The Ultimate AP Calculus BC Cheat Sheet

Limits & Differentiation Rules (Units 1–3)

Limit Foundations

Limit: $\lim_{x\to a} f(x) = L$ iff for all $\varepsilon > 0$, $\exists\delta > 0$ s.t. $0 < |x - a| < \delta \to |f(x) - L| < \varepsilon$.
Continuity at $x = a$: $\lim_{x\to a} f(x) = f(a)$. Check: defn, left/right limits match & exist.
IVT: f cts. on [a,b] & k b/w f(a), f(b) $\Longrightarrow \exists c \in (a,b)$ where $f(c) = k$. Use on MC to prove zeros, critical pts.
Jump disc. (left ≠ right limit); **removable** (hole, not value); **infinite** (vert. asymptote).

Derivative Rules & Strategies

Power, sum, const. mult. (on sheet). **Product rule**: $(uv)' = u'v + uv'$. **Chain**: $(f\circ g)' = f'(g)\cdot g'$.
Quotient: $(u/v)' = (u'v - uv')/v^2$. **When to use**: quotient rule slower than rewriting as $u\cdot v^{-1}$.
Implicit diff.: diff. both sides wrt x, collect dy/dx terms, solve. **Use** when y not explicit or relation is implicit.
Inverse fn.: $(f^{-1})'(x) = 1 / f'(f^{-1}(x))$. **Check**: $f'(f^{-1}(a))$ in denominator.
Log diff.: ln both sides, diff., solve y′. Use on products/quotients of many terms.

Derivative	Rule
e^u	$u'e^u$
$\ln\lvert u\rvert$	u'/u
sin u	u′ cos u
cos u	−u′ sin u
tan u	$u' \sec^2 u$
a^u	$u' a^u \ln a$

Common Traps

Chain rule: (sin 2x)′ = 2 cos 2x, not cos 2x. Always include inner deriv.

Implicit: solve for dy/dx algebraically. Never leave dy/dx on both sides.

Applications of Differentiation (Units 4–5)

Optimization & Shape

MVT: f cts. on [a,b], diff. on (a,b) $\Longrightarrow \exists c$ where $f'(c) = [f(b) - f(a)] / (b - a)$. Use to justify crit. pts.
Crit. pt. test: $f'(x) = 0$ or undef. **1st deriv. test**: sign of f′ before/after → max/min. **2nd deriv. test**: $f''(c) < 0$ → max; > 0 → min.
Concavity: $f''(x) > 0$ → concave up; $f''(x) < 0$ → concave down. **Inflection pt.**: f″ changes sign.
Optimization: set up constraint, express obj. fn. in 1 var., find crit. pts., test endpoints & crit. pts.
Related rates: diff. constraint wrt time, substitute known rates, solve for unknown rate. Draw & label diagram.

Behavior & Asymptotes

Horiz. asymptote: check $\lim_{x\to\pm\infty} f(x)$. **Vert. asymptote**: $f' \to \pm\infty$ or f undef. while near pt.
End behavior: ratio of leading coeffs (rational); dom. term (poly/exp). **Increasing**: $f' > 0$; **decreasing**: $f' < 0$.
Graph interpretation: f′ tells slope, f″ tells concavity. On FRQ, always state f′ = 0 & verify with 2nd test.

Analysis	Check
Max/min	f′ = 0 or undef.
Inflection	f″ = 0 or undef.
Concav. up	f″ > 0
Concav. down	f″ < 0
Vert. asym.	denom = 0, num ≠ 0

FRQ Strategy

Always interpret f′, f″ in context: "rate of change of...", "accel of...".

Optimization: show f′ = 0, verify with 2nd deriv. test, state answer w/ units.

Related rates: sketch → label vars → diff. wrt time → substitute → solve.

Integration Techniques (Unit 6)

Core Antiderivatives & Rules

Power: $\int x^n\,dx = x^{n+1}/(n+1) + C$ ($n \neq -1$). **Exp**: $\int e^x\,dx = e^x + C$. **1/x**: $\int 1/x\,dx = \ln|x| + C$.
Trig: $\int \sin x\,dx = -\cos x + C$; $\int \cos x\,dx = \sin x + C$; $\int \sec^2 x\,dx = \tan x + C$.
u-sub: choose u = inner fn., du = u′ dx, rewrite integral in u, integrate, back-sub. **Check**: u′ divides integrand.

