrease refers to a decrease in parts of a number per 100. For example, 20% decrease of a number n is $100 - 20 = 80\%n$ or $\frac{80}{100}n$ or $0.8n$.
- A percent increase refers to an increase in parts of a number per 100. For example, 20% increase of a number n is $100 + 20 = 120\%n$ or $\frac{120}{100}n$ or $1.2n$. Similarly, 250% increase of number n is $100 + 250 = 350\%n$ or $3.5n$.
- Multiple increases and decreases can be multiplied together. For example, a 20% increase of n, followed by a 25% decrease is $(1.2 \times 0.75)n$. The order of the increase/decrease does not matter. For example, $(1.2 \times 0.75)n$ is same as $(0.75 \times 1.2)n$.
- It is important to remember the distinction between percent of a number and percent increase/decrease of a number.

The Original Number before Percent Increase/Decrease

- When a number is modified by percent increase or decrease (as described above), the original number (before the increase or decrease) can be determined as follows, where end number is the number after percent increase/decrease.

$$\text{original number} = \frac{\text{end number}}{1 \pm \text{percent increase or decrease, as decimal}}$$

- For example, if the price of a shirt after 30% decrease is \$21, then end number = 21 and denominator $= 1 - 0.3 = 0.7$.

$$\text{original price of shirt} = \frac{21}{0.7} = \$30$$

  Similarly, if the price of a shirt after 30% increase is \$21, then denominator $= 1 + 0.3 = 1.3$.
- Multiple increases and decreases can be combined. For example, if a number is increased by 30% and then decreased by 12%, then the denominator to determine the number before increase/decrease $= (1 + 0.3)(1 - 0.12) = (1.3)(0.88)$.
- For questions on price/cost in dollars, remember that % discount is decrease in price and % sales tax is increase in price.

A Number Percent of Another Number

- If x and y are two numbers, then x is what percent of y (also worded as what percent of y is x) can be determined as

$$\frac{x}{y} \times 100$$

Percent Change

- Percent change is the percent by which a number is changed (increased or decreased). The number before the change is referred to as the "old value" and the number after the change is referred to as the "new value". A positive percent change indicates an increase, and a negative percent change indicates a decrease.

$$\%\ \text{change} = \frac{\text{new value} - \text{old value}}{\text{old value}} \times 100$$

  For example, if the price of a shirt decreased from \$20 to \$12, then old value = 20 and new value = 12.

$$\%\ \text{change} = \frac{12 - 20}{20} \times 100 = -40\% = 40\%\ \text{decrease in price}$$

Section 12 – Exponents

Key Points

- When a number is multiplied by itself several times, such as $5 \times 5 \times 5 \times 5$, the number is called the base and the number of times it is multiplied by itself is called the exponent. Since 5 is multiplied by itself 4 times, 5 is the base and 4 is the exponent. It can be written as 5 to the power of $4 = 5^4$. The same applies for an expression. For example, $5xy \times 5xy \times 5xy$ can be written as $(5xy)^3$, where $5xy$ is the base and 3 is the exponent. All the numbers and expressions within parentheses are the base of the exponent.
- Following are the exponent rules to remember.
  - $x^0 = 1$
  - $x^a \times x^b = x^{a+b}$
  - $\frac{x^a}{x^b} x^{a-b}$
  - $x^{a^b} = x^{ab}$
  - $(xy)^a = x^a y^a$
  - $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^b}$
  - $x^{-a} = \frac{1}{x^a}$
- When a number or an expression is within a root (square root, cube root, and so on), the root can be removed and replaced with a fractional exponent. The value of the root is the denominator of the fraction. For example, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt[a]{x} = x^{\frac{1}{a}}$.

  The entire expression within the root must be to the power of the fractional exponent. For example, $\sqrt{3xy} = (3xy)^{\frac{1}{2}}$.

  If the expression within the root has exponent, then the exponent is the numerator of the fractional exponent. For example, $\sqrt[4]{y^3} = x^{\frac{3}{4}}$ and $\sqrt[3]{xy^3} = (xy^3)^{\frac{1}{3}} = x^{\frac{1}{3}} y^{\frac{3}{3}} = x^{\frac{1}{3}} y$.
- When the bases of two numbers or expressions are the same, the exponents can be equated. For example, if $(xy)^a = (xy)^5$, then $a = 5$. If the bases are not same but are multiple of the same number, then they may be rewritten with the same base. For example, the equation $3^a = 27$ can be written as $3^a = 3^3$. The exponents can now be equated. $a = 3$.

  See few examples of multiples below.
  - Base 2 or 4: $4 = 2^2$, $8 = 2^3$, $16 = 4^2 = 2^4$, $32 = 2^5$, $64 = 4^3 = 2^6$.
  - Base 3 or 9: $9 = 3^2$, $27 = 3^3$, $81 = 9^2 = 3^4$.
  - Base 5: $25 = 5^2$, $125 = 5^3$.

Tips

Mental Math:

- In the equation $3^a = 27$, the value of a is apparent. Since 27 is 3^3, $a = 3$.
- The expression $\frac{x^2y^2}{xy^{-3}}$ can be simplified as xy^5. x from the denominator when moved to numerator is x^{-1}, resulting in $x^{-1+2} = x$. Similarly, y^{-3} moved to the numerator is y^3, resulting in $y^{2+3} = y^5$.

Section 13 – Exponential Growth and Decay

Key Points

Exponential Growth and Decay versus Linear Increase and Decrease

- A linear increase or decrease occurs at a constant rate. Exponential growth starts with a slow increase followed by a rapid increase. Exponential decay starts with a rapid decrease followed by a slower decrease. See examples below.
  - If the number 400 is increased by 2 each day, then the increase each day is the same. For example, on the first day of increase the number will be 402 and then 404 the day after, 406 the day after, 408 the day after, and so on. This is linear increase (or increasing linear). If the number 400 is doubled each day, then the increase each day is significantly greater. For example, on the first day of increase the number will be 800 and then 1,600 the day after, 3,200 the day after, 6,400 the day after, and so on. This is exponential growth (increase).
  - If the number 400 is decreased by 2 each day, then the decrease each day is same. For example, on the first day of decrease the number will be 398 and then 396 the day after, 394 the day after, 392 the day after, and so on. This is linear decrease (or decreasing linear). If the number 400 is decreased by half each day, then the decrease each day is significantly greater. For example, on the first day of decrease the number will be 200 and then 100 the day after, 50 the day after, 25 the day after, and so on. This is exponential decay (decrease).
  - Words/phrases like "double", "half", or "x% more than the preceding year" in a question refer to exponential growth/decay.
- The graph of linear increase and linear decrease is a straight line (Fig. 1 and Fig 2 below, respectively).The graph of exponential growth rises slowly from left to right followed by a sharp curve upwards (Fig. 3 below). The graph of exponential decay drops sharply from left to right as a curve followed by a slower decrease (Fig. 4 below).

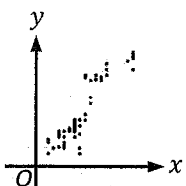


Fig. 1

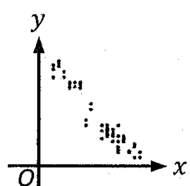


Fig. 2

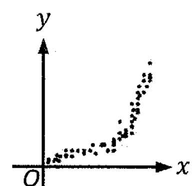


Fig. 3

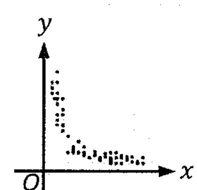


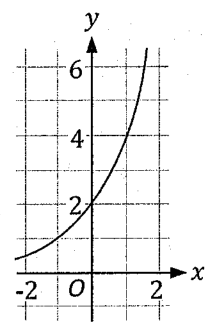
Fig. 4

Exponential Growth and Decay

- The formula for exponential growth or decay is $y = a(b)^x$. The components of the equation are:
  - a is the initial number and is greater than 0. (On a graph, this is the y-intercept.)
  - b is the rate of change in the value of a.
  - x is the number of time intervals.
  - y is the accumulated number after x time intervals.
- The rate of change in exponential growth and decay can be calculated as follows:
  - In exponential growth, $b = 1 + r$, where r is the percent growth rate as decimal. The value of b is greater than 1. For example, if the growth rate is 20%, then $b = 1 + 0.2 = 1.2$. The equation is $y = a(1.2)^x$.
  - In exponential decay, $b = 1 - r$, where r is the percent decay rate as decimal. The value of b is less than 1 but greater than 0. For example, if the decay rate is 20%, then $b = 1 - 0.2 = 0.8$. The equation is $y = a(0.8)^x$.
- It is important to interpret the time interval correctly. For example, if an increase/decrease occurs twice per year, then the time interval is $2t$ per year. If an increase/decrease occurs once in every 2 years, then the time interval is $\frac{t}{2}$ per year. Note that time interval could be any unit of time. For example, year, quarter, month, week, day, hour, minute, and so on.

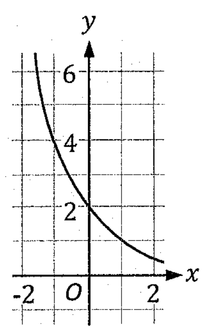
Graph of Exponential Functions

- In the xy-plane, the graph of an exponential growth or decay function is defined by $f(x) = a(b)^x$, where $f(x)$ is the y value for a particular value of x.
  - The coordinates of the y-intercept are $(0, a)$, where a is the y-coordinate of the y-intercept.
  - The graph always passes through the points $(0, a)$ and $(1, ab)$. When $a = 1$, these points are $(0, 1)$ and $(1, b)$.
- To match the equation of a given exponential function with the graphs in the answer choices, determine the points $(0, a)$ and $(1, ab)$ and look for the graph with these points. For example, in $y = 2(2)^x$, $(0, a) = (0, 2)$ and $(1, ab) = (1, 2 \times 2) = (1, 4)$. The corresponding graph, shown below, will pass through these points.



