CALCULUS BC

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. $\int_1^2 \left(4x^3 - x\right) dx =$

(A) $\frac{27}{2}$ (B) 27 (C) 36 (D) 57

$[x^4 - \frac{1}{2}x^2]_1^2$

$16 - 2 - \left(1 - \frac{1}{2}\right)$

$14 - \frac{1}{2}$

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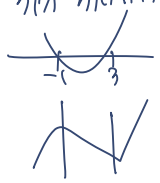
2. Let f be the function defined by $f(x) = x^3 - 3x^2 - 9x + 11$. At which of the following values of x does f attain a local minimum?

(A) 3 (B) 1 (C) -1 (D) -3

$f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$



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3. $\dfrac{d}{dx}\left(2(\sin\sqrt{x})^2\right) =$

(A) $4\cos\left(\dfrac{1}{2\sqrt{x}}\right)$ (B) $4\sin\sqrt{x}\cos\sqrt{x}$ (C) $\dfrac{2\sin\sqrt{x}}{\sqrt{x}}$ (D) $\dfrac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

$4\sin\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$

$\frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

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4. The position of a particle is given by the parametric equations $x(t) = \ln\left(t^2+1\right)$ and $y(t) = e^{3-t}$. What is the velocity vector at time $t = 1$?

(A) $\langle 1, e^2\rangle$ (B) $\langle 1, -e^2\rangle$ (C) $\left\langle \frac{1}{2}, e^2\right\rangle$ (D) $\left\langle \frac{1}{2}, -e^2\right\rangle$

$x'(t) = \frac{2t}{t^2+1}$ $y'(t) = -e^{3-t}$

$\frac{2}{2}$ $-e^2$

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5. $\sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$ is

(A) $\frac{\pi}{\pi - e}$ (B) $\frac{e}{\pi - e}$ (C) $\frac{e}{\pi \ln\left(\frac{\pi}{e}\right)}$ (D) divergent

$\left(\frac{e}{\pi}\right)^n$ $\frac{a}{1-r}$ $\frac{\frac{e}{\pi}}{1-\frac{e}{\pi}} = \frac{\frac{e}{\pi}}{\frac{\pi - e}{\pi}} = \frac{e}{\pi - e}$

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x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
2	4	-3	3	-2	5	1

6. The table above gives values of the twice-differentiable functions f and g and their derivatives at $x = 2$. If h is the function defined by $h(x) = \frac{f'(x)}{g(x)}$, what is the value of $h'(2)$?

(A) $\frac{9}{4}$ (B) $\frac{3}{5}$ (C) $-\frac{3}{2}$ (D) $-\frac{21}{4}$

$h'(x) = \frac{f''(x)g(x) - f'(x)g'(x)}{(g(x))^2}$

$= \frac{f''(2)g(2) - f'(2)g'(2)}{(g(2))^2}$

$= \frac{3 \cdot (-2) - (-3)(5)}{4}$

$= \frac{-6+15}{4}$

$= \frac{9}{4}$

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7. Which of the following is the Maclaurin series for $x\cos\left(x^2\right)$?

(A) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \frac{x^{13}}{6!} + \cdots$

(B) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(C) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \frac{x^{15}}{7!} + \cdots$

(D) $x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{7!} + \cdots$

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8. $\lim_{x\to\infty} \frac{10 - 6x^2}{5 + 3e^x}$ is

(A) -2 (B) 0 (C) 2 (D) nonexistent

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9. The function f is not differentiable at $x = 5$. Which of the following statements must be true?

(A) f is not continuous at $x = 5$.

(B) $\lim_{x \to 5} f(x)$ does not exist.

(C) $\lim_{x \to 5} \dfrac{f(x) - f(5)}{x - 5}$ does not exist.

(D) $\int_0^5 f(x)\,dx$ does not exist.

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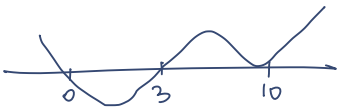
10. The second derivative of a function f is given by $f''(x) = x(x - 3)^5(x - 10)^2$. At which of the following values of x does the graph of f have a point of inflection?

(A) 3 only

(B) 0 and 3 only

(C) 3 and 10 only

(D) 0, 3, and 10



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11. $\displaystyle\int_0^{\pi} \frac{e^x - 1}{e^x - x}\,dx =$

(A) $e^{\pi} - \pi - 1$ (B) $\ln(e^{\pi} - \pi) - 1$ (C) $\pi - \ln \pi$ (D) $\ln(e^{\pi} - \pi)$

$e^x - x = t \quad (e^x - 1)\,dx = dt.$

$\displaystyle\int_1^{e^{\pi}-\pi} \left(\frac{1}{t}\right) dt = [\ln t]_1^{e^{\pi}-\pi}$

$\ln(e^{\pi} - \pi) =$

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x	0	4	8	12	16
$f(x)$	8	0	2	10	1

12. The table above gives selected values for the differentiable function f. In which of the following intervals must there be a number c such that $f'(c) = 2$?

