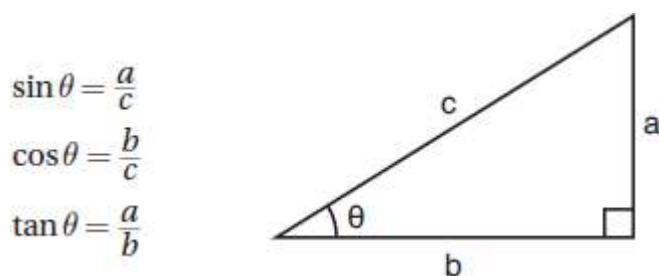


The popular acronym SOH-CAH-TOA will help you remember which sides to use when calculating *sine*, *cosine*, and *tangent*.

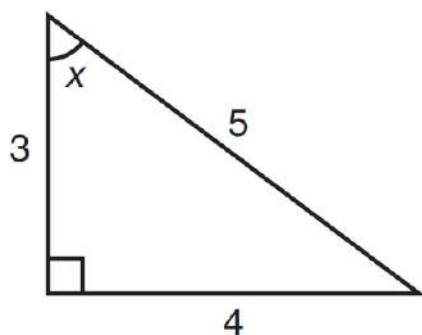
S-O-H	C-A-H	T-O-A
$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Let's take a look at an example to see what the different trigonometry values would be.



► Example

What is the tangent for angle x in the right triangle below?



✓ Solution

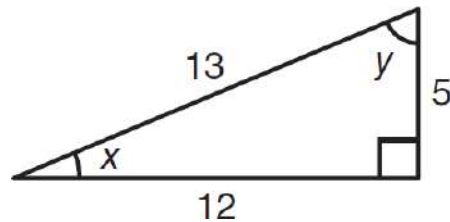
To find the tangent, we need the opposite side and the adjacent side. The side of length 4 is opposite $\angle x$, and the side of length 3 is adjacent to $\angle x$. So, calculate the tangent of $\angle x$ as follows:

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} \rightarrow$$

$$\tan x = \frac{4}{3}$$

Practice

Consider the triangle below:



What is $\sin x$?

What is $\cos y$?

What is $\tan y$?

In a 45-45-90 triangle, which values will be identical?

I. $\cos 45$

II. $\sin 45$

III. $\tan 45$

Solutions

$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\cos y = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\tan y = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{12}{5}$$

In a 45-45-90 triangle, the sides have a ratio of $s:s:s\sqrt{2}$. I and II are equal since $\cos(45)$ and $\sin(45)$ are both $\frac{1}{\sqrt{2}}$. III is not equal to the others since $\tan(45)$ is just 1.

Area of an Equilateral Triangle

While it is manageable to solve for the area of an equilateral triangle without knowing this formula, memorizing it will save you time should you come across a problem that asks about it.

The area, A , of an equilateral triangle with sides of length a is:

$$A = \frac{\sqrt{3}}{4} a^2$$

► Example

What is the area of an equilateral triangle with sides that are 4 inches long?

✓ Solution

Plug 4 in for a into the equilateral triangle area formula:

$$\begin{aligned} A &= \frac{\sqrt{3}}{4} a^2 \rightarrow \\ A &= \frac{\sqrt{3}}{4} (4)^2 = \frac{\sqrt{3}}{4} (16) = 4\sqrt{3} \end{aligned}$$

So, the area of the triangle would be $4\sqrt{3}$ square inches.

Sine and Cosine of Complementary Angles

Complementary angles add up to 90° . The relationship of the sine and cosine of complementary angles is:

$$\sin(x) = \cos(90^\circ - x)$$

► Example

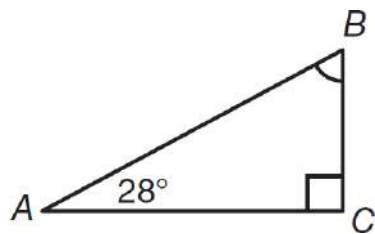
If the sine of 30° is 0.5, what is the cosine of 60° ?

✓ Solution

While you could use a calculator for a problem like this on the calculator-permitted test section, if this were on the non-calculator test section you would have to know an alternative way to approach it. Use the relationship between the sine and cosine of complementary angles to solve. Since $30 + 60 = 90$, 30° and 60° are complementary. Because $\sin(x) = \cos(90^\circ - x)$, $\sin(30) = \cos(60)$, and the cosine of 60° will also equal 0.5.

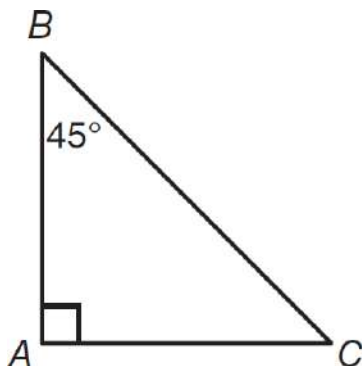
Practice Drill—Right Triangles and Trigonometry

(If timing, take about 12 minutes to complete.)



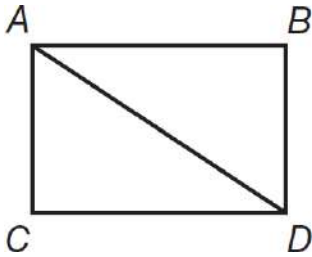
Triangle ABC is portrayed above. If triangle DEF is similar to triangle ABC , and angle B corresponds to angle E , what is the measure of angle E ?

- a) 28°
- b) 52°
- c) 62°
- d) 84°



In $\triangle ABC$, $\angle A$ is 90° and $\angle B$ is 45° . If $\overline{BC}$ has a length of 4 units, what is the length of $\overline{AC}$?

- a) 2
- b) $2\sqrt{2}$
- c) $2\sqrt{3}$
- d) 4



rectangle $ABCD$ has a side length of 7 for AC and a side length of 24 for D , what is the length of the diagonal AD ?

- a) 14
- b) 25
- c) 28
- d) 35

or an equilateral triangle with a side length of 8 units, what is the triangle's area in square units?

- a) $8\sqrt{2}$
- b) $8\sqrt{3}$
- c) $16\sqrt{2}$
- d) $16\sqrt{3}$

triangle DEF is similar to triangle ABC , with vertex D corresponding to vertex A , vertex E corresponding to vertex B , and vertex F corresponding to vertex C . The measure of $\angle C$ is 90° and the measure of $\angle B$ is 30° . What is the sine of $\angle D$?

- a) $\frac{1}{3}$
- b) $\frac{1}{2}$

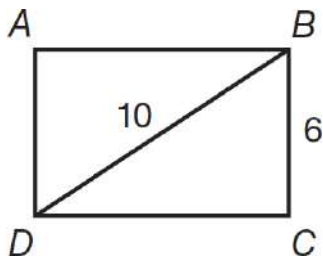
- c) $\frac{2}{3}$
- d) $\frac{3}{4}$

What is the difference between $\sin(x)$ and $\cos(90^\circ - x)$ if x is an angle between 0 and 30 degrees?

- a) 0
- b) $\frac{1}{2}$
- c) $\frac{3}{5}$
- d) $\frac{3}{4}$

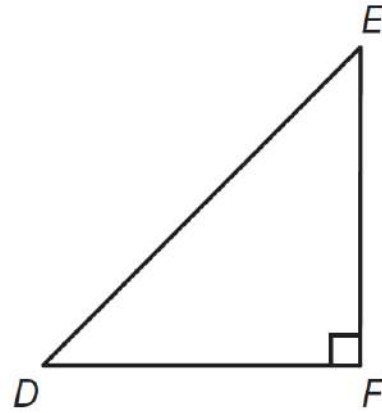
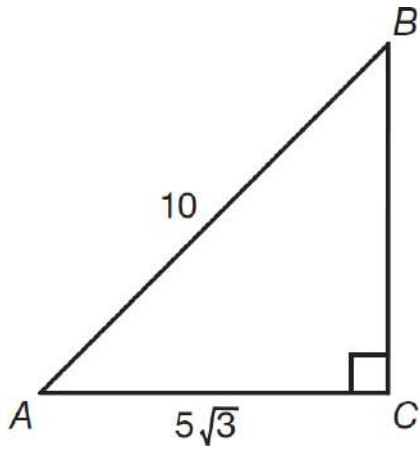
What is the sine of the smallest angle in a right triangle with sides 7 , 24 , and 25 ?