- Shifted Graphs: The graph of $f(x) = a(b)^x$ can be shifted vertically upwards or downwards. The equation of a graph shifted upwards by c units is $f(x) = a(b)^x + c$ and the equation of a graph shifted downwards by c units is $f(x) = a(b)^x - c$. The y-coordinate of the y-intercept of the graph will shift proportionally.

  For example, if the above graph is shifted downwards by 3 units, then the y-intercept of the shifted graph would be $(0, 2 - 3) = (0, -1)$.
- Graphs with negative exponent: When the function has a negative exponent, for example $f(x) = a(b)^{-x}$, then the point $(1, ab)$ will be $(-1, ab)$. See the graph below of $f(x) = 2(2)^{-x}$. Point $(1, ab)$ is $(-1, 4)$.



Tips

Mental Math:

- The components of an exponential equation are apparent by looking at the equation. In the equation $y = 50(1.06)^t$, since $b > 1$, the equation is of exponential growth. The components of the equation are: $a = 50$, percent growth rate = 6% (1.06 is 6% increase), and the time interval $x = t$ is once each year.
- In the equation $y = 500(0.96)^{\frac{t}{5}}$, since $b < 1$, the equation is of exponential decay. The components of the equation are: $a = 500$, percent decay rate = 4% (0.96 is 4% decrease), and the time interval $x = t$ is once every 5 years.
- The components of an exponential function are apparent by looking at the equation as shown in the Key Points. An equation can be matched to the corresponding graph and vice versa by looking at the points $(0, a)$ and $(1, ab)$.

Section 14 – Manipulating Expressions and Equations

Key Points

Expressions under Square Root

- When an equation contains a square root expression, the square root can be removed by squaring both sides of the equation. After removing the square root, the value of the variable in the equation can be solved. See the example below.

$$\sqrt{2x+1} = \sqrt{x+3} \rightarrow \left(\sqrt{2x+1}\right)^2 = \left(\sqrt{x+3}\right)^2 \rightarrow 2x+1 = x+3 \rightarrow x = 2$$

It is important to check that a solution is not an extraneous solution.

- If both sides have integers, then collect them on one side before squaring. See the example below.

$$\sqrt{x+2} - 3 = 6 \rightarrow \sqrt{x+2} = 9 \rightarrow \left(\sqrt{x+2}\right)^2 = 9^2 \rightarrow x+2 = 81 \rightarrow x = 79$$

- The entire expression on both sides must be squared. For example, $\sqrt{2x+1} = 5 + x \rightarrow \left(\sqrt{2x+1}\right)^2 = (5+x)^2$.

Expressions with Factors

- The following factors are helpful to remember.
  - $(x+y)(x+y) = x^2 + 2xy + y^2$
  - $(x-y)(x-y) = x^2 - 2xy + y^2$
  - $(x-y)(x+y) = x^2 - y^2$

For example, the expression $\frac{x+1}{x^2-1}$ can be simplified as $\frac{x+1}{(x-1)(x+1)} = \frac{\cancel{x+1}}{(x-1)\cancel{(x+1)}} = \frac{1}{x-1}$.

Fractions with Expressions in the Denominator

- When an equation has fractions with expressions in the denominator, determine a strategy that will simplify or remove the expressions from the denominators. See examples below.
  - In the equation $\frac{x^2-y^2}{x-y} = 2$, the value of $x + y$ can be solved as shown below.

$$\frac{(x-y)(x+y)}{x-y} = 2 \rightarrow \frac{\cancel{(x-y)}(x+y)}{\cancel{x-y}} = 2 \rightarrow x+y = 2$$

  - Below is an example of fractions with different denominators. A common denominator can be created as shown.

$$\frac{5}{x+1} + \frac{2}{x+3} = y \rightarrow \frac{5(x+3)}{(x+1)(x+3)} + \frac{2(x+1)}{(x+1)(x+3)} = y \rightarrow \frac{5x+15+2x+2}{(x+1)(x+3)} = y \rightarrow \frac{7x+17}{(x+1)(x+3)} = y$$

Combining Like Terms in Expressions

- Like terms have the same variable(s) with the same exponent. For example, in the expression $4x^2 - xy + x^2 + 9xy$, $4x^2$ and x^2 are like terms since they have the same variable and exponent, and xy and $9xy$ are like terms since they have the same two variables, and the exponent is 1. Like terms can be added or subtracted as shown below.

$$4x^2 - xy + x^2 + 9xy = 5x^2 + 8xy$$

Expressing a Variable in Terms of Other Variables

- The variables in an equation can be rearranged to isolate one variable from the other variables. For example, in the equation $a + b = s(m+n)$, the variable n can be isolated as shown below.

$$a+b = s(m+n) \rightarrow a+b = sm+sn \rightarrow a+b-sm = sn \rightarrow \frac{a+b-sm}{s} = n \rightarrow \frac{a+b}{s} - m = n$$

Tips

Desmos Graphing Calculator:

- Equations with square roots can be solved by typing the equation in the Desmos graphing calculator and reading the value of x where the line corresponding to the solution intersects the x-axis. If the variable is y, then read the value of y where the line intersects the y-axis.
  Note that the Desmos graphing calculator does not graph an extraneous solution. Hence, no need to check for an extraneous solution.

Section 15 – Probability

Key Points

- Probability is the likelihood of achieving certain or desired outcomes from the total possible outcomes, at random.

$$\text{probability} = \frac{\text{number of certain or desired outcomes}}{\text{number of total possible outcomes}}$$

- It is important to identify the outcomes correctly. The table below shows 2 brands of pencils in 3 different colors.

	Green	Red	Black	Total
Brand A	2	5	3	10
Brand B	3	2	5	10

- The probability of randomly selecting a red color Brand A pencil out of all the pencils is

$$\frac{\text{All Brand A red color pencils}}{\text{All pencils}} = \frac{5}{20} \text{ or } \frac{1}{4} \text{ or } 0.25$$

  Note that all the red color Brand A pencils must be considered in the desired selection. The total outcomes must include all the pencils.

- The probability of randomly selecting a red color or a black color pencil out of all the Brand B pencils is

$$\frac{\text{All Brand B red color pencils} + \text{All Brand B black color pencils}}{\text{All Brand B pencils}} = \frac{2+5}{10} = \frac{7}{10} \text{ or } 0.7$$

  Note that all the red color Brand A pencil and all the black color Brand B pencil must be considered in the desired selection. Since the selection is out of Brand B pencils, the total outcomes must include all Brand B pencils and exclude Brand A pencils.

- The probability of randomly selecting a red color Brand A pencil or a black color Brand B pencil out of all the pencils is

$$\frac{\text{All Brand A red color pencils} + \text{All Brand B black color pencils}}{\text{All pencils}} = \frac{5+5}{10} = \frac{10}{20} \text{ or } \frac{1}{2} \text{ or } 0.5$$

  Note that all the red color Brand A pencil and all the black color Brand B pencil must be considered in the desired selection. The total outcomes must include all the pencils.

Section 16 – Reading Tables and Graphs

Key Points

Reading Graphs

- A graph illustrates the relationship between two variables plotted as data points. One variable is represented on the horizontal axis and the other variable is represented on the vertical axis. Each data point is represented as a dot and has a value on the horizontal axis and a value on the vertical axis.
- The value of a data point on a graph can be determined by reading the value of that dot on the horizontal and the vertical axes. Use best approximation when reading data points not on the graph grid lines.
  - In Fig. 1 below, the data points connect to form a straight line. The number of products sold (vertical axis) are shown based on the package size of the product (horizontal axis). For example, for package size = 2 ounces, number of products sold = 70 and for package size = 6 pounds, number of products sold = 40, see the arrows.
  - In Fig. 2 below, the data points are connected by line segments. Fig. 3 below, shows the same graph represented as a curve model. For each year (horizontal axis), the lemon production (vertical axis) can be read from the graph. For example, in 2002, lemon production = 10 metric tonnes and in 2003, lemon production = approximately 35 metric tonnes.