(A) $(0, 4)$ (B) $(4, 8)$ (C) $(8, 12)$ (D) $(12, 16)$

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13. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + 2y$ with initial condition $f(0) = 2$. What is the approximation for $f(-0.4)$ obtained by using Euler's method with two steps of equal length starting at $x = 0$?

(A) 0.76 (B) 1.20 (C) 1.29 (D) 3.96

$\frac{dy}{dx} = x+2y$. $(0,2)$ 기울기

$y = 4(x-0)+2$

$= 4x+2$.

2.2. $(-0.2, 1.2)$

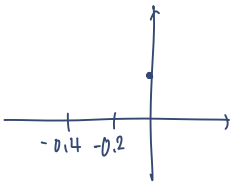
$y = 2.2(x+0.2)+1.2$

$= 2.2x + 0.44 + 1.2$

$= 2.2x + 1.64$

$= 2.2(-0.4) + 1.64$

$= -0.88 + 1.64 = 0.76$



$y = -0.8x + 2$

$(-0.2, 1.2)$

$(-0.2) + (2.4)$

$= 2.2$

14. What is the slope of the line tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $\left(\frac{9}{4}, \frac{1}{4}\right)$?

(A) -3 (B) $-\frac{1}{3}$ (C) 1 (D) $\frac{4}{3}$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y' = 0$

$\frac{1}{\sqrt{y}} y' = -\frac{1}{\sqrt{x}}$

$y' = -\frac{\sqrt{y}}{\sqrt{x}}$ $\quad y' = -\frac{\frac{1}{2}}{\frac{3}{2}} = -3$

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15. $\displaystyle\int_1^{\infty} \frac{6}{(x+3)^{3/2}}\,dx$ is

(A) $\frac{3}{4}$ (B) 3 (C) 6 (D) divergent

$\int_1^{\infty} 6\cdot(x+3)^{-\frac{3}{2}}\,dx$

$\left[-12(x+3)^{-\frac{1}{2}}\right]_1^{\infty}$

$3x^2 \to x^3$

$\frac{1}{3}\cdot 3\cdot x^3$

$\left[\frac{-12}{\sqrt{x+3}}\right]_1^{\infty} = 0-\left(\frac{-12}{\sqrt{4}}\right)$

$= \frac{12}{2} = 6$

---

16. If $\frac{dy}{dx} = 2 - y$, and if $y = 1$ when $x = 1$, then $y =$

(A) $2 - e^{x-1}$ (B) $2 - e^{1-x}$ (C) $2 - e^{-x}$ (D) $2 + e^{-x}$

(1,1)

$y' = 2-y$

$y = 2x - \frac{1}{2}y^2 + C$

$1 = 2 - \frac{1}{2} + C$

$1 = \frac{3}{2} + C$

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17. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{1-n}{n}\right)$

(B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+1}{2n}\right)$

(C) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n^2}{3\sqrt{n}}\right)$

(D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\sqrt{n}}{n}\right)$

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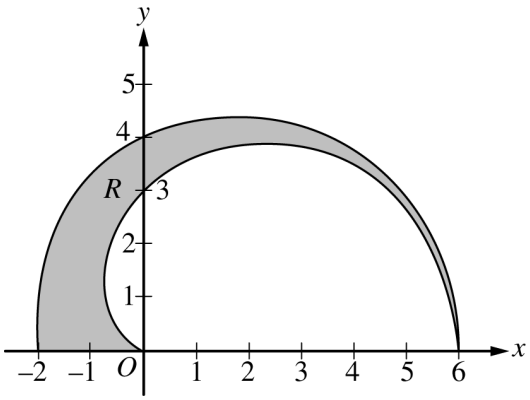

18. Let R be the region in the first and second quadrants between the graphs of the polar curves $f(\theta) = 3 + 3\cos\theta$ and $g(\theta) = 4 + 2\cos\theta$, as shaded in the figure above. Which of the following integral expressions gives the area of R ?

(A) $\int_{-2}^{6} (g(\theta) - f(\theta))\, d\theta$

(B) $\int_{0}^{\pi} (g(\theta) - f(\theta))\, d\theta$

(C) $\frac{1}{2}\int_{0}^{\pi} (g(\theta) - f(\theta))^2\, d\theta$

(D) $\frac{1}{2}\int_{0}^{\pi} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$

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19. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is true?

(A) The series can be shown to diverge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series can be shown to diverge by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series can be shown to converge by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(D) The series can be shown to converge by the alternating series test.

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20. If $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$, which of the following cannot be determined from the information given?