- A) $\frac{7}{25}$
- B) $\frac{7}{24}$
- C) $\frac{24}{7}$
- D) $\frac{25}{7}$



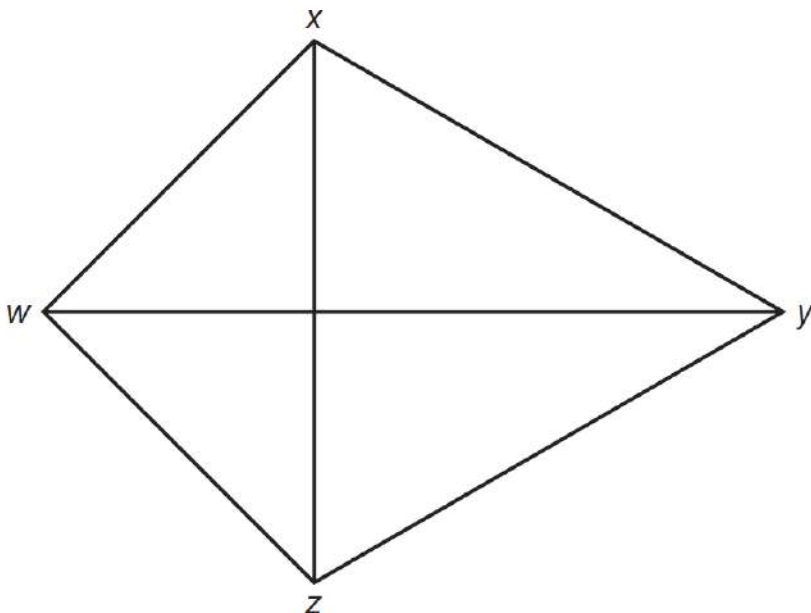
In rectangle $ABCD$ above, diagonal $\overline{DB}$ is 10 units long, and side $\overline{BC}$ is 6 units long. What is the area of the rectangle $ABCD$?

- a) 30 square units
- b) 36 square units
- c) 48 square units
- d) 60 square units



Right triangles ABC and DEF are similar to each other. What is the tangent of $\angle EDF$?

- a) $\frac{1}{\sqrt{3}}$
- b) $\frac{\sqrt{2}}{3}$
- c) $\frac{1}{2}$
- d) 2



In the figure above, $\overline{XZ}$ and $\overline{WY}$ are perpendicular, and $\overline{WY}$ bisects $\overline{XZ}$. $\overline{XY}$ and $\overline{YZ}$ are of equal length, $\overline{WX}$ and $\overline{WZ}$ are of equal length, and $\angle XWY = 45^\circ$. If $\overline{XZ}$ is 4 and WX is $2\sqrt{2}$, what is the length of WY ?

- a) $2\sqrt{2}$
- b) $2\sqrt{3}$
- c) 4
- d) $2 + 2\sqrt{3}$

Solutions

1. **(C)** Calculate $\angle B$ as follows:

$$90 - 28 = 62$$

Since $\angle B$ and $\angle E$ correspond to one another, $\angle E$ will have the same measure of 62° .

2. **(B)** In a 45-45-90 triangle, the hypotenuse is $\sqrt{2}$ multiplied by the length of one of the legs. So, in this situation, solve for the length of one of the legs by using a proportion:

$$\begin{aligned}\frac{\overline{BC}}{\overline{AC}} &= \frac{\sqrt{2}}{1} \rightarrow \\ \frac{4}{\overline{AC}} &= \frac{\sqrt{2}}{1} \rightarrow \\ \overline{AC} &= \frac{4}{\sqrt{2}} = 2\sqrt{2}\end{aligned}$$

3. **(B)** Use the Pythagorean theorem to solve for the length of the diagonal:

$$\begin{aligned}a^2 + b^2 &= c^2 \rightarrow \\ 7^2 + 24^2 &= c^2 \rightarrow \\ 625 &= c^2 \rightarrow \\ \sqrt{625} &= 25 = c\end{aligned}$$

Alternatively, you could recognize that this is a Pythagorean triple of 7-24-25, thereby avoiding the need to calculate.

4. **(D)** Plug 8 in for a into the area formula for an equilateral triangle:

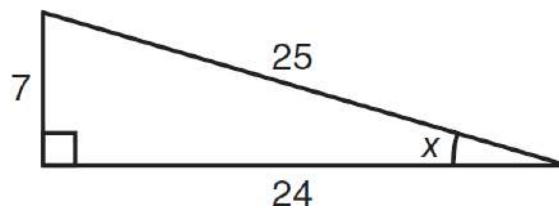
$$A = \frac{\sqrt{3}}{4} a^2 \rightarrow$$

$$A = \frac{\sqrt{3}}{4} (8)^2 = \frac{\sqrt{3} \times 64}{4} = 16\sqrt{3}$$

5. **(B)** $\angle D$ will have a measure of 60° , since it corresponds to $\angle A$ in the other triangle. Calculate the cosine of $\angle D$ using your calculator (be sure your calculator is set to calculate in degrees, not radians), or realize that this is a 30-60-90 triangle so the side adjacent to 60° will be like 1 and the hypotenuse will be like 2. Since the cosine equals the adjacent divided by the hypotenuse, its value will be $\frac{1}{2}$.
6. **(A)** Given that $\sin(x) = \cos(90^\circ - x)$, the two expressions will be equal to one another.

Therefore, the value when the $\cos(90^\circ - x)$ is subtracted from $\sin(x)$ will be zero, regardless of what particular value you would select for x .

7. **(A)** The smallest angle is marked below—it is the one across from the smallest side, 7.



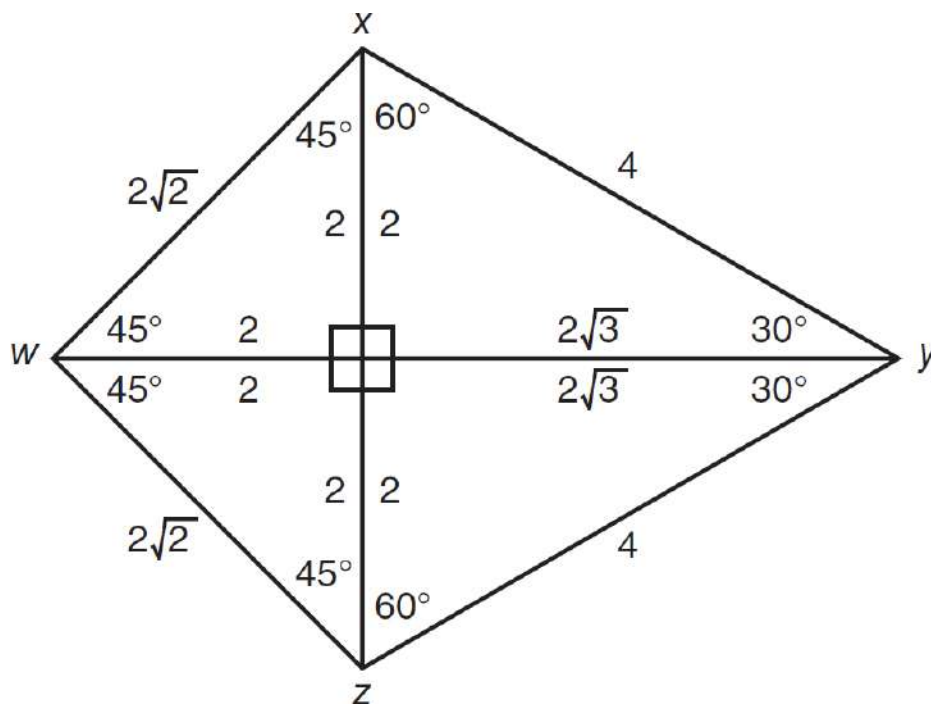
The sine of this angle is equal to the opposite side divided by the hypotenuse: $\frac{7}{25}$.

8. **(C)** Solve for the length of the rectangle so that you can then multiply by the width of 6 to find the rectangle's area. Save time by recognizing that the numbers 10 and 6 are double the values found in the 3-4-5 Pythagorean triple. So, the length will be twice 4: 8. Then, multiply 8 by 6 to find the area of the rectangle: $8 \times 6 = 48$.

9. **(A)** Because the triangles are similar to one another, their corresponding sides will be proportional. The tangent of $\angle EDF$ will be the same as the tangent of $\angle BAC$. Recognize that triangle ABC is a special right triangle: 30-60-90 with sides relating to each other as x , $x\sqrt{3}$, and $2x$. Therefore, the unknown side in the triangle ABC will be 5. You can then calculate the tangent of $\angle BAC$ by dividing the opposite side of 5 by the adjacent side of $5\sqrt{3}$:

$$\frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$$

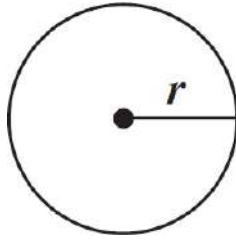
10. **(D)** Label the figure below to find the angle and side values based on the 30-60-90 triangles and 45-45-90 triangles within:



The length of $\overline{WY}$ is therefore $2 + 2\sqrt{3}$.