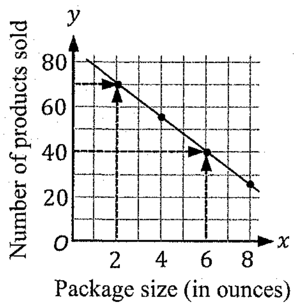


Fig. 1

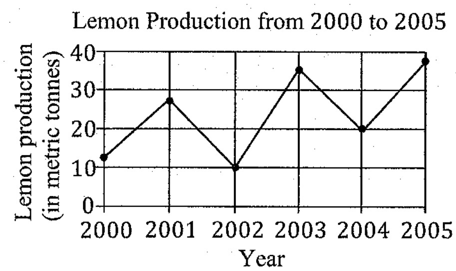


Fig. 2

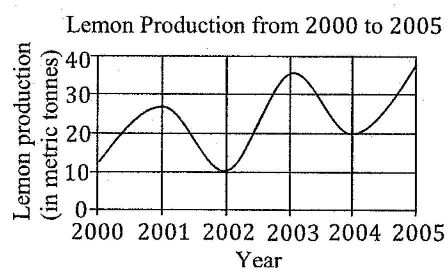


Fig. 3

Scatter Plots

- A scatter plot shows the correlation between two variables plotted as data points on a graph.
  - In a positive correlation, the increase in the value of one variable increases the value of the other variable (Fig. 4 below).
  - In a negative correlation, the increase in the value of one variable decreases the value of the other variable (Fig. 5 below).
  - When the data points are scattered, then there is no correlation between the two variables (Fig. 6 below).
- A line of best fit is a straight line drawn through the maximum number of data points on a scatter plot. If a question does not have a line of best fit drawn on the scatter plot, then use best estimation to draw a line passing through the maximum data points.
  - When the data points are concentrated around the line of best fit, the correlation is high (strong) (Fig. 4 below). When the data points are spread out around the line of best fit, the correlation is low (weak) (Fig. 5 below).
  - The actual value of a data point is the value of the dot on a scatter plot. The predicted value of the same data point is the value on the line of best fit.
  - The points that lie on the line of best fit are the points for which the predictions are most accurate. The data points below the line are overestimated since their value on the line is higher than the actual value. The data points above the line are underestimated since their value on the line is lower than the actual value.

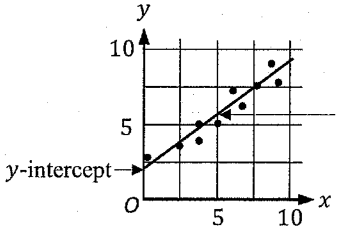


Fig. 4

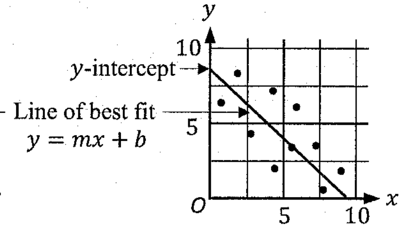


Fig. 5

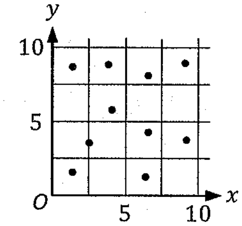
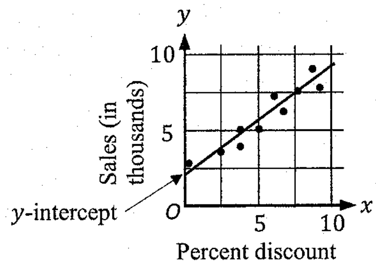


Fig. 6

- The equation of the line of best fit is the slope-intercept equation, $y = mx + b$. The slope of the line of best fit can be determined using the slope formula (refer to Section 2 of this book).
- The point where the line of best fit intersects the y-axis (vertical axis) is the y-intercept of the line. At y-intercept, x (horizontal axis) $= 0$. For example, if x represents the percent discount offered for a certain brand of perfume and y represents the sales (in thousands of dollars), then y-intercept is the sales (in thousands of dollars) when 0 percent discount is offered; in other words, when no percent discount is offered. See the figure below.



Tips

Mental Math:

- In Fig.1 in the Key Points, the difference between the number of products sold based on the package size can be determined by reading the graph. For example, 70 products are sold for package size = 2 ounces and 40 are products sold for package size = 6 ounces. The difference is 30.
- The equation for the line of best fit is often apparent by looking at the scatter plot. The y-intercept is the value of b in the $y = mx + b$ equation. The slope can be read as rise/run from the graph.

No Calculation Required:

- The data points can be directly read from a graph. For example, in Fig. 2 in the Key Points, for any given year the lemon production for that year can be directly read from the graph.

  On a scatter plot graph, if the x value of a point is given, then the corresponding y value can be read from the graph and vice versa.
- The difference between the actual and the predicted value of a data point on a scatter plot is apparent from the graph.

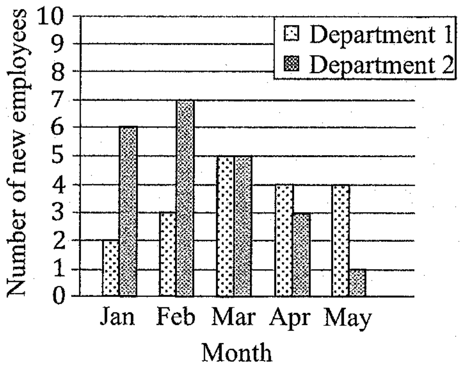
Section 17 – Histograms, Bar Graphs, and Dot Plots

Key Points

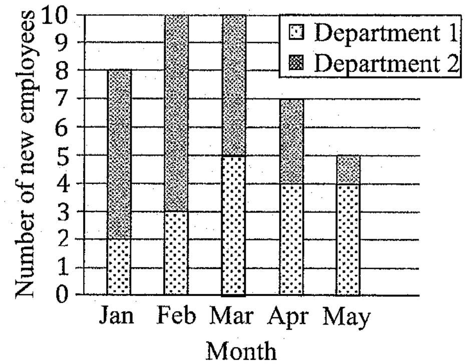
Bar Graphs

- A bar graph shows data grouped into categories. For example, data grouped by book genre, year, color, height, and so on. The number of data points in each category are represented by a rectangular bar. The height of a bar can be determined by reading the difference between the bottom value and the top value of the bar.
- A bar graph may contain sub-groups within each category. See example below of departments at a company. Sub-groups may be displayed side by side as bars within each category (left bar graph below) or may be stacked on each other as a column (right bar graph below). Note that in a stacked bar graph, the height of the top bar is read from the top of the bottom bar not from the bottom of the graph. For example, in the stacked bar graph below, in Jan. for Department 1 the height of the bar is from 0 to 2 = 2 employees and for Department 2, the height of the bar is from 2 to 8 = 6 employees.

Number of New Employees in 2 Departments



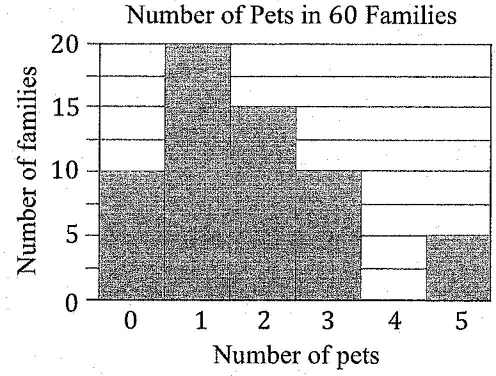
Number of New Employees in 2 Departments



Histograms

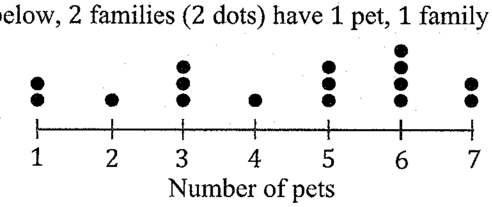
- A histogram shows the continuous distribution of data in groups, a main difference from the bar graphs. Groups are generally represented on the horizontal axis and the number of data points (also known as frequency) on the vertical axis. The number of data points (or frequency) in each group are represented by a rectangular bar. For example, in the histogram below, the number of pets are in groups from 0 to 5. The height of each bar represents the number of families in that group. Out of 60 families, 10 families have no pets, 20 families have 1 pet, 15 families have 2 pets, 10 families have 3 pets, no family has 4 pets and 5 families have 5 pets.
- The sum of the data in a histogram can be determined by multiplying the value of a group by the frequency of the group. In this example, the total number of pets in the 60 families $= (0 \times 10) + (1 \times 20) + (2 \times 15) + (3 \times 10) + (5 \times 5)$.

Number of Pets in 60 Families



Dot Plots

A dot plot shows the distribution of relatively small data sets, as compared to a histogram. Data points are represented using dots. The horizontal axis represents the group and the number of dots on each group represent the number of data points in that group. For example, in the dot plot below, 2 families (2 dots) have 1 pet, 1 family (1 dot) has 2 pets, and so on.



Section 18 – Mean, Median, Mode, Range, and SD

Key Points

Mean

- The mean is the sum of numbers in a data set divided by the total count of numbers in the data set. It is same as average.

$$\text{mean} = \frac{\text{sum of numbers}}{\text{count of numbers}}$$

- When the count and the mean of a set of numbers is given, the sum of the numbers = mean × count of numbers. For example, if 8.5 is the mean of a set of 5 numbers, then the sum of the numbers is $5 \times 8.5 = 42.5$.

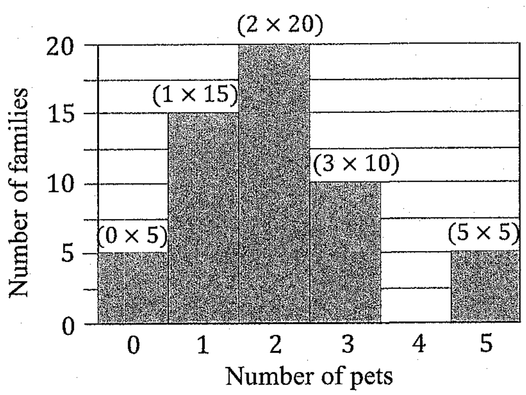
  Similarly, if the mean of 3 numbers 10, 7, and x is 24, then x can be determined by setting the mean and equating it to 24.

$$\text{mean} = \frac{10 + 7 + x}{3} = 24 \rightarrow 17 + x = 24 \times 3 \rightarrow 17 + x = 72 \rightarrow x = 55$$

Mean in Grouped Data

- When grouped numbers are given, the sum of numbers in each group must be determined before calculating the mean of the group. For example, if 7 students are 50 inches tall and 3 students are 65 inches tall, then the total height of $7 + 3 = 10$ students is $(7 \times 50) + (3 \times 65)$. Dividing this total by 10 will give the mean height of the 10 students.
- In a histogram or a dot plot, the mean is the sum of the data in all groups divided by the number of data points. See below.