Integration by Parts & Fractions

By parts: $\int u\,dv = uv - \int v\,du$. **LIATE**: ln, inv. trig, alg., trig, exp. (pick u by priority). Useful for uv, $x\cdot e^x$, $x\cdot \ln x$.
Partial fractions: decompose rational fn. into sum of simpler fractions. **Steps**: factor denom., set up form, solve for coeff., integrate each term.
Improper integrals: $\int_a^\infty = \lim_{t\to\infty} \int_a^t$. Converges if limit is finite. Also handle at x = c where f(c) undef.

Accumulation & Area

FTC I: $d/dx \int_a^x f(t)\,dt = f(x)$. **FTC II**: $\int_a^b f(x)\,dx = F(b) - F(a)$ ($F' = f$).
Net area: $\int_a^b f(x)\,dx$ = area above − area below x-axis. **Total area**: $\int_a^b |f(x)|\,dx$.
Riemann sums: Left/right/mid approximate ∫ with rectangles. **Trap. rule**: $(b-a)/n \cdot [f(x_0)/2 + f(x_1) + \dots + f(x_n)/2]$.
Acc. fns: $F(x) = \int_a^x f(t)\,dt$. $F'(x) = f(x)$; $F''(x) = f'(x)$. Use to define & evaluate antiderivatives.

u-Sub Type	Example
Power inside	$\int (3x+1)^5\cdot 3dx$
Exp/ln	$\int e^{2x}dx$
Trig in chain	$\int \sin(x^2)\cdot 2x\,dx$

+C on indefinite. Improper: watch for discontinuities in [a,b].

u-sub: if u′ doesn't divide integrand cleanly, wrong u choice.

Differential Equations & Integration Apps (Units 7–8)

Separable DEs & Slope Fields

Separable: $dy/dx = f(x)g(y)$. Separate: $g(y)\,dy = f(x)\,dx$. Integrate both sides, solve for y.
Slope field: plot (x,y) & direction dy/dx. Solutions follow field. **Use**: match DE to field by checking slopes.
Euler's method: $y_{n+1} = y_n + \Delta y$; $\Delta y = f(x_n, y_n)\cdot h$ (h = step size). Approximate solution via iterations.

Growth Models

Exponential: $dy/dt = ky$. Solution: $y(t) = y_0 e^{kt}$. **Half-life**: $t = (\ln 2)/k$. **Double time**: $t = (\ln 2)/k$.
Logistic: $dy/dt = ky(L-y)/L$ (L = carrying capacity). Solution: $y(t) = L / (1 + Ae^{-kt})$. Approaches L asymptotically.
Linear DE: $dy/dt + P(t)y = Q(t)$. Integrating factor: $e^{\int P(t)\,dt}$. Multiply by integrating factor, integrate.

Volumes & Solids

Disk method: $V = \int_a^b \pi[R(x)]^2\,dx$. **Washer**: $V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)\,dx$.
Shell method: $V = 2\pi \int_a^b$ (radius)(height) $dx = 2\pi \int x\cdot f(x)\,dx$ (rotate around y-axis).
Arc length: $L = \int_a^b \sqrt{1 + (dy/dx)^2}\,dx$.
Surface area: $S = 2\pi \int_a^b y\sqrt{1 + (dy/dx)^2}\,dx$.
Work: $W = \int F(x)\,dx$.
Avg. value: $f_{avg} = 1/(b-a)\cdot \int_a^b f(x)\,dx$.
Mean Value Thm for Integrals: $\exists c$ where $f(c) = f_{avg}$.

Method	Formula
Disk	$\pi\int R^2dx$
Washer	$\pi\int (R^2-r^2)dx$
Shell	$2\pi\int xf(x)dx$
Arc len	$\int \sqrt{1+f'^2}dx$

Always sketch region, label R(x)/r(x), set up bounds. Rotate → disk; along side → shell.

Arc len hard to integrate; often leave in integral form or use approx/calc.