Circles

At the beginning of each math test section, you are provided with these facts and formulas about circles:

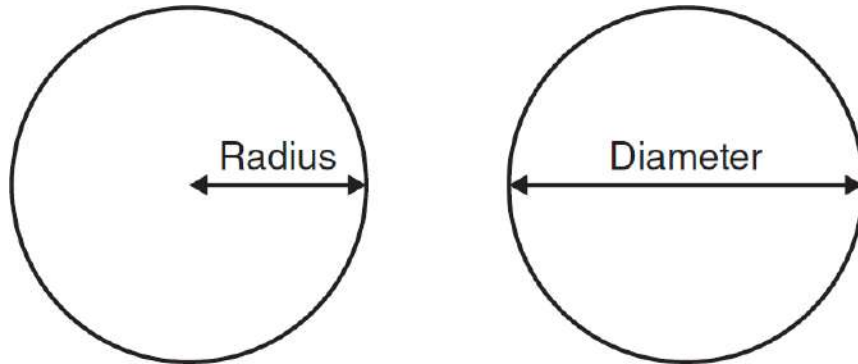


- **r = Radius of the Circle**
- **Area of a Circle = πr^2**
- **Circumference of a Circle = $2\pi r$**
- **A circle has 360 degrees.**

Memorize these facts so you do not have to flip back to the beginning of the section. Here is how to apply these essential concepts.

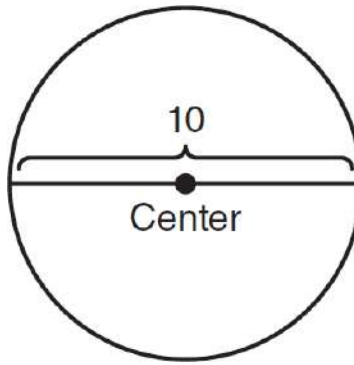
Radius and Diameter

- The *radius* goes the distance from the center of the circle to the circle itself, and is half the length of the diameter.
- The *diameter* goes from one side of a circle to another side through the center, and is twice the measure of the radius.



> Example

What is the radius and the diameter in the circle drawn below?



✓ Solution

The diameter is 10 and the radius will be half the diameter, so the radius is 5.

Area and Circumference

The *area* of a circle is calculated using the radius, r :

$$\text{Area} = \pi r^2$$

The *circumference* of a circle is calculated using the radius, r :

$$\text{Circumference} = 2\pi r$$

Keep these two concepts clear by remembering that the area will give you square units, while the circumference will give you units of length.

Note: It is fine to use the formula $\text{Circumference} = \pi \times \text{Diameter}$ if you would prefer.

› Example

What is the area and the circumference of a circle with a radius of 5 inches?

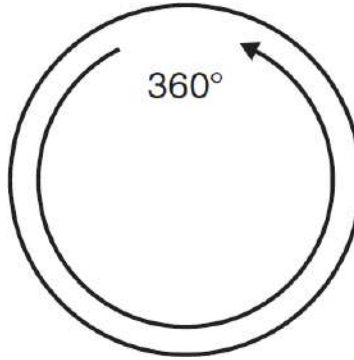
✓ Solution

Calculate each value using the above formulas, plugging 5 in for r .

$$\text{Area} = \pi r^2 \rightarrow \pi 5^2 = 25\pi \text{ square inches}$$

$$\text{Circumference} = 2\pi r = 2 \times \pi \times 5 = 10\pi \text{ inches}$$

360° in a Circle



A full rotation of a circle will be 360° .

► Example

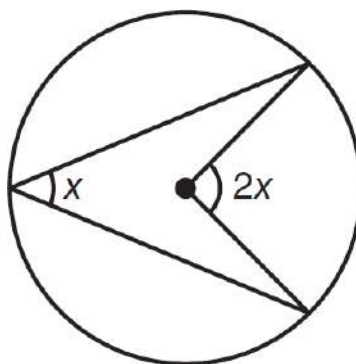
A circle can be divided into how many right angles?

✓ Solution

Since there are a total of 360° in a circle, and a right angle is 90° , a circle can be divided into $\frac{360}{90} = 4$ right angles.

Inscribed Angle

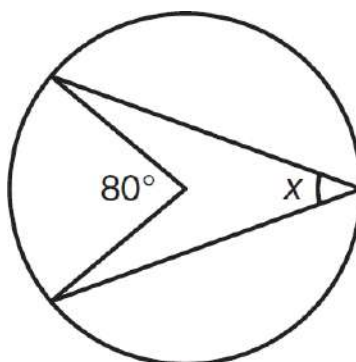
This is a less common concept related to degree measure that is good to understand, just in case a problem about it comes up.



In the graph above, $2x$ corresponds to a central angle and x corresponds to an inscribed angle. When the inscribed angle and the central angle have the same arc, the central angle is *twice* the measure of the inscribed angle.

► Example

In the circle graphed below with a central angle of 80° , what is the measure of $\angle x$?



✓ Solution

Given that $\angle x$ is an inscribed angle that shares the same arc as the central angle of 80° , $\angle x$ will be half the measure of 80° :

$$\frac{80^\circ}{2} = 40^\circ$$

Practice

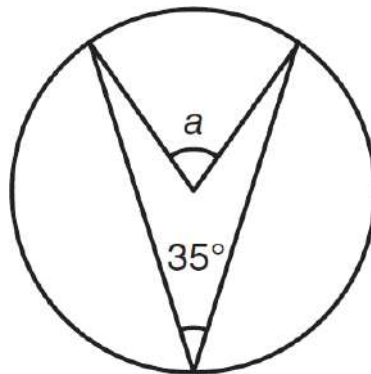
the radius of a circle is x units, what will be the diameter of the circle in terms of x ?

What is the area of a circle with a radius of 10 inches?

What is the circumference of a circle with a diameter of 4 inches?

A circular pie is to be divided into 12 equal pieces, what will be the measure of the angle formed at the central tip of each piece of pie after the pieces are cut?

What is the measure of the central angle a of the sector drawn in the figure below?



Solutions

The diameter of a circle is twice that of the radius, so the diameter will be $2x$.

$$\text{Area} = \pi r^2 \rightarrow \pi 10^2 = 100\pi$$

Be sure you use 2 for the radius, not 4. Circumference $= 2\pi r \rightarrow 2 \times \pi \times 2 = 4\pi$
Take 360° and divide it by 12 to find the angle:

$$\frac{360^\circ}{12} = 30^\circ$$

The central angle will be twice the measure of the inscribed angle, 35° . So $35^\circ \times 2 = 70^\circ$.

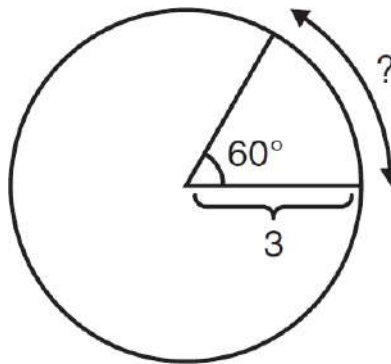
The SAT will have you apply the general concepts about circle area, circumference, and degree measure to calculate the length of an arc or the area of a sector.

Arc Length

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Angle}}{360^\circ} = \frac{\text{Length of Arc}}{\text{Circumference}}$$

► Example

In a circle of radius 3 with an arc of 60° , what is the length of the arc?



✓ Solution

$$\frac{\text{Part}}{\text{Whole}} = \frac{60^\circ}{360^\circ} = \frac{1}{6} = \frac{\text{Length of Arc}}{\text{Circumference}} = \frac{x}{2 \times \pi \times 3}$$

So, set up a proportion to solve for the arc length:

$$\frac{1}{6} = \frac{x}{6\pi} \rightarrow \frac{6\pi}{6} = x \rightarrow \pi = x$$

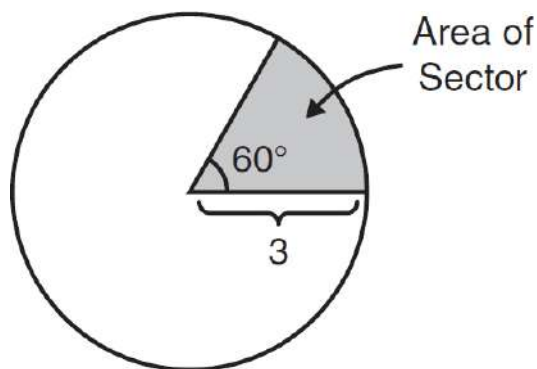
The length of the arc is therefore π units.

Sector Area

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Angle}}{360^\circ} = \frac{\text{Area of Sector}}{\text{Area of Circle}}$$

► Example

In a circle with a radius of 3 and a sector of 60° , what is the area of the sector?