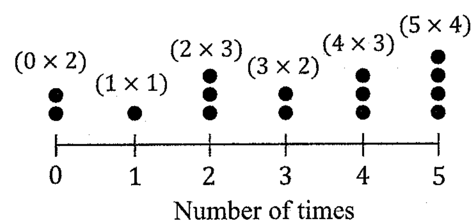
Number of Pets in 55 Families



$$\frac{(0 \times 5) + (1 \times 15) + (2 \times 20) + (3 \times 10) + (5 \times 5)}{55} =$$

$$\frac{0 + 15 + 40 + 30 + 25}{55} = \frac{110}{55} = 2 \text{ pets}$$

Number of Times 15 Students Played Sports per Week



$$\frac{(0 \times 2) + (1 \times 1) + (2 \times 3) + (3 \times 2) + (4 \times 3) + (5 \times 4)}{15} =$$

$$\frac{0 + 1 + 6 + 6 + 12 + 20}{15} = \frac{45}{15} = 3 \text{ times}$$

Median

- In a data set of sorted numbers, the median is determined based on the count of numbers in the data set. If the data set is not sorted, then it must be sorted before determining the median.
  - If the data set contains an odd count of numbers, then the median is the middle number. For example, in a set of 7 numbers, the 4^{th} number is the middle number and the median. In the data set 4, 8, 19, 32, 44, 60, 71, the median is 32.
  - If the data set contains an even count of numbers, then the median is the average of the two middle numbers. For example, there are 8 numbers in the data set 5, 7, 12, 15, 21, 22, 30, 31. The median is the average of 15 and 21 = 18.

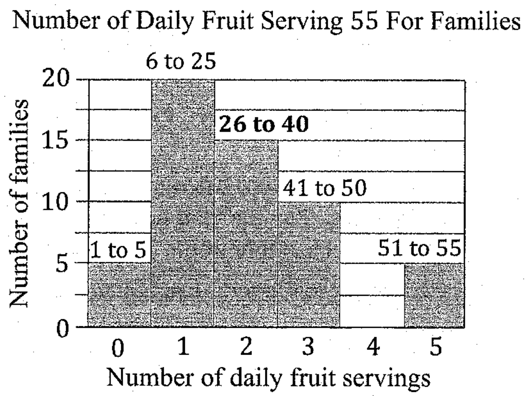
Median in Grouped Data

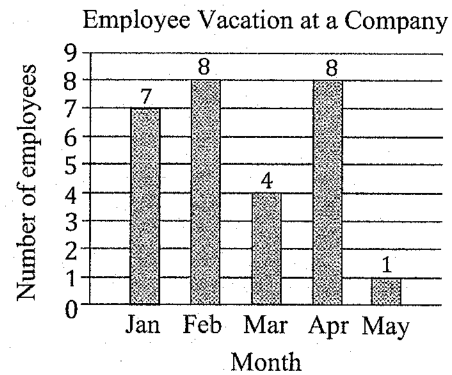
- In a grouped set of data, the group that includes the median of the data points is the median group. For example, the table below shows the number of vacation days per year for 29 employees at a company. The vacation days are grouped in increments of 5.

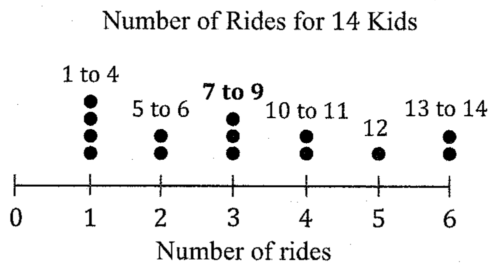
  The median of 29 employees (29 data points) is the 15^{th} employee (15^{th} data point). The median number of vacation days is in the group that contains the 15^{th} employee. Do a running count of employees from left to right till the 15^{th} employee is reached. This falls in $11 - 15$ vacation days. The median number of vacation days can be any integer from 11 to 15.

Number of vacation days	1 – 5	6 – 10	11 – 15
Number of employees	5	8	16
Running count	1^{st} to 5^{th}	6^{th} to 13^{th}	**14^{th} to 29^{th}**

- In a histogram, the bar that includes the median data point is the median group. In the histogram below, there are 55 families (data points). The medium is in the bar that includes the 28th family (median of 55 data points). The running count of the number of families from left to right is shown on the top of each bar. The 28th family is in number of daily fruit servings = 2. Hence, the median number of daily servings of 55 families is 2. Note that there are 15 families in the median bar. Hence, the number of families for the median number of daily fruit servings is 15.
- In a dot plot, the median is in the group that includes the dot corresponding to the median data point. In the dot plot below, there are 14 kids (data points). The median is in the group that includes the 7th and the 8th dot (median of 14 data points). The running count of the number of kids from left to right is shown. This is in the group for number of rides = 3. Hence, the median number of rides is 3.
- In a bar graph, the median is the middle number of the height of the bars, ordered from least to greatest. In the bar graph below, the number of employees ordered from least to greatest are 1, 4, 7, 8, 8. The median number of employees is 7.







Mode

- The mode is the number that occurs most often in a data set. For example, in the data set 4, 5, 19, 32, 5, the mode is 5 since it occurs 2 times. Other numbers occur once. If a data set does not contain repeated numbers, then there is no mode.

Range

- The range is the difference between the lowest number and the highest number in a data set.

Standard Deviation (SD in the title)

- The standard deviation is the spread of numbers in a data set from the mean. The comparative spread of numbers in 2 or more data sets is apparent by looking at the numbers in the data sets. Calculating the mean is not required. For example, in the two data sets shown below, it is apparent that the spread of numbers in Set 1 is higher.

Set 1	4.5	6.8	50.5	60.5	70.2	80.8
Set 2	42.3	35.8	32.5	65.9	46.4	45.6

Tips

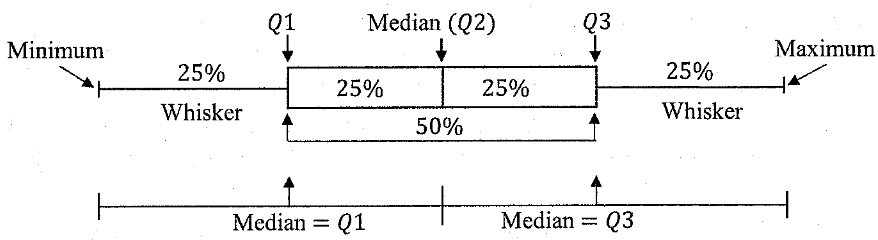
Mental Math:

- The median in a dot plot can be determined by counting the dots from left to right till the dot corresponding to the median data point is reached.
- The median in a table or a histogram can be determined by doing a running count of data points till the median data point is reached.
- The standard deviation in two or more data sets can be compared by observing the spread of numbers.

Section 19 – Median and Range in Box Plots

Key Points

- A box plot (also known as box and whisker plot) distributes the numbers in a data set from least to greatest into 4 sections. Each section contains approximately 25% of the numbers. See the figure below.



  - The middle 50% of the numbers are represented in a box. The median is the middle number of the data set in the box and is shown by a vertical line. It can be read directly from the box plot.
  - The number at the left end of the box is known as the First Quartile ($Q1$). It is the median of the 50% numbers to the left of the median.
  - The number at the right end of the box is known as the Third Quartile ($Q3$). It is the median of the 50% numbers to the right of the median.
  - The minimum is the lowest number in a data set and the maximum is the highest number in a data set. They can be directly read from the box plot.
- The range is the difference between the maximum and the minimum.
- The actual numbers in a data set are not known in a box plot. Hence, the mean cannot be determined from a box plot.

Tips

<u>No Calculation Required:</u>

- The median, and the minimum and the maximum values in a data set can be read from a box plot.

Section 20 – Studies and Data Interpretation

Key Points

- Studies and surveys are conducted using a random sample from the sample population.

  For example, if a survey is to be conducted on the satisfaction of train service at a certain train station, then the sample population is all the people coming in and out of that train station. Since it is not practical to survey every single person coming in and out of that train station a random sample of appropriate size will be surveyed. The sample must be large enough to represent the true average of the sample population. If the sample is not random, or is small, or deviates from the sample population, then the study or the survey is flawed.

  In the above example, if a survey is conducted on the first 50 people at the train station each day, then the sample is not random and is small. It will not represent the true average of the sample population. Similarly, if random people at a nearby restaurant are also included in the survey, then the sample no longer represents the correct sample population.

- A margin of error is the variability by which the results obtained from a random sample are expected to differ from that of the sample population. It is the positive and the negative deviation from the results. Increasing the random sample size can reduce the margin of error.

  In the above example, if 65% of the people surveyed in a random sample were satisfied with a margin of error of 3%, then it is plausible that between $65\% - 3\% = 62\%$ and $65\% + 3\% = 68\%$ of the people at that train station will respond similarly.

  For example, if the random sample is 5,000, then it is plausible that between $5{,}000 \times 62\% = 5{,}000 \times 0.62 = 3{,}100$ and $5{,}000 \times 67\% = 5{,}000 \times 0.67 = 3{,}350$ people at that train station will respond similarly.