Parametric Equations & Vector Functions (Unit 9 Part 1)

Parametric Curves

Parametric: $x = f(t)$, $y = g(t)$ trace curve as t varies. **dy/dx** = (dy/dt) / (dx/dt). **d^2y/dx^2** = d/dx(dy/dx).
Eliminate parameter: solve one eq. for t, substitute into other. Use if curve has simple form (ellipse, line).
Velocity: = (dx/dt, dy/dt). **Speed**: $\|\| = \sqrt{(dx/dt)^2 + (dy/dt)^2}$.
Acceleration: $= (d^2x/dt^2, d^2y/dt^2)$.
Arc length (param): $L = \int_{t_1}^{t_2} \sqrt{(dx/dt)^2 + (dy/dt)^2}\,dt$.
Area under curve: $A = \int_{t_1}^{t_2} y(t)\cdot x'(t)\,dt$.

Vector-Valued Functions

Vector fn.: r(t) = . **Derivative**: r′(t) = . **Integral**: $\int r(t)\,dt$ = <∫f(t) dt, ∫g(t) dt>.
Magnitude: $|r(t)| = \sqrt{f(t)^2 + g(t)^2}$. **Dot product**: $u\cdot v = u_1v_1 + u_2v_2$. **Orthogonal**: $u\cdot v = 0$.
Unit tangent: $T(t) = r'(t) / |r'(t)|$. **Normal accel.**: $a_N = |a\cdot N|$. **Tangent accel.**: $a_T = (v\cdot a) / |v|$.
Projectile motion: $x(t) = x_0 + v_0 \cos\theta\cdot t$; $y(t) = y_0 + v_0 \sin\theta\cdot t - (1/2)gt^2$. Solve y = 0 for landing time.
Position fn.: r(t) satisfies r′(t) = velocity, r″(t) = acceleration. Initial conditions: r(0), r′(0).

Concept	Formula
Speed	$\sqrt{(dx/dt)^2+(dy/dt)^2}$
Arc len	$\int\sqrt{(dx/dt)^2+(dy/dt)^2}dt$
Unit tangent	$r'(t)/\lvert r'(t)\rvert$
Dot prod	$u_1v_1+u_2v_2$

2nd deriv. of param: $d^2y/dx^2 = d/dt(dy/dx) / (dx/dt)$.
Concavity: >0 → concave up; <0 → concave down.

Param arc len easy to set up, hard to integrate. On MC: use calc or approx.

Speed vs. velocity: speed is magnitude (scalar); velocity is vector w/ direction.

Polar Coordinates & Curves (Unit 9 Part 2)

Polar to Cartesian

Polar form: (r, θ) where $x = r\cos\theta$, $y = r\sin\theta$. **Inverse**: $r = \sqrt{x^2 + y^2}$, $\theta = \arctan(y/x)$ [adjust quadrant].
Polar curve: $r = f(\theta)$. Trace as θ varies. **dy/dx** = $(r'\sin\theta + r\cos\theta) / (r'\cos\theta - r\sin\theta)$ where $r' = dr/d\theta$.
Tangent line at θ: use formula above or convert to parametric $x = r\cos\theta$, $y = r\sin\theta$, find dy/dx.
Horizontal tangent: $dy/d\theta = 0$ (not dy/dx). **Vertical tangent**: $dx/d\theta = 0$. **Tangent passes origin**: r = 0.

Area & Arc Length

Polar area: $A = (1/2) \int_\alpha^\beta r^2\,d\theta$. **Setup**: sketch region, identify θ bounds, set up integral.
Area b/w curves: $A = (1/2) \int_\alpha^\beta (r_1^2 - r_2^2)\,d\theta$. Assume $r_1 > r_2$ in region.
Polar arc length: $L = \int_\alpha^\beta \sqrt{r^2 + (dr/d\theta)^2}\,d\theta$. **Use**: only if integrable; else approx. or leave answer.

Common Polar Curves

Circle: r = a (center origin); **spiral**: r = aθ; **rose**: r = a sin(nθ) or a cos(nθ).
Limaçon: r = a ± b cos θ; **cardioid**: r = a(1 ± cos θ) [loop if inner]; **lemniscate**: $r^2 = a^2 \cos 2\theta$.

Curve	Petals/Shape
r=a sin nθ	n petals (n odd)
r=a cos nθ	n petals (n even)
r=a(1+cosθ)	cardioid
$r^2=a^2\cos 2\theta$	lemniscate
r=aθ	spiral

Polar area has (1/2) factor! $A = (1/2)\int r^2\,d\theta$, not $\int r^2\,d\theta$.

Check: does r = 0 in [α, β]? If yes, region includes origin. Sketch!