✓ Solution

$$\frac{\text{Part}}{\text{Whole}} = \frac{60^\circ}{360^\circ} = \frac{1}{6} = \frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{x}{\pi 3^2} = \frac{x}{9\pi}$$

So, set up a proportion to solve for the sector area:

$$\frac{1}{6} = \frac{x}{9\pi} \rightarrow \frac{9\pi}{6} = x \rightarrow \frac{3}{2}\pi = x$$

The area of the sector is therefore $\frac{3}{2}\pi$ square units.

Practice

What is the length of a 45° arc in a circle that has a radius of 5 inches?

What is the area of a 30° arc in a circle with a radius of 4 inches?

A circular pizza has a surface area of 64π square inches. If the pizza is divided into 8 equal pieces made by cutting along the diameter through the center point of the pizza, what is the length of the crust on the edge of each piece?

Solutions

1.

$$\frac{\text{Part}}{\text{Whole}} = \frac{45^\circ}{360^\circ} = \frac{1}{8} = \frac{\text{Length of Arc}}{\text{Circumference}} = \frac{x}{2 \times \pi \times 5}$$

So, set up a proportion to solve for the arc length:

$$\frac{1}{8} = \frac{x}{10\pi} \rightarrow \frac{10\pi}{8} = x \rightarrow \frac{5}{4}\pi = x$$

2.

$$\frac{\text{Part}}{\text{Whole}} = \frac{30^\circ}{360^\circ} = \frac{1}{12} = \frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{x}{\pi 4^2} = \frac{x}{16\pi}$$

So, set up a proportion to solve for the sector area:

$$\frac{1}{12} = \frac{x}{16\pi} \rightarrow \frac{16\pi}{12} = x \rightarrow \frac{4}{3}\pi = x$$

3. First, solve for the radius of the pizza by using its area:

$$64\pi = \pi r^2 \rightarrow 64 = r^2 \rightarrow 8 = r$$

Next, find the angle for the sector formed by each of the 8 pieces:

$$\frac{360^\circ}{8} = 45^\circ$$

Then, calculate the length of a sector in a circle with a radius of 8 inches and a sector angle of 45° :

$$\frac{\text{Part}}{\text{Whole}} = \frac{45^\circ}{360^\circ} = \frac{1}{8} = \frac{\text{Length of Arc}}{\text{Circumference}} = \frac{x}{2 \times \pi \times 8}$$

So, set up a proportion to solve for the arc length:

$$\frac{1}{8} = \frac{x}{16\pi} \rightarrow \frac{16\pi}{8} = x \rightarrow 2\pi = x$$

So, the length of the crust on each piece would be approximately 6.3 inches.

Circle Formula

To graph a circle in the xy -coordinate plane, use this formula:

$$(x - h)^2 + (y - k)^2 = r^2$$

$(h, k) = \text{Center}$

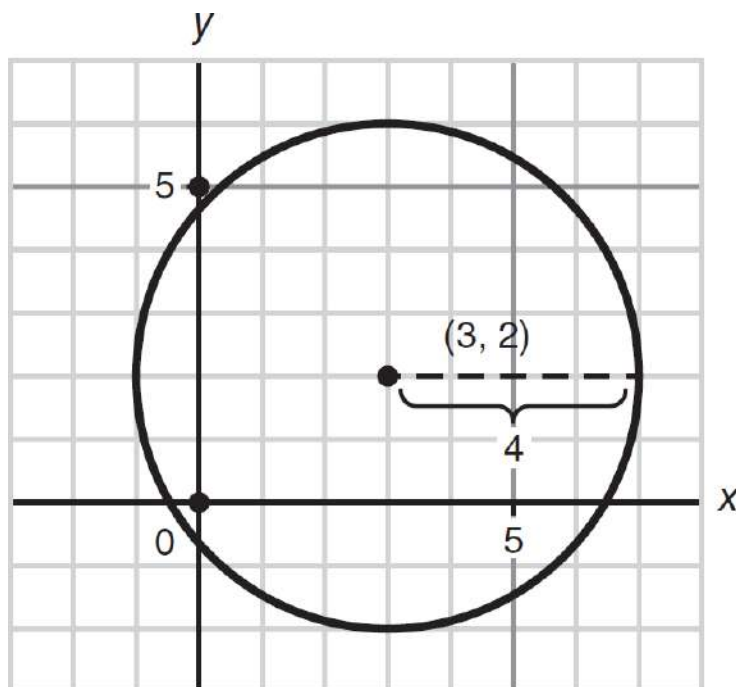
$r = \text{Radius}$

➤ Example

What is the graph of a circle with the equation $(x - 3)^2 + (y - 2)^2 = 16$?

✓ Solution

Based on the equation, the center of the circle will be $(3, 2)$, and the radius will be the square root of 16, which is 4. Here is the graph of this circle:

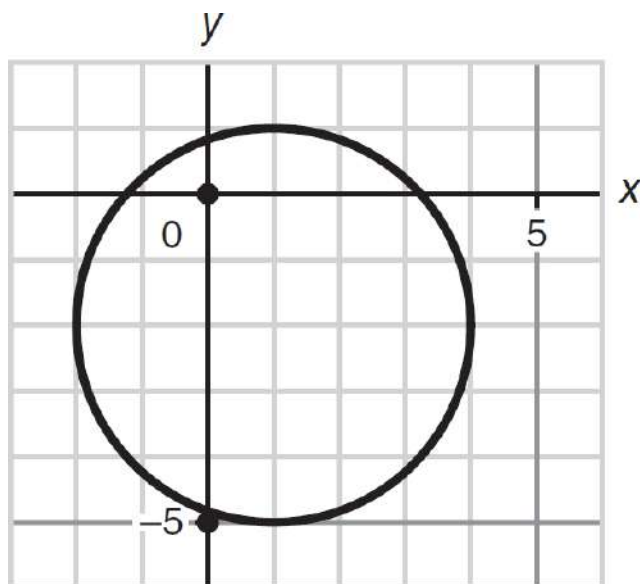


Practice

What is the radius of a circle with the equation $(x - 2)^2 + (y + 5)^2 = 81$?

What is the center of a circle with the equation $(x + 5)^2 + (y - 6)^2 = 14$?

What is the equation of the circle graphed below?



Solutions

Take the square root of 81 to find the radius:

$$\sqrt{81} = 9$$

Be careful with the negative signs, keeping in mind that in the original circle formula, there are negative signs in front of the h and k . So, +5 will give an x -coordinate for the center of -5 and -6 will give a y -coordinate for the center of 6. Therefore, the center point (h, k) will be $(-5, 6)$.

The center of the circle is at $(1, -2)$ and the radius is 3 units. So, the equation for the circle will be $(x - 1)^2 + (y + 2)^2 = 9$.

Radians

A radian is a different way of measuring angles. The SAT says the following at the beginning of the test section:

- **There are 2π radians in a circle.**

Since there are 360° in a circle, 360° corresponds to 2π radians. Divide this by 2 to find:

$$180^\circ = \pi \text{ radians}$$

While you could figure out radian to degree conversion based just on what the SAT provides at the beginning of the section, it might be easier for you to memorize this conversion formula:

$$\frac{\text{Radians}}{\pi} = \frac{\text{Degrees}}{180}$$

› Example

How many radians are in 90 degrees?