- A confidence level is how often the results obtained from a random sample will deviate from another random sample from the same population for the same study or survey.

  For example, 95% confidence level of a team conducting a survey indicates that the team is 95% confident that if the survey is repeated as many times, the same results will be obtained 95% of the time and will reflect the true average of the sample population.

- Generalization is applying the results obtained from a random sample to the sample population. The largest population the results can be generalized to is the sample population. In the above example, the largest population the survey can be generalized to is all the people at that train station. It would be incorrect to generalize the results to people at any other train station.

Section 21 – Circles

Key Points

Circle and Angles

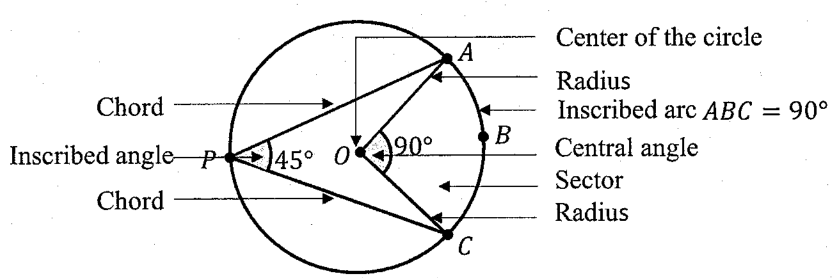
- The radius of a circle is any line segment from the center of the circle to a point on the circle. In the figure below, AO and CO are two radii of a circle.
- The diameter of a circle is any line segment from one point on the circle to another point on the circle that passes through the center of the circle. It divides the circle into two equal halves, each half is known as a semicircle.
- A chord in a circle is any line segment from one point on the circle to another point on the circle that does not pass through the center of the circle. See the figure below for chords AP and CP.
- An angle formed between two radii of a circle is known as a central angle. See angle AOC in the figure below.
- An angle formed between two chords originating from the same point on a circle is known as an inscribed angle. See angle APC in the figure below. An inscribed angle is half the measure of its corresponding central angle.
- An angle in π radians can be converted to degrees by multiplying it with $\frac{180}{\pi}$. For example, $\frac{2}{3}\pi \times \frac{180}{\pi} = 120°$.
- An angle in degrees can be converted to π radians by multiplying it with $\frac{\pi}{180}$. For example, $120° \times \frac{\pi}{180} = \frac{2}{3}\pi$ radians.

Circumference and Arc of a Circle

- The circumference of a circle is $2\pi r$, where r is the radius of the circle and π is approximately 3.14.
- In degrees, the circumference of a full circle is 360°.
- An arc is the portion of the circumference enclosed between the endpoints of two chords or two radii on the circumference. In the figure below, arc ABC is enclosed between the endpoints of radii AO and CO and the endpoints of chords AP and CP.
- The degree measure of an arc is equal to the degree measure of its central angle and twice the degree measure of its inscribed angle. See arc ABC in the figure below. Note that an arc can also be written with an arc accent. For example, the arc ABC can be written as $\overset{\frown}{ABC}$.
- An arc less than a semicircle is known as a minor arc. An arc greater than a semicircle is known as a major arc.
- The proportion relationship between the length of an arc and the circumference of the circle is

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{degree measure of central angle}}{360°}$$

- When the central angle is given in radians, the length of the arc can be determined as $s = r\theta$, where s is the length of the arc, r is the radius of the circle, and θ is the central angle in π radians.
- When the central angle is given in degrees, the length of the arc can be determined as $s = 2\pi r \times \frac{\theta}{360}$, where s is the length of the arc, r is the radius of the circle, and θ is the central angle in degrees.



Area of a Circle, a Semicircle, and a Sector of a Circle

- The area of a circle is πr^2, where r is the radius of the circle and π is approximately 3.14.
- The area of each semicircle is half the area of the circle.
- If the area of a circle is given, then the radius can be determined by equating it to πr^2. For example, if the area of a circle is 16π, then the radius can be determined as shown below.

$$\pi r^2 = 16\pi \rightarrow r^2 = 16 = 4^2 \rightarrow r = 4$$

- The area enclosed between two radii is known as a sector.
  - When the central angle is given in radians, the area of the sector can be determined as $\frac{1}{2}r^2\theta$, where r is the radius of the circle and θ is the central angle in π radians.
  - When the central angle is given in degrees, the area of the sector can be determined as $\pi r^2 \times \frac{\theta}{360}$, where r is the radius of the circle and θ is the central angle in degrees.

Standard Form Equation of a Circle

- The equation of a circle in the standard form is $(x-h)^2+(y-k)^2=r^2$, where r is the radius of the circle, (h, k) are the coordinates of the center of the circle, and (x, y) are the coordinates of any point on the circumference of the circle. For example, in the equation $(x-2)^2+(y+5)^2=9$, the x-coordinate of the center is 2, the y-coordinate of the center is -5, and $r^2=9$. Since $r^2=9$, $r=3$ and the diameter is $2\times 3=6$.
- The radius of a circle can be determined when the center of the circle and any point on the circumference of the circle are given. Plug the center as (h, k) and the given point as (x, y) in $(x-h)^2+(y-k)^2=r^2$ and determine r^2.

General Form Equation of a Circle

- The equation of a circle in the general form is $x^2+y^2+Ax+By+C=0$, where A, B, and C are constants. A and B are also the coefficients of x and y, respectively. The general form equation can be converted to the standard form. See below.

  Completing the Square
  - Pre-step, if needed: If the equation contains a common multiple in all the terms, then simplify the equation before proceeding. Keep the value of C to the right side of the equation. For example, in $2x^2+8x+2y^2-12y+8=0$ move 8 to the right side and divide the equation by the common multiple 2.

$$2x^2+8x+2y^2-12y+8=0 \rightarrow x^2+4x+y^2-6y=-4$$

  - Step 1: Determine the factors: The factors for x and y can be determined by dividing their coefficient by 2. In the above equation, the coefficient of $x=4$ and the coefficient of $y=-6$. The corresponding factor are $(x+2)^2$ and $(y-3)^2$.
  - Step 2: Balance the equation: Replace the left-side of the general form equation with the above factors and individually add the square of 2 and -3 to the right-side of the equation.

$$(x+2)^2+(y-3)^2=-4+2^2+(-3)^2 \rightarrow (x+2)^2+(y-3)^2=-4+4+9 \rightarrow (x+2)^2+(y-3)^2=9$$

Circles and Other Geometric Figures

- A triangle formed within a circle with two sides as radii is an isosceles triangle (Fig. 1 below).
- A triangle formed within a circle with one side as the diameter of the circle and the other two sides as chords, has a right angle where the two chords meet (Fig. 2 below).
- An angle formed by the radius of a circle and a line segment tangent to the circle is a right angle (Fig. 3 below). Two such tangent line segments form a quadrilateral (Fig. 3 below).

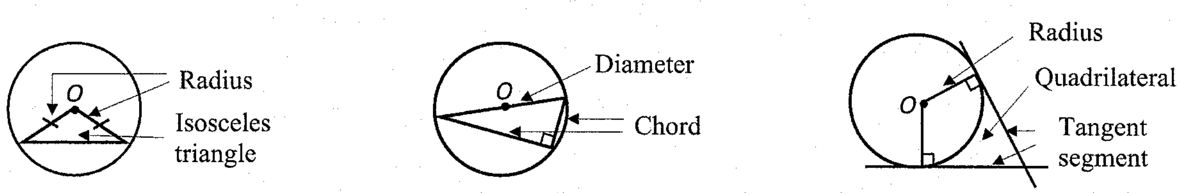


Fig. 1 Fig. 2 Fig. 3

Tips

No Calculation Required:

- The coordinates of the center of the circle can be directly read from the standard form equation.

Desmos Graphing Calculator:

- From a general form equation of a circle, the coordinates of the center of a circle and the length of radius and diameter of a circle can be determined by typing the equation of the circle in the Desmos graphing calculator.
  The diameter is the distance between the two opposite vertical points or two opposite horizontal points on the circumference of the circle. The midpoint of the opposite horizontal points is the x-coordinate of the center and the midpoint of the opposite vertical points is the y-coordinate of the center. The length of the radius is half of the diameter.

Section 22 – Lines and Angles

Key Points

- At any point on a line, the sum of angles is 180°. For example, in the Fig. 1 below, at the point P, $\angle 1 + \angle 2 + \angle 3 = 180°$.
- Two angles that add up to 180° are known as supplementary angles.
- Two angles that add up to 90° are known as complimentary angles.
- Two intersecting lines form four angles. Each opposite pair are called vertical angles and are congruent. For example, in the Fig. 2 below, lines c and p are intersecting lines. $\angle 1 = \angle 4$ and $\angle 2 = \angle 3$. Similarly, lines d and p are intersecting lines. $\angle 5 = \angle 8$ and $\angle 6 = \angle 7$.
- When a straight line cuts across two parallel lines, several types of angles are formed. In Fig. 2 below, parallel lines c and d are intersected by line p. The types of angles and their position identified by numbers is shown in the figure.
  - Corresponding angles are congruent. $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, and $\angle 4 = \angle 8$.
  - Alternate interior angles are congruent. $\angle 4 = \angle 5$ and $\angle 3 = \angle 6$.
  - Alternate exterior angles are congruent. $\angle 1 = \angle 8$ and $\angle 2 = \angle 7$.
  - Same side interior angles are supplementary angles. $\angle 3 + \angle 5 = 180°$ and $\angle 4 + \angle 6 = 180°$.
  - Same side exterior angles are supplementary angles. $\angle 1 + \angle 7 = 180°$ and $\angle 2 + \angle 8 = 180°$.
- When two or more parallel lines are intersected by two or more non-parallel lines, the ratio of any two segments on one non-parallel line is equal to the ratio of the corresponding segments on the other non-parallel line.