Sequences & Series Fundamentals (Unit 10 Pt. 1)

Sequences & Limits

Sequence: ordered list $\{a_n\}$. **Limit**: $\lim_{n\to\infty} a_n = L$ iff given ε > 0, ∃N where $|a_n - L| < \varepsilon$ for n > N.
Convergent: limit exists (finite). **Divergent**: limit = ±∞ or oscillates. **Monotonic**: ↑ or ↓; **bounded**: $|a_n| \le M$.
Monotonic + bounded $\Longrightarrow$ convergent. **Find limit**: evaluate $\lim_{n\to\infty}$ (use L-Hôpital on indeterminate forms).

Infinite Series Basics

Series: $\Sigma a_n = a_1 + a_2 + \dots$ **Partial sum**: $S_N = \Sigma_{n=1}^N a_n$. **Series conv.** iff $\{S_N\}$ converges.
Geometric: $\Sigma ar^{n-1} = a/(1-r)$ if $|r| < 1$.
p-series: $\Sigma 1/n^p$ conv. iff p > 1.
Divergence test: if $\lim a_n \neq 0$, series diverges. If $\lim a_n = 0$, test is inconclusive (need convergence test).
Telescoping: $(b_n - b_{n+1})$. Partial sums collapse: $S_N = b_1 - b_{N+1}$. Take lim as N → ∞.

Series Convergence Tests

Integral test: $\Sigma a_n \equiv \int f(x)\,dx$ (both conv. or both div.). f must be pos., cont., decr.
Comparison: if $0 \le a_n \le b_n$ & Σb_n conv., then Σa_n conv.
Limit comparison: $\lim (a_n/b_n) > 0$ & finite.
Ratio test: $\lim |a_{n+1}/a_n| = L$. L < 1 → conv.; L > 1 → div.; L = 1 → inconclusive.
Root test: $\lim |a_n|^{1/n} = L$. L < 1 → conv.; L > 1 → div.; L = 1 → inconclusive.
Alternating series: $\Sigma (-1)^n a_n$ conv. if $a_n \downarrow 0$ & $\lim a_n = 0$ (AST). **Absolute**: $\Sigma |a_n|$ conv. $\Longrightarrow \Sigma a_n$ conv.

Test	Condition
p-series	conv. iff p>1
Integral	∫converges
Ratio	L<1→conv
Root	L<1→conv
Alt. series	$a_n\downarrow$ to 0

Always check divergence test FIRST (easiest

Convergence Tests & Power Series + FRQ (Unit 10 Part 2)

Power Series & Taylor

Power series: $\Sigma c_n(x - a)^n$. **Radius of conv.** R from ratio test: $|x - a| < R$. **Interval**: (a − R, a + R); test endpoints.
Taylor at x=a: $f(x) = \Sigma f^{(n)}(a)/n! \cdot (x - a)^n$. **Maclaurin (a=0)**: $f(x) = \Sigma f^{(n)}(0)/n! \cdot x^n$.
Key series: $e^x = \Sigma x^n/n!$; $\sin x = \Sigma (-1)^n x^{2n+1}/(2n+1)!$; $\cos x = \Sigma (-1)^n x^{2n}/(2n)!$.
Geometric: $1/(1-x) = \Sigma x^n$ ($|x| < 1$); **ln(1+x)** $= \Sigma (-1)^{n+1} x^n/n$ ($-1 < x \le 1$).

Error Bounds & Lagrange

Lagrange error: $|R_n(x)| \le M/(n+1)! \cdot |x - a|^{n+1}$ where $|f^{(n+1)}(c)| \le M$ on interval.
Alternating series error: |error| ≤ |first omitted term|. **Use** to bound approximation error or find n.

Operations on Series

Diff./integrate power series term-by-term (within radius). Derive sin x from cos x via integration.
Substitution: replace x in known series. E.g. e^{2x} from e^x: $\Sigma (2x)^n/n! = \Sigma 2^n x^n/n!$.
Mult./div. series (rare on BC): multiply out, collect like terms.

FRQ Series & Calc Strategies

Series Q: name test, state conditions, show work (lim, ratio, integral). State convergence clearly.

Radius/interval: use ratio test, find R, test endpoints separately (AST? p-series?).

Power series: recognize fn., write series from memory or derive, substitute, integrate/diff. if needed.

Taylor error: bound with Lagrange; alt. series: omitted term. Never just guess magnitude.

Calc: know when allowed (Part B MC, Part A FRQ). Use to verify answers, solve transcendental eqs.