✓ Solution

$$\frac{\text{Radians}}{\pi} = \frac{90}{180} \rightarrow$$

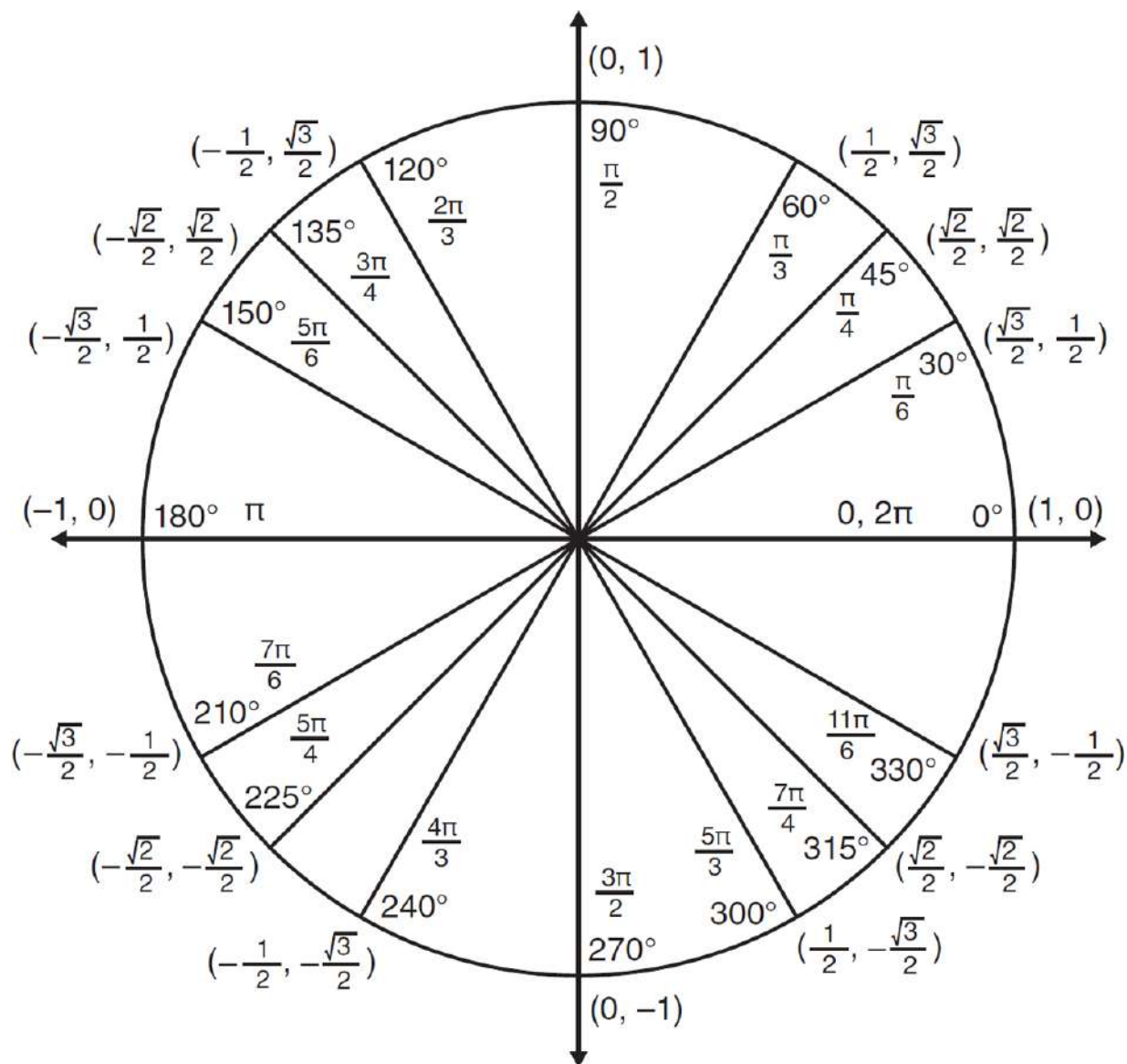
$$\frac{\text{Radians}}{\pi} = \frac{1}{2} \rightarrow$$

$$\text{Radians} = \frac{\pi}{2}$$

So, there are $\frac{\pi}{2}$ radians in 90° .

In addition to doing this conversion, you can memorize several common unit circle points and radian to degree conversions as given in the graph of the *unit circle* below.

Common Unit Circle Points and Degree to Radian Conversions:



Practice

How many radians are in 270°?

How many degrees are in π radians?

How many total radians would there be in two circles?

Solutions

1.

$$\frac{\text{Radians}}{\pi} = \frac{270}{180} \rightarrow$$

$$\frac{\text{Radians}}{\pi} = \frac{3}{2} \rightarrow$$

$$\text{Radians} = \frac{3\pi}{2}$$

You could also use the graph of the circle with the radian to degree conversions.

2.

$$\frac{\text{Radians}}{\pi} = \frac{\text{Degrees}}{180} \rightarrow$$

$$\frac{\pi}{\pi} = \frac{\text{Degrees}}{180} \rightarrow$$

$$1 = \frac{\text{Degrees}}{180} \rightarrow$$

$$180 = \text{Degrees}$$

You could also memorize that 180° are in π radians.

3. A single circle has 2π radians, so two circles will have twice this: 4π radians.

Practice Drill—Circles

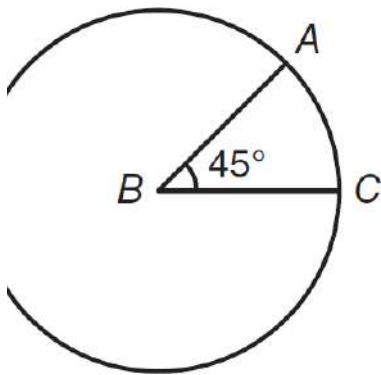
(If timing, take about 12 minutes to complete.)

a circle has an area of 64π units, how many units long is its circumference?

- a) 8π
- b) 12π
- c) 16π
- d) 32π

Consider the circle defined by the equation $(x + 4)^2 + (y - 8)^2 = 144$. What is the radius of this circle?

- A) 4
- B) 8
- C) 12
- D) 72



The above circle has points A and C and a center of B . What percentage of the circle's area is within the sector designated by the central angle $\angle ABC$?

- A) 8%
- B) 12.5%
- C) 14.5%
- D) 25%

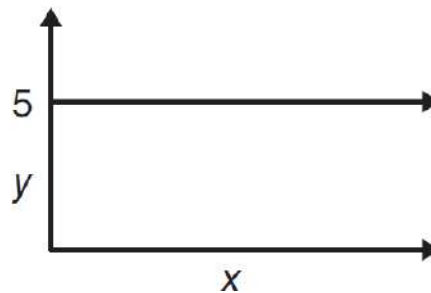
A square with a perimeter of 16 inches is inscribed within a circle, all of the square's vertices intersecting the circle itself. What is the diameter of the circle?

- A) 4
- B) $4\sqrt{2}$
- C) $4\sqrt{3}$
- D) 8

Circle A has a radius of 2 units and an area of x square units. If circle B has a radius twice that of circle A , and the area of circle B is expressed in the form nx , in which n is a constant, what is the value of n ?

- a) 1
- b) 2
- c) 3
- d) 4

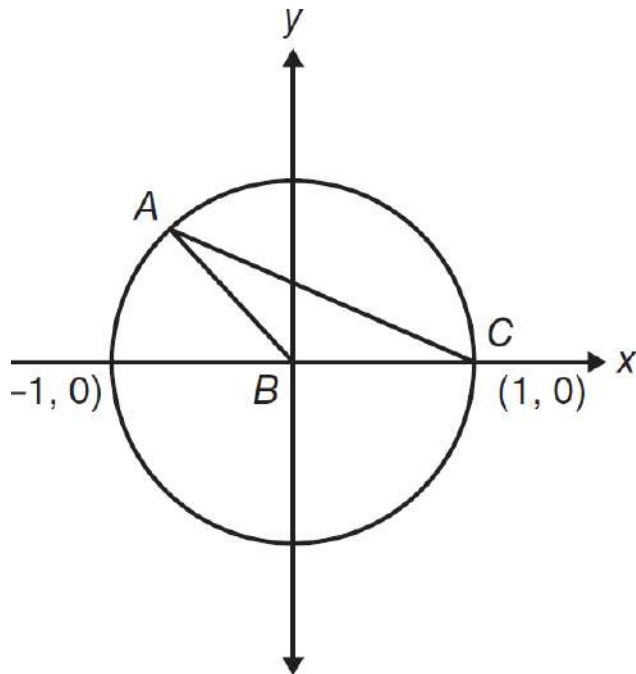
A circle has a diameter of 10 units. The function graphed below could represent what about the circle?



- a) The distance y from a point x on the circle to its center.
- b) The area of the circle, y , depending on its radius, x .
- c) The circumference of the circle, y , depending on its diameter, x .
- d) The diameter of the circle measured at different points in the coordinate plane, (x, y) .

What is the radius of the circle with the equation $x^2 - 4x + y^2 + 2y = 11$?

- a) 3
- b) 4
- c) 6
- d) 7

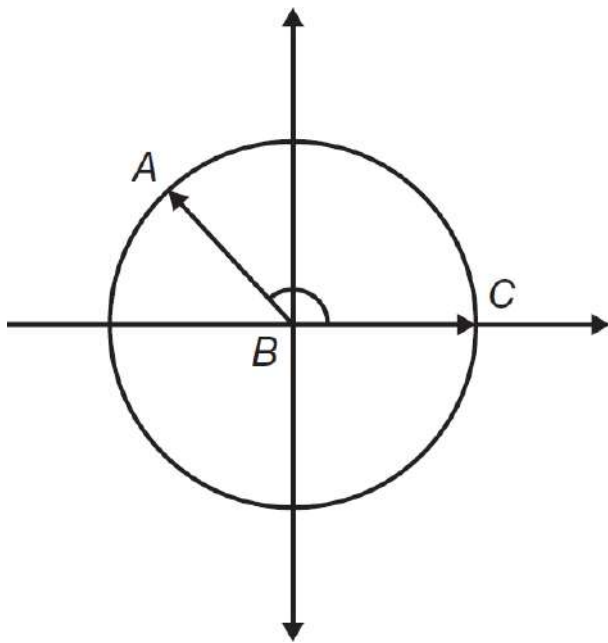


The circle above is centered at the origin with a radius of 1 unit; $\overline{AB}$ and $\overline{BC}$ are both radii of the circle. In triangle ABC above, $\angle BAC$ is 40° . What is the measure of $\angle ACB$ in radians?