  For example, in the Fig. 3 below, lines l, m, and n are parallel lines and lines p and q are non-parallel intersecting lines. Hence, $\frac{AC}{CE} = \frac{BD}{DF}$, $\frac{AC}{AE} = \frac{BD}{BF}$, and $\frac{CE}{AE} = \frac{DF}{BF}$.

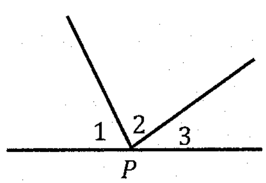


Fig. 1

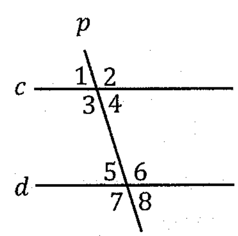


Fig. 2

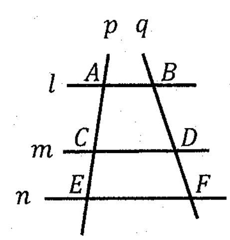


Fig. 3

Tips

Mental Math:

- In Fig. 1 in the Key Points, if $\angle 1 = 2x$, $\angle 2 = 2x$, and $\angle 3 = x$, then $5x$ is 180 and x is 36.
- In Fig. 2 in the Key Points, if $\angle 4 = 75°$, then $\angle 1$ is also 75° and $\angle 6$ is 180 minus 75 equal to 105.

Section 23 – Triangles

Key Points

Triangle Sides and Angles

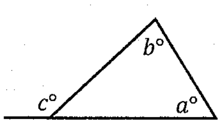
- A triangle has three sides, three angles, and three vertices (corners).
  - The sum of the degree measure of the three angles is 180°.
  - The sum of the lengths of any two sides of a triangle is greater than the length of the third side. For example, if x, y, and z are the three sides of a triangle, then $x + y > z$, $x + z > y$, and $y + z > x$.
  - An exterior angle is formed when any side of a triangle is extended. It is equal to the sum of the two non-adjacent angles. In the Fig. 1 below, c is the exterior angle, and a and b are the two non-adjacent angles. $c° = a° + b°$.

Types of Triangles

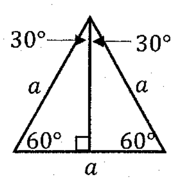
- An equilateral triangle has all three sides and all three angles equal. Since the total degree measure of the three angles of a triangle is 180°, each angle of an equilateral triangle is 60°.
  - A perpendicular line segment from any corner to the opposite side divides the equilateral triangle into two equal triangles that have angles with degree measure shown in the Fig. 2 below.
- An isosceles triangle has two sides of equal length. The angles opposite to these sides are equal (Fig. 3 below).
- A right triangle has one of the angles as 90°, also known as the right angle. The other two angles are acute angles, and their sum is 90° (Fig. 4, Fig. 5, and Fig. 6 below). The side opposite to the right angle is known as the hypotenuse and is the longest side. The other two sides are known as the legs and meet at the 90° angle.

Special Right Triangles

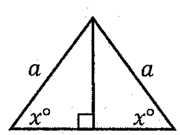
- The right triangles that have angles with degree measure of 45°-45°-90° and 30°-60°-90° have known side length relationship. These are helpful to remember.
- 45°-45°-90°: In the Fig. 5 below, a are the sides opposite to 45° angles and $a\sqrt{2}$ (hypothenuse) is the side opposite to 90° angle. The ratio of the side lengths in relation to the opposite angles is 45°:45°:90° $\rightarrow$ $a{:}\,a{:}\,a\sqrt{2}$.
  If the length of one of the sides is given, then the length of the other two sides can be determined. For example, if the side corresponding to $a\sqrt{2}$ is $5\sqrt{2}$, then $a = 5$. Similarly, if the side corresponding to a is 4, then $a\sqrt{2}$ is $4\sqrt{2}$.
- 30°-60°-90°: In the Fig. 6 below, a is the side opposite to 30° angle, $a\sqrt{3}$ is the side opposite to 60° angle, and $2a$ (hypothenuse) is the side opposite to 90° angle. The ratio of the side lengths in relation to the opposite angles is 30°:60°:90° $\rightarrow$ $a{:}\,a\sqrt{3}{:}\,2a$. If $a\sqrt{3}$ is $2\sqrt{3}$, then $a = 2$ and $2a = 4$. Similarly, if a is 4, then $a\sqrt{3} = 4\sqrt{3}$ and $2a = 8$.



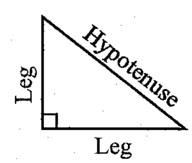
Exterior angle
Fig. 1



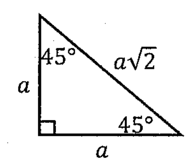
Equilateral triangle
Fig. 2



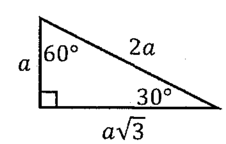
Isosceles triangle
Fig. 3



Right triangle
Fig. 4



45°-45°-90°
Fig. 5

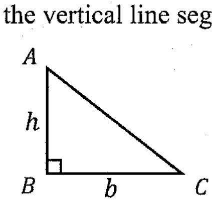


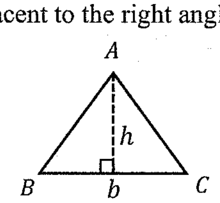
30°-60°-90°
Fig. 6

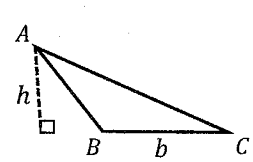
Remember that in a question on a right triangle, if the length of one of the legs is given with $\sqrt{3}$, then it is most likely a 30°-60°-90° triangle and if the length of the hypothenuse is given with $\sqrt{2}$, then it is most likely a 45°-45°-90° triangle.

Area of a Triangle

- The area of any triangle can be determined as $\frac{1}{2}bh$, where b is the length of the base and h is the height. The height of a triangle is the length of a perpendicular line segment from the highest vertex of the triangle to the base. In a right triangle, the height is the vertical line segment adjacent to the right angle. See the figures below.



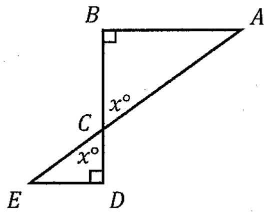




- The area of an equilateral triangle can also be determined as $\frac{\sqrt{3}}{4}a^2$, where a is the length of each side.

Similar Triangles

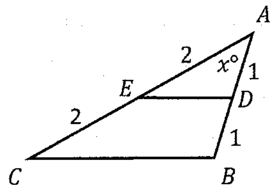
- In two similar triangles, the corresponding angles are congruent, and the corresponding sides are in proportion. Two triangles are similar when they meet one of the criteria shown below.



Three angles are congruent

In the triangles ABC and EDC, angles x are congruent vertical angles and the right angles B and D are congruent. Since the sum of three angles of any triangle is 180°, the third angle must be congruent.

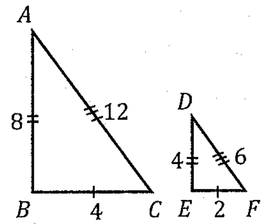
The corresponding sides are between the corresponding angles. Hence, the corresponding sides are AB and ED, BC and DC, and AC and EC.



Two sides are in proportion and share the same angle

In the triangles ABC and ADE, the ratios $AB{:}AD$ $(2{:}1)$ and $AC{:}AE$ $(4{:}2 = 2{:}1)$ are the same and the sides share the same angle x.

The sides sharing the same angle are corresponding sides. Hence, the corresponding sides are AB and AD, AC and AE, and BC and DE.



Three sides are in proportion

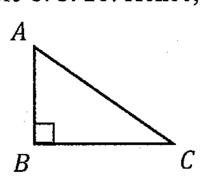
In the triangles ABC and DEF, the ratios $AB{:}DE$ $(8{:}4 = 2{:}1)$, $BC{:}EF$ $(4{:}2 = 2{:}1)$, and $AC{:}DF$ $(12{:}6 = 2{:}1)$ are the same.

The corresponding sides will be between the corresponding vertices. In this example, vertices A, B, and C correspond to vertices D, E, and F, respectively. Hence, the corresponding sides are AB and DE, BC and EF, and AC and DF.

- When a triangle is modified by proportionally shrinking or enlarging each side, the original and the modified triangles are similar triangles. The perimeter and the area of the modified triangle will change, and all the three sides will proportionally decrease or increase in length. However, the corresponding angles and the side proportions in the original and the modified triangles will remain the same.

Pythagorean Theorem and Pythagorean Triples

- Pythagorean Theorem: If the lengths of any the two sides of a right triangle are known, then the length of the third side can be calculated using the Pythagorean Theorem. Pythagorean Theorem is written as $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the two legs of the right triangle.
- Pythagorean Triples: When the lengths of all three sides of a right triangle are integers, their lengths are collectively known as Pythagorean triples. Common Pythagorean triples are $3{:}4{:}5$, $5{:}12{:}13$, $8{:}15{:}17$ and $7{:}24{:}25$ or any of their multiples. The largest ratio corresponds to the hypotenuse and the smallest ratio corresponds to the smallest side.
  For example, in the triangle ABC shown below, if the hypotenuse $AC = 10$, and one of the legs $AB = 6$, then the ratio of $AB{:}AC = 6{:}10$ is the ratio of the Pythagorean triple $6{:}8{:}10$. Hence, $AB{:}BC{:}AC = 6{:}BC{:}10 = 6{:}8{:}10 \rightarrow BC = 8$.