(A) $\int_4^1 g(x)\,dx$

(B) $\int_1^4 3f(x)\,dx$

(C) $\int_1^4 3f(x)g(x)\,dx$

(D) $\int_1^4 (3f(x) + g(x))\,dx$

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21. Which of the following is the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 5^{n+1}}$?

(A) $[-9, 1)$ (B) $[-5, 5)$ (C) $[1, 9)$ (D) $(-\infty, \infty)$

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22. What is the slope of the line tangent to the polar curve $r = 3\theta$ at the point where $\theta = \frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) 0 (D) 3

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23. The definite integral $\int_0^4 \sqrt{x}\ dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 4 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $L < \int_0^4 \sqrt{x}\ dx < T < R$

(B) $L < T < \int_0^4 \sqrt{x}\ dx < R$

(C) $R < \int_0^4 \sqrt{x}\ dx < T < L$

(D) $R < T < \int_0^4 \sqrt{x}\ dx < L$

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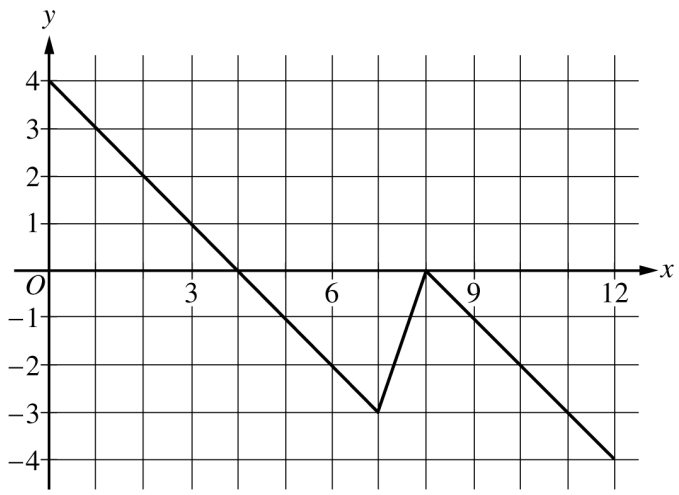

Graph of f

$-4-(4)$

24. The graph of the piecewise linear function f is shown above. What is the value of $\int_0^{12} f'(x)\,dx$?

(A) -8 (B) -6 (C) 0 (D) 22

$f(12)-f(0)$

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25. The function f has a continuous derivative. If $f(0) = 1$, $f(2) = 5$, and $\int_0^2 f(x)\,dx = 7$, what is $\int_0^2 x \cdot f'(x)\,dx$?

(A) 3 (B) 6 (C) 10 (D) 17

$\int_0^2 x f'(x)\,dx$

$[x f(x)]_0^2 - \int_0^2 f(x)\,dx$

$2f(2) - 7$

---

26. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$

(A) I only (B) II only (C) II and III only (D) I, II, and III

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27. Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{x-1}$, the x-axis, and the vertical line $x = 10$. Which of the following integrals gives the volume of the solid generated by revolving R about the y-axis?

(A) $\pi \int_1^{10} (x-1)\, dx$

(B) $\pi \int_1^{10} (100 - (x-1))\, dx$

(C) $\pi \int_0^3 \left(10 - \left(y^2+1\right)\right)^2 dy$

(D) $\pi \int_0^3 \left(100 - \left(y^2+1\right)^2\right) dy$

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28. If $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = \sin\left(t^2\right)$, then $\frac{d^2y}{dx^2}$ is

(A) $2t\cos\left(t^2\right)$ (B) $\frac{2t\cos\left(t^2\right)}{5}$ (C) $\frac{2t\cos\left(t^2\right)}{25}$ (D) undefined

$\frac{dy}{dx} = \frac{1}{5}(\sin(t^2))$

$\frac{1}{5}(\cos(t^2)\cdot 2t$

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29. Which of the following expressions is equal to $\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{2+\frac{1}{n}}+\frac{1}{2+\frac{2}{n}}+\frac{1}{2+\frac{3}{n}}+\cdots+\frac{1}{2+\frac{n}{n}}\right)$?

(A) $\int_1^2 \frac{1}{x}\,dx$

(B) $\int_0^1 \frac{1}{2+x}\,dx$

(C) $\int_0^2 \frac{1}{2+x}\,dx$

(D) $\int_2^3 \frac{1}{2+x}\,dx$

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30. A function f has a Maclaurin series given by $2 + 3x + x^2 + \frac{1}{3}x^3 + \cdots$, and the Maclaurin series converges to $f(x)$ for all real numbers x. If g is the function defined by $g(x) = e^{f(x)}$, what is the coefficient of x^2 in the Maclaurin series for g ?

(A) $\frac{1}{2}e^2$ (B) e^2 (C) $\frac{5}{2}e^2$ (D) $\frac{11}{2}e^2$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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