- A) $\frac{2}{9}\pi$
- B) $\frac{3}{4}\pi$
- C) π
- D) $\frac{7}{6}\pi$

At what point will a circle with the equation $(x - 4)^2 + (y - 5)^2 = 9$ and the line $y = -2$ intersect?

- A) $(-5, -2)$
- B) $(3, -2)$
- C) $(4, 5)$
- D) They will not intersect.



the cosine of $\angle ABC$ in the unit circle above is $-\frac{1}{\sqrt{2}}$, what is the tangent of ABC ?

- a) -1
- b) -0.5
- c) 1
- d) 1.8

Solutions

1. **(C)** Use the formula for area to find the radius of the circle:

$$\begin{aligned}\pi r^2 &= 64\pi \rightarrow \\ r^2 &= 64 \rightarrow r = 8\end{aligned}$$

Now, use the formula for circumference, using 8 as the radius:

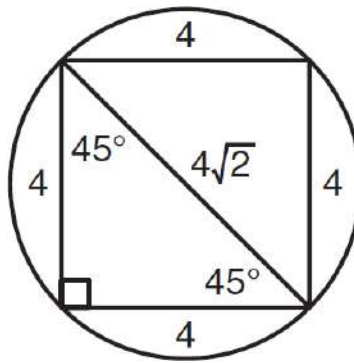
$$2\pi r \rightarrow 2 \times \pi \times 8 = 16\pi$$

2. **(C)** In the circle formula, the right side is the radius squared. So, find the square root of 144 to find the radius: $\sqrt{144} = 12$.

3. **(B)** There are 360° in a circle, so find the percentage that 45° is out of 360° :

$$\frac{45}{360} \times 100 = 12.5\%$$

4. **(B)** If the square has a perimeter of 16 inches, each side on the square is 4 inches. The square is inscribed within the circle, making the situation look like this:



The triangle formed by two of the sides and the diameter of the circle is a 45-45-90 triangle, so the diameter will be $4\sqrt{2}$ units.

5. **(D)** Circle A will have an area of 4π square units, since it has a radius of 2 and the area of a circle is πr^2 . Circle B has twice the radius of circle A , meaning 4 units, thereby having 16π square units of area. Comparing the two areas, if $x = 4\pi$, and the area of B is 16π , then n must equal 4 since $\frac{16\pi}{4\pi} = 4$.
6. **(A)** Since the diameter of the circle is 10, the radius is 5. From any point on the circle, the distance from the point to the center of the circle will be 5. The graph does portray this relationship. It is not going to be choice (B) because the area of this circle would be 25π . It is not going to be choice (C) because the circumference of the circle would be 10π . And it is not going to be choice (D) because the diameter is 10, not 5.

7. **(B)** Complete the square for both the x and y components to get the equation into the format of the circle formula, enabling you to see the radius:

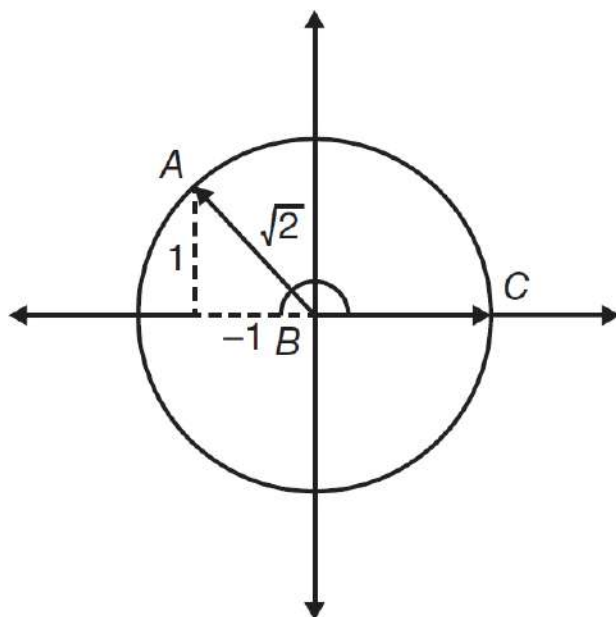
$$\begin{aligned}x^2 - 4x + y^2 + 2y &= 11 \rightarrow \\x^2 - 4x + 4 + y^2 + 2y + 1 &= 11 + 4 + 1 \rightarrow \\(x - 2)^2 + (y + 1)^2 &= 16\end{aligned}$$

The radius is the square root of 16, 4.

8. **(A)** Sides $\overline{AB}$ and $\overline{BC}$ are equivalent, since they are both radii of the circle. Therefore, triangle ABC is isosceles, with $\angle BAC$ and $\angle ACB$ both 40° . So, we need to convert 40° to radians using the radian conversion formula:

$$\begin{aligned}\frac{\text{Radians}}{\pi} &= \frac{\text{Degrees}}{180} \rightarrow \\\frac{\text{Radians}}{\pi} &= \frac{40}{180} \rightarrow \\\frac{\text{Radians}}{\pi} &= \frac{2}{9} \rightarrow \\\text{Radians} &= \frac{2}{9}\pi\end{aligned}$$

9. **(D)** The circle has a center at $(4, 5)$ and a radius of 3. So, it has no points that will be below the x -axis, and therefore will never intersect the line $y = -2$.
10. **(A)** We can label the graph as following given the information in the problem, since sides of this ratio will form a 45-45-90 triangle:



The tangent of $\angle ABC$ will therefore be the opposite divided by the adjacent, which is $\frac{-1}{1} = -1$. You could also calculate the value by using your calculator and solving $\tan(135) = -1$.

Advanced Drills

Use these drills to push yourself with the toughest types of problems you are likely to encounter on SAT test day. Each drill has ten questions, with solutions that follow.

- Algebra Drill 1
- Algebra Drill 2
- Advanced Math Drill 1
- Advanced Math Drill 2
- Problem Solving and Data Analysis
- Geometry and Trigonometry
- Mixed Problems

If you are doing a drill under timed conditions, allow about 15 minutes to complete (longer than the chapter drills because of the increased average

difficulty of the problems). Good luck!

Algebra Drill 1

Solve for x :

$$1\frac{7}{8}x + \frac{5}{32} = 3\frac{3}{4}x - 1\frac{1}{4}$$

- a) $\frac{3}{4}$
- b) $2\frac{17}{32}$
- c) $\frac{15}{16}$
- d) $-\frac{45}{76}$

When 2 times a number is subtracted from 14, the result is 2 greater than the number. What is the number in question?

- a) $\frac{16}{3}$
- b) 4
- c) 12
- d) 16

In 2015, Andre had 210 coins in his collection. If Andre adds 5 new coins a year starting in 2015 through the end of 2022 and then adds 8 coins a year starting in 2023, how many coins will he have in his collection at the end of 2045?

- a) 224
- b) 421
- c) 429
- d) 434

How many pairs (x, y) satisfy both $x - y > 3$ and $y - 5 > x$?

- a) 0
- b) 1
- c) 2
- d) Infinitely many

What is the solution (x, y) to the following set of equations?

$$4x - 3y = \frac{11}{3} \text{ and } -\frac{2}{3}x + \frac{1}{4}y = -\frac{13}{18}$$

- a) $(\frac{4}{3}, \frac{5}{4})$
- b) $(\frac{5}{4}, \frac{4}{9})$
- c) $(-\frac{37}{12}, -\frac{16}{3})$
- d) $(\frac{7}{4}, 10)$

How many ordered pairs (x, y) satisfy the following system of equations?

$$(x - 2)(y + 5) = 0 \text{ and } 3x + y = 1$$

- a) 0
- b) 1
- c) 2
- d) Infinitely many

What are the solutions to the following series of equations?

$$\frac{3}{8}a + \frac{2}{3}b = 4.3 \text{ and } -12.9 + 1.125a = -2b$$

- a) $a = 0.8$ and $b = 6$
- b) $a = 16.8$ and $b = -3$
- c) No solutions
- d) Infinitely many solutions

andidate M and Candidate N are the only candidates running for city mayor. the total number of votes the two candidates receive is 50,000 and if candidate M receives 3 times as many votes as Candidate N, what is the total number of votes Candidate N receives?

- a) 12,500
- b) 16,667
- c) 21,500
- d) 37,500

Jennifer's yearly salary, S , is modeled using the equation $S = 2,500Y + 40,000$, where Y represents how many years she has been working at the company. What does the number 2,500 represent in this equation?