Tips

Mental Math:

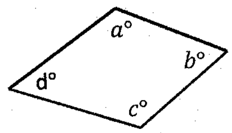
- In Fig. 1 in the Key Points, if $a = 60°$ and $b = 80°$, then the exterior angle c is 60 plus 80 equal to 140°.
- In a triangle ABC, if angle A is 50° and angle B is 70°, then the degree measure of angle C is 180 minus the sum of 50 and 70 equal to 180 minus 120 equal to 60.
- As shown in the Key Points, the side lengths of 45°-45°-90° triangle and 30°-60°-90° triangle are apparent if the length of one of the sides is given.
- As shown in the Key Points, if the ratio of two sides of a right triangle is a Pythagorean triple, then the length of the third side is apparent if you know the Pythagorean triples.

Section 24 – Quadrilaterals

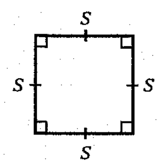
Key Points

Quadrilaterals

- A quadrilateral has 4 sides (edges) and 4 angles. Various quadrilaterals are given below.
- The sum of the interior angles of a quadrilateral is 360°. For example, in the quadrilateral below, the sum of angles a, b, c, and d is 360°.



Square, Rectangle, Trapezoid, and Parallelogram

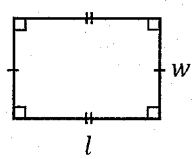


Square

All 4 sides are equal, and all angles are 90°.

- Perimeter $= 4s$.
- Area $= s^2$.
- Diagonal $= s\sqrt{2}$.

  s is the side.

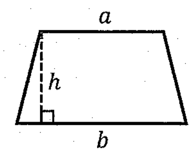


Rectangle

The opposite facing sides are equal. All angles are 90°.

- Perimeter $= 2(l + w)$.
- Area $= lw$.

  l is the length and w is the width.

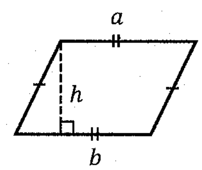


Trapezoid

One pair of opposite sides are parallel.

- Area$= \frac{1}{2}(a + b)h$.

  a and b are the two parallel bases and h is the height.



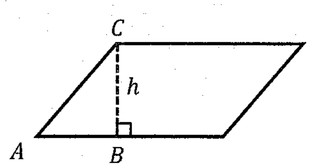
Parallelogram

Opposite sides are parallel and equal in length.

- Area $= bh$.

  b is the base and h is the height.

- In a trapezoid and a parallelogram, the height is the perpendicular distance between the parallel bases. If the height is not given, then it can most likely be determined using the ratio of Pythagorean triples or special right triangles. For example, in the parallelogram shown below, if the length of $\overline{AB}$ is 6 and $\overline{AC}$ is 10, then $BC = h = 8$ (ratio of Pythagorean triple).



Tips

Mental Math:

- If each side of a square is 9, then the area is 9 times 9 equal to 81. Similarly, if the area of a square is 144, then each side is square root of 144 equal to 12.
- The height of a trapezoid or a parallelogram may be apparent as described in the Key Points.

Section 25 – Three-Dimensional Figures

Key Points

Cube and Right Rectangular Prism

- A cube is bounded by 6 square faces (Fig. 1 below). For a cube of side s, see the following formulas.
  - Surface area $= 6s^2$.
  - Surface area of each square face $= s^2$.
  - Volume $= s^3$.
  - Diagonal $= s\sqrt{3}$.
- A right rectangular prism has 6 rectangular faces, for example, a rectangular box (Fig. 2). For a right rectangular prism with length l, width w, and height h, see the following formulas.
  - Surface area $= 2(lw + lh + wh)$.
  - Volume $= lwh$.

Sphere, Right Circular Cone, Right Pyramid, and Right Cylinder

- The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere (Fig. 3 below). Example of a sphere is a circular ball.
- The volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the cone (Fig. 4 below).
  - The volume can also be written as $\frac{1}{3} \times$ area of the circular base $(\pi r^2) \times h$.
- The volume of a right pyramid with a rectangular base is $\frac{1}{3}lwh$, where l is the base length, w is the base width, and h is the height of the pyramid (Fig. 5 below).
  - The volume can also be written as $\frac{1}{3} \times$ area of the rectangular base $(lw) \times h$.
  - If the base is a square, then l and w are same.
- The volume of a right cylinder is $\pi r^2 h$, where r is the radius and h is the height of the cylinder (Fig. 6 below).
  - The volume can also be written as area of the circular base $(\pi r^2) \times h$.

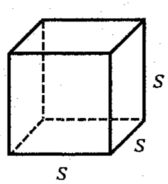


Fig. 1

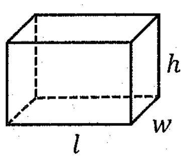


Fig. 2

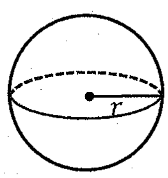


Fig. 3

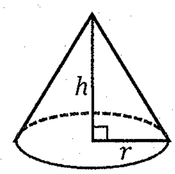


Fig. 4

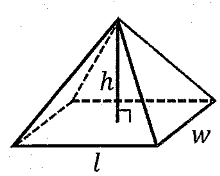


Fig. 5

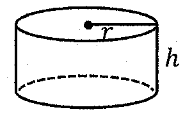


Fig. 6

- If the volume is given, the radius or the height can be determined, depending on the question. For example, if the volume of a right circular cone is 12π and the height is 1, then radius can be calculated as follows.

$$V = \frac{1}{3}\pi r^2 h \rightarrow 12\pi = \frac{1}{3}\pi r^2 h \rightarrow 12 = \frac{1}{3}r^2 \times 1 \rightarrow r^2 = 12 \times 3 \rightarrow r^2 = 36 = 6^2 \rightarrow r = \pm 6 = 6$$

Tips

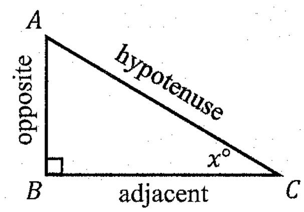
Mental Math:

- If each side of a cube is 5, then the volume is 5 times 3 equal to 125. Similarly, if the volume of a cube is 64, then each side is cube root of 64 equal to 4. For larger numbers, calculator will be required.

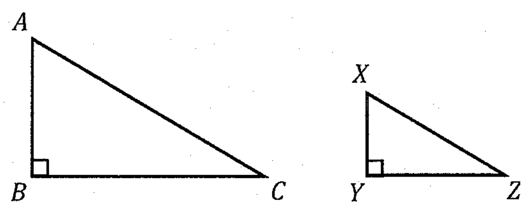
Section 26 – Trigonometry

Key Points

- The three main trigonometric functions are the sine (sin), the cosine (cos), and the tangent (tan). The definitions can be remembered as SOH-CAH-TOA.
- In the figure below, triangle ABC is a right triangle with angle B as the right angle. For the acute angle x, AB is the opposite side, BC is the adjacent side and AC is the hypotenuse.



  - SOH: sine equals opposite over hypotenuse. For angle x,
$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{AC}$$
  - CAH: cosine equals adjacent over hypotenuse. For angle x,
$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AC}$$
  - TOA: tangent equals opposite over adjacent. For angle x,
$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC}$$
- For two acute angles x and y in a right triangle, the following facts are helpful to remember.
  - $\sin x° = \cos y°$ and $\sin y° = \cos x°$ (sine of one acute angle equals the cosine of the other acute angle and vice versa).
  - $\sin x° = \cos(90° - x°)$ and $\sin y° = \cos(90° - y°)$ (sine of an angle equals to the cosine of its complement).
  - $\cos x° = \sin(90° - x°)$ and $\cos y° = \sin(90° - y°)$ (cosine of an angle equals to the sine of its complement).
  - tan of the two acute angles are inverse of each other. For example, if $\tan x = \sqrt{3}$, then $\tan y = \frac{1}{\sqrt{3}}$.
- In similar right triangles, the sine, the cosine, and the tangent at the corresponding vertices are the same. For example, in similar right triangles ABC and XYZ shown below, vertices A, B, and C correspond to vertices X, Y, and Z, respectively.



$\sin A = \sin X$, $\cos A = \cos X$, and $\tan A = \tan X$
$\sin C = \sin Z$, $\cos C = \cos Z$, and $\tan C = \tan Z$

- The side ratio of a right triangle may be the ratio of a Pythagorean triple, or the ratio of a 45°-45°-90° or 30°-60°-90° triangle. For example, in the triangle ABC above, $BC = 8$, and $AC = 10$. To determine $\sin C = \frac{AB}{AC}$, the value of AB is required. $BC : AC = 8 : 10$ is the ratio of the Pythagorean triple $6 : 8 : 10$. Hence, $AB : BC : AC = 6 : 8 : 10 \rightarrow AB = 6$.
$$\sin C = \frac{AB}{AC} = \frac{6}{10}$$

Tips

Mental Math:

- For two acute angles x and y of a right triangle, if $\sin x$ is given, then $\cos(90° - x°)$ and $\cos y$ will have the same value. Similarly, if $\cos x$ is given, then $\cos(90° - x°)$ and $\sin y$ will have the same value.
- In the similar triangles ABC and XYZ in the Key Points, if $AB = 6$, and $AC = 10$, then BC is 8. sin, cos, and tan of any acute angle can be determined using mental math.