- a) The amount Jennifer's salary increases for each year she has been working
- b) Jennifer's starting salary
- c) The amount of money Jennifer has made in year Y
- d) The number of hours Jennifer has worked in year Y

Avinash reads 2 fiction articles per day and 14 nonfiction articles per week, which expression models the total number of articles he would read in w weeks?

- a) $2w + 14$
- b) $16w$
- c) $28w$
- d) $112w$

Solutions

- a) To make solving this problem a bit easier, let's convert all of our mixed numbers into improper fractions:

$$\frac{15}{8}x + \frac{5}{32} = \frac{15}{4}x - \frac{5}{4}$$

We eventually want to combine our x terms and combine our constant terms. To do this, our x terms need a common denominator (8), and our constant terms need a common denominator (32). Thus, let's rewrite our fractions with these common denominators. Our full equation is now:

$$\frac{15}{8}x + \frac{5}{32} = \frac{30}{8}x - \frac{40}{32}$$

To avoid dealing with negatives, let's bring the constants to the left by adding $\frac{40}{32}$ to both sides:

$$\frac{15}{8}x + \frac{45}{32} = \frac{30}{8}x$$

We then want to get all x terms on the right by subtracting $\frac{15}{8}x$ from both sides:

$$\frac{45}{32} = \frac{15}{8}x$$

To solve for x , divide both sides by $\frac{15}{8}$ (in other words, multiply both sides by $\frac{8}{15}$):

$$\frac{45(8)}{32(15)} = x \text{ so } \frac{360}{480} = x$$

This reduces to $\frac{3}{4} = x$, which is choice (A).

b) Let x be the number we are trying to find.

"2 times a number is subtracted from 14" can be written as $14 - 2x$. (We are subtracting 2 times a number, or $2x$, from the 14.)

"The result is" means an equals sign.

"2 greater than the number" can be written as $x + 2$. Thus, the whole sentence can be written as:

$$14 - 2x = x + 2$$

add $2x$ to both sides to get all x terms on the right. Then subtract 2 from both sides to get all constants on the left:

$$12 = 3x$$

Dividing by 3 tells us that $x = 4$, which is answer **(B)**.

D) Between 2015 and 2022, Andre adds 5 coins 8 times:

2015, 2016, 2017, 2018, 2019, 2020, 2021, and 2022

Adding 5 coins 8 times means he added a total of $5(8) = 40$ coins in those years.

Between 2023 and 2045, he added 8 coins 23 times:

2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036,
2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, and 2045

Thus, from 2023 to 2045, he added a total of $8(23) = 184$ coins.

Andre started with 210 coins. So the sum of the coins in his collection will be the original 210 plus the number of coins he added:

$$210 + 40 + 184 = 434$$

This is choice **(D)**.

A) Since we have two inequalities, we can use elimination to get rid of one variable. Let's add both equations together:

$$\begin{array}{r} x - y > 3 \\ + \quad y - 5 > x \\ \hline x - 5 > 3 + x \end{array}$$

Subtracting x from both sides and adding 5 to both sides leaves you with $0 > 8$. Because we know that under no circumstances is 0 greater than 8, there are no

solutions.

b) In a system of two equations, we can either use substitution or elimination solve.

we multiply the second equation by 12 and add the equations together, our y terms will cancel out. Start by multiplying the second equation by 12:

$$12\left(-\frac{2}{3}x + \frac{1}{4}y = -\frac{13}{18}\right) = -8x + 3y = -\frac{26}{3}$$

now we can add this equation to the first equation:

$$\begin{array}{r} 4x - 3y = \frac{11}{3} \\ + -8x + 3y = -\frac{26}{3} \\ \hline -4x = -\frac{15}{3} \end{array}$$

to solve for x, divide by -4:

$$x = \frac{15}{12} = \frac{5}{4}$$

this is enough to narrow down the answer to choice (B). However, you could plug this x value into one of your functions to solve for y if you wanted.

Alternatively, you could have used substitution by solving for x in the first equation and then plugging that equation back into the second equation.

to solve for x, first add 3y to both sides:

$$4x = 3y + \frac{11}{3}$$

then divide both sides by 4 to isolate x:

$$x = \frac{3y}{4} + \frac{11}{12}$$

now you can plug this expression into the second equation for x:

$$-\frac{2}{3}\left(\frac{3y}{4} + \frac{11}{12}\right) + \frac{1}{4}y = -\frac{13}{18}$$

ext, distribute the $-\frac{2}{3}$

$$-\frac{6}{12}y - \frac{22}{36} + \frac{1}{4}y = -\frac{13}{18}$$

ome of these fractions can be reduced. For our purposes, though, it doesn't ally matter. Next, we need to combine our y terms, but first we need a mmon denominator.

et's convert $\frac{1}{4}y$ to $\frac{3}{12}y$:

$$-\frac{6}{12}y - \frac{22}{36} + \frac{3}{12}y = -\frac{13}{18}$$

ow combine like terms:

$$-\frac{1}{4}y - \frac{22}{36} = -\frac{13}{18}$$

o add $\frac{22}{36}$ to both sides, we need to first convert $-\frac{13}{18}$ to $-\frac{26}{36}$ so that our nstants have a common denominator:

$$\begin{aligned} -\frac{1}{4}y - \frac{22}{36} &= -\frac{26}{36} \\ -\frac{1}{4}y &= -\frac{1}{9} \end{aligned}$$

o solve for y , divide both sides by $-\frac{1}{4}$ (multiply both sides by -4):

$$y = \frac{4}{9}$$

his is enough to narrow it down to choice **(B)**. However, we can solve for x / plugging the y value into our equation for x :

$$x = \frac{3}{4}y + \frac{11}{12} = \frac{3}{4}\left(\frac{4}{9}\right) + \frac{11}{12} = \frac{12}{36} + \frac{11}{12} = \frac{12}{36} + \frac{33}{36} = \frac{45}{36} = \frac{5}{4}$$

nally, we could have solved this problem by plugging each answer choice to the two equations to see which set of points work. If using this method, be

remember to check that the points satisfy BOTH equations. For instance, choice A) works when plugged into the first equation but not the second.

B) In the first equation, one of those factors must equal 0 for the whole equation to equal 0. Therefore, we will find the possible solutions by setting each factor equal to 0:

$$x - 2 = 0$$

Adding 2 to both sides tells us that $x = 2$ is a possible solution:

$$y + 5 = 0$$

Subtracting 5 from both sides tells us that $y = -5$ is another possible solution.

Plugging these values into the second equation will tell us the solutions to it:

$$3(2) + y = 1$$

$$6 + y = 1.$$

Subtracting 6 from both sides tells us that $y = -5$, which is the original y value we got from the first equation.

Let's try the y value that we already determined:

$$3x + -5 = 1$$

So $3x = 6$. Dividing by 3 tells us that $x = 2$. However, this is the x solution that we already tried. Therefore, there is only one solution:

$$(2, -5)$$

This is choice **(B)**.

D) Let's use substitution. Solve for b in the second equation by dividing both sides by -2 :

$$6.45 - 0.5625a = b$$

Let's now plug the left side of this equation into the first equation for b :

$$\frac{3}{8}a + \frac{2}{3}(6.45 - 0.5625a) = 4.3$$

Distributing the $\frac{2}{3}$ leaves us with:

$$\frac{3}{8}a + 4.3 - 0.375a = 4.3$$

Let's get that $\frac{3}{8}a$ into decimal form so that we can easily combine our like terms:

$$0.375a + 4.3 - 0.375a = 4.3$$

Combining both a terms results in:

$$4.3 = 4.3$$

We know that this is always true, meaning we have an infinite number of solutions.

Alternatively, you may have noticed that if you multiply the second equation by $-\frac{1}{3}$; and convert any fractions in the two equations into decimals, the two equations are exactly the same. Since they are the same line, there are infinitely many solutions.

1) Let's turn what we know into equations. Let m equal the number of votes that Candidate M receives and let n equal the number of votes that Candidate N receives.

First, we know that the sum of the votes that the two candidates receive is 50,000. In other words:

$$m + n = 50,000$$

Additionally, we know that the number of votes Candidate M receives is 3 times the number of votes that Candidate N receives. This can be expressed by the equation

$$m = 3n$$

Now we can plug in $3n$ for m in the first equation:

$$3n + n = 50,000$$

Combining like terms results in:

$$4n = 50,000$$

We can solve for n by dividing both sides by 4:

$$n = 12,500$$

This is choice (A).

(A) We're told that Y is the number of years Jennifer has worked at the company. We can see that each time Y increases by 1, her salary goes up \$2,500. For instance, when $Y = 0$, Jennifer's salary is \$40,000. When $Y = 1$, her salary is \$42,500. When $Y = 2$, her salary is \$45,000. Therefore, choice (A) is correct.

Choice (B) is incorrect because it can be shown that Jennifer's starting salary (when $Y = 0$) is \$40,000.