Digital SAT Math Practice Tests

Instructions

Instructions

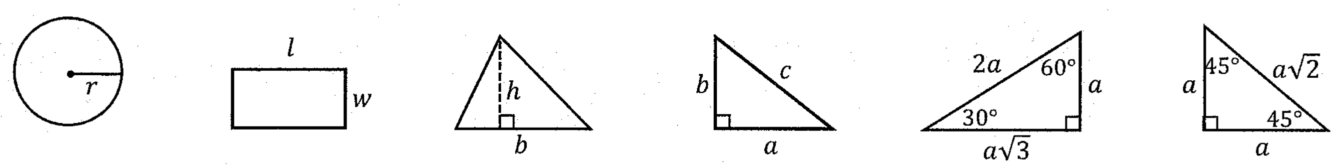
Use of calculator is permitted for all the questions.

For multiple-choice questions, solve each problem, choose the correct answer from the choices provided, and then circle your answer in this book. Circle only one answer for each question. If you change your mind, completely erase the circle.

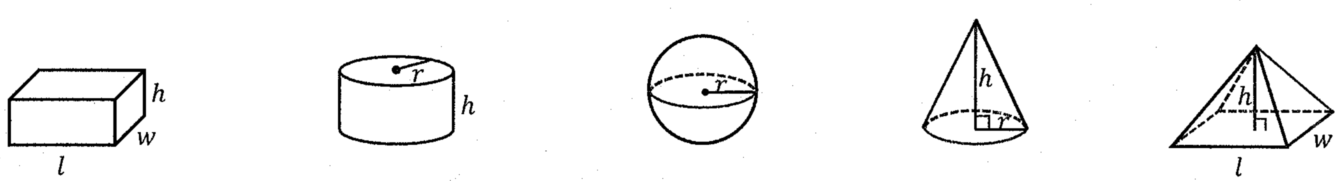
For student-produced response questions, solve each problem and write your answer next to or under the question in the book as described below.

- If you find more than one correct answer, write and circle only one answer.
- Your answer can be up to 5 characters for a positive answer and up to 6 characters (including the negative sign) for a negative answer, but no more.
- If your answer is a fraction that is too long (over 5 characters for positive, 6 characters for negative), write the decimal equivalent.
- If your answer is a decimal that is too long (over 5 characters for positive, 6 characters for negative), truncate it or round at the fourth digit. If your answer is a mixed number (such as $3\frac{1}{2}$), write it as an improper fraction $\left(\frac{7}{2}\right)$ or its decimal equivalent (3.5).
- Don't include symbols.

Reference



$A = \pi r^2$ $C = 2\pi r$ | $A = lw$ | $A = \frac{1}{2}bh$ | $c^2 = a^2 + b^2$ | Special Right Triangles



$V = lwh$ | $V = \pi r^2 h$ | $V = \frac{4}{3}\pi r^3$ | $V = \frac{1}{3}\pi r^2 h$ | $V = \frac{1}{3}lwh$

The number of degrees in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

Digital SAT Math Practice Test 1

Module 2.1

Test duration - 35 min
Total questions - 22

Time the test: If you have not completed all the questions in this module within 35 minutes, then leave the remaining questions and proceed to Module 2.2. You can complete the remaining questions after you have completed checking the answers.

1

In the equation $5x - 9 = 2x$, what is the value of x?

A) 3

B) 5

C) 9

D) 15

2

Which of the following expression is equivalent to $(3x^4 - x^2) + x(x + 2x^2)$

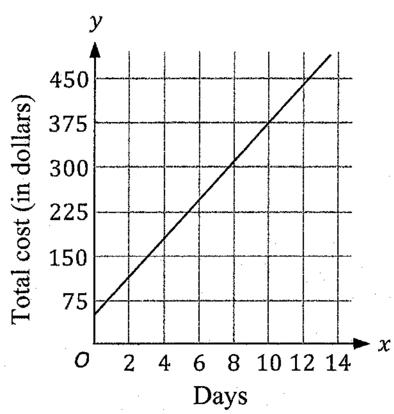
A) $3x^3 + 2x^3 + 2x^2$

B) $3x^4 + 2x^3 - x^2$

C) $3x^4 + x^3$

D) $3x^4 + 2x^3$

3



The line graph above shows the relationship between the total cost y, in dollars, for renting a certain type of a car from a certain car rental agency and the number of days the car is rented x. Based on the graph, what is the total cost of renting a car for 10 days?

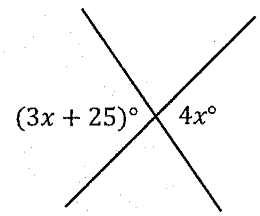
A) \$70

B) \$150

C) \$375

D) \$450

4



Note: Figure not drawn to scale

The figure above shows two intersecting lines. What is the value of $4x$?

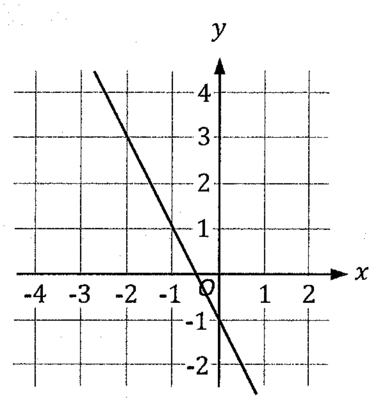
A) 25

B) 45

C) 60

D) 100

5



What is the equation of the graph shown?

A) $y = -0.5x - 1$

B) $y = -2x - 1$

C) $y = 2x - 1$

D) $y = 3x - 1$

6

$$n = 490 + 14m$$

The above equation determines the total number of books n in a library m months after the library was renovated. How many new books were added each month to the library for m months after renovation?

A) 5

B) 14

C) 35

D) 490

7

$$2x + 5y = \frac{45}{2z}$$

In the given equation, x, y, and z are positive numbers. Which equation expresses $2z$ in terms of x and y?

A) $2z = 2x + 5y - 45$

B) $2z = 2x + 5y + 45$

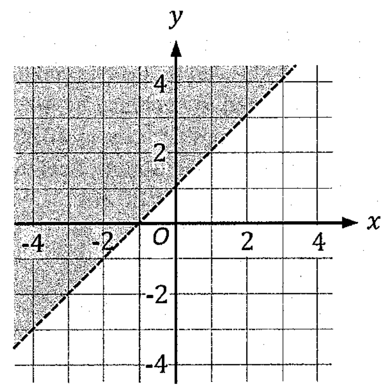
C) $2z = \frac{2x+5y}{45}$

D) $2z = \frac{45}{2x+5y}$

8

The expression $5(k + x) + 7k$ is equivalent to the expression $5x - 1$. What is the value of k, where k is a constant?

9



In the xy-plane, which of the following inequality models the shaded region in the above graph?

A) $y = 2$

B) $y < x + 1$

C) $y > x + 1$

D) $y > -x + 2$

10

What value of x satisfies the equation $\sqrt{7x - 3} = \sqrt{2x + 17}$, where $x > 0$?

A) 4

B) 7

C) 9

D) 14

11

A survey conducted by an independent research company in 2020 showed that the population of a certain city is expected to decrease by 4% every 3 years. If the population of the city in 2020 is 35,000, which of the following equation models the decrease in population, P, in t years after 2020?

A) $P = 35{,}000(0.96)^{\frac{t}{3}}$

B) $P = 35{,}000(0.96)^{t}$

C) $P = 35{,}000(0.96)^{3t}$

D) $P = 35{,}000(1.04)^{t}$

12

If $\sqrt{a} = b^{-\frac{1}{2}}$, what is b in terms of a, where a and b are positive real numbers?

A) $\frac{1}{a^2}$

B) $\frac{1}{a}$

C) $\sqrt{a}$

D) a

13

$$0.1x + 0.25y = 10$$
$$2x + 5y = 1$$

In the xy-plane, which of the following must be true about the lines graphed by the above system of equations?

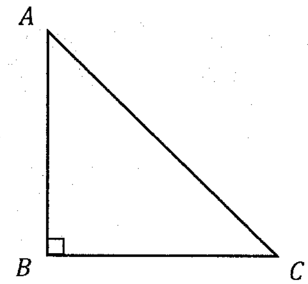
A) They are the same line.

B) They are parallel lines that have distinct y-intercepts.

C) They are perpendicular lines that intersect at one point.

D) The information given is inconclusive.

14



Note: Figure not drawn to scale

Triangle ABC given above is an isosceles right triangle with angle $B = 90°$. Based on the given information, the lengths of which of the following can be used to determine the length of $\overline{BC}$?

A) $\overline{AB}$ only.

B) $\overline{AC}$ only.

C) $\overline{AB}$ or $\overline{AC}$.

D) Neither $\overline{AB}$ nor $\overline{AC}$.

15

Year	Annual number of miles
2012	92,956
2013	117,560
2014	97,820
2015	114,844
2016	80,055
2017	80,550

The table above shows the annual number of miles a certain truck driver drove from 2012 to 2017. What is the median annual number of miles the truck driver drove from 2012 to 2017?

A) 37,010

B) 95,388

C) 97,820

D) 106,332