Choice (C) is incorrect because the amount of money made is given by the variable S .

Choice (D) is incorrect because the equation tells us nothing about the number of hours Jennifer worked.

(2) We want to know how many articles Avinash reads in w weeks. So we first need to figure out how many articles he reads per week. We can then multiply this number by w to give the number of articles he reads in w weeks.

Avinash reads 2 fiction articles per day. Since there are 7 days in a week, he reads $2(7) = 14$ fiction articles per week. He also reads 14 nonfiction articles per week. So Avinash reads $14 + 14 = 28$ fiction and nonfiction articles per week.

Therefore, the number of articles Avinash reads in w weeks is given by the expression $28w$, which is choice (C).

Algebra Drill 2

$-(2x - 4) + 3(x - 5) = -4$, what is the value of x ?

- A) -3
- B) 5
- C) 7
- D) 15

$g(x + 2) = 5x - 4$, what is the value of $g(7)$?

- A) 21
- B) 29
- C) 31
- D) 41

John is having an undetermined number of people over for dinner. He needs to have 6 serving utensils (used by everyone collectively), plus a knife, fork, and spoon for each diner. Which of the following equations correctly models the total number of utensils, U , John will need for x number of diners, himself included?

- A) $U = 3x$
- B) $U = 9x$
- C) $U = 3x + 6$
- D) $U = 6x + 3$

$f(x) = 4x + 7$ and if $g(x) = -3x + 2$, what is the value of $f(g(3))$?

- A) -27
- B) -21

- c) -7
- d) 19

Under a new state law, a massage therapist will be required to charge sales tax on her services. If the sales tax rate is 7%, by what ratio would she need to multiply the current price of her services to determine the new total amount customers will pay under the new law?

- a) $\frac{7}{100}$
- b) $\frac{7}{10}$
- c) $\frac{107}{100}$
- d) $\frac{170}{100}$

$6(2a - b) = 4b$, what is the ratio of b to a ?

- a) $\frac{2}{3}$
- b) $\frac{5}{6}$
- c) $\frac{6}{5}$
- d) $\frac{12}{5}$

The total operational costs C for a restaurant are modeled by the equation $C = M + 50,000$, where M represents the number of meals served. What does the 50,000 represent in the equation?

- a) The total operational costs
- b) The fixed operational costs
- c) The cost per meal
- d) The minimum number of meals served

The total costs C to operate a factory are represented by the function $C(n) = an + b$, where n is the number of days the factory is operational. If the daily operations costs were to increase beyond the given rate and if the initial startup

ists were to decrease beyond the given rate, how would this affect the constants a and b ?

- a) a would increase, and b would increase.
- b) a would increase, and b would decrease.
- c) a would decrease, and b would increase.
- d) a and b would remain the same.

ow will the function $f(x) = 4x - 5$ be affected by the translation

$$g(x) = f(x - 1) + 2?$$

- a) It will be shifted up 2 units and 1 unit to the left.
- b) It will be shifted down 1 unit and 2 units to the left.
- c) It will be shifted up 2 units and 1 unit to the right.
- d) It will be shifted down 1 unit and 2 units to the right.

company is conducting an online campaign to increase its social media followers. The number of social media followers, N , is estimated by the equation $N = 30W + 250$, where W represents the number of weeks of the campaign (and $W > 0$). What does the number 250 represent in the equation?

- a) The number of weeks of the campaign
- b) The number of new social media followers each week
- c) The number of social media followers at the start of the campaign
- d) The number of social media followers at the end of W weeks

Solutions

c) First, we need to distribute. Don't forget to distribute the negative sign to the $(2x - 4)$ term. Distributing gives us:

$$-2x + 4 + 3x - 15 = -4$$

Combining like terms on the left side gives:

$$x - 11 = -4$$

Adding 11 to both sides tells us that $x = 7$, which is choice (C).

Note that you could also plug each answer choice into the equation to see which one gives the correct equality, but doing this could be more time-consuming.

1) First, we must determine what number we should plug in for x . Our original function is $g(x + 2)$, and we're looking for $g(7)$. That means that we want x such that $x + 2 = 7$. Subtracting 2 from both sides gives $x = 5$. So to find $g(7)$, we simply plug in 5 for x in the original function:

$$g(5 + 2) = 5(5) - 4 = 25 - 4 = 21$$

The correct answer is choice (A).

2) John needs 6 utensils no matter how many people come, so +6 will be a constant. He also needs 3 utensils per person, which can be represented as $3x$. Thus, he needs $3x + 6$ utensils, choice (C).

Choice (A) is incorrect because although it correctly depicts the 3 utensils needed for each person, it neglects the 6 serving utensils.

Choice (B) is incorrect because it states that each person needs 9 utensils.

Choice (D) is incorrect because it states that there are 3 utensils needed no matter how many people come, rather than the 6 utensils actually needed.

Alternatively, you could have imagined a scenario in which John invited 1 other person over, making a total of 2 people at dinner. They need 6 serving utensils, plus each one of them needs a knife, spoon, and fork. This makes a total of 2 knives, 2 spoons, and 2 forks, or 6 more utensils. The total number of utensils needed in this case is $6 + 6 = 12$.

You could have then plugged in 12 for U , plugged in 2 for x , and chosen the answer choice that worked, which is choice (C):

$$12 = 3(2) + 6$$

b) First, we have to find the value of $g(3)$ by plugging in 3 wherever there's an x in $g(x)$:

$$g(3) = -3(3) + 2 = -9 + 2 = -7$$

Now we have to find $f(-7)$ by plugging in -7 wherever there is an x in $f(x)$:

$$f(-7) = 4(-7) + 7 = -28 + 7 = -21$$

The answer is choice **(B)**.

c) Let's imagine that a massage therapist currently charges \$100 for a massage. If the tax rate is 7%, or 0.07, tax on that service will be:

$$0.07(\$100) = \$7$$

Therefore, the price of the massage including the sales tax will be:

$$\$100 + \$7 = \$107$$

The ratio of the new price to the old price is:

$$\frac{107}{100}$$

The massage therapist can find the new prices for all of her services by multiplying the old prices by this ratio.

d) First, distribute the 6:

$$12a - 6b = 4b$$

Bring the b terms to the right side by adding $6b$ to both sides:

$$12a = 10b$$

To find the ratio of b to a , we want to solve for $\frac{b}{a}$. First, divide both sides by a :

$$12 = 10\frac{b}{a}$$

To isolate $\frac{b}{a}$, we need to divide both sides by 10:

$$\frac{b}{a} = \frac{12}{10} = \frac{6}{5}$$

The correct ratio is choice **(C)**.

(B) This \$50,000 is some sort of cost that stays the same whether the restaurant serves 0 meals or serves 1,000 meals. Because the \$50,000 doesn't vary with the variable meals, it's a fixed cost, which is choice **(B)**.

Choice (A) is incorrect because C represents the total costs, and C varies with the number of meals served.

Choice (C) is incorrect because 2 is the cost per meal, as shown by the $2M$ in the equation.

Choice (D) is incorrect because the minimum number of meals served could be any positive integer.

(B) Total cost is found by adding together the variable costs and the fixed costs. In this problem, the variable cost is an since this value depends on n , the number of days the factory is operational. Since an is a cost and since n is a number of days, it follows that the daily operational cost is given by a . Therefore, if the daily operational cost increases, a will increase.

The initial startup costs, or fixed costs, are given by variable b because b is a constant that isn't affected by the variable n . Therefore, if the startup costs increase, b will decrease.

The correct scenario is depicted in choice **(B)**.

(C) In general, if $f(x)$ is our original function and if c is a constant, then:

$f(x - c)$ shifts $f(x)$ to the right by c units.

$f(x + c)$ shifts $f(x)$ to the left by c units.

$f(x) + c$ shifts $f(x)$ up by c units.

$f(x) - c$ shifts $f(x)$ down by c units.

this particular problem, $g(x) = f(x - 1) + 2$. Using the above properties, $f(x - 1)$ tells us that $f(x)$ is shifted to the right by 1 unit. The $+2$ in the expression tells us that $f(x)$ is shifted up by 2 units. This responds to choice (C).

) The number 250 is in the equation regardless of the value of W . Therefore, it makes sense that 250 would be the initial number of followers the campaign had. We can see this by plugging 0 in for W in the equation. This gives us the initial number of followers before the company starts campaigning:

$$N = 30(0) + 250 = 0 + 250 = 250$$

) we can see that when $W = 0$, $N = 250$.

Advanced Math Drill 1

An element's half-life is the amount of time that it takes for the element to decay by half. If there is x amount of element Z initially, which of the following represents the amount A of Z that would remain after n whole half-lives of Z had passed?

- a) $A = \frac{x}{2n}$
- b) $A = \frac{x}{2^n}$
- c) $A = \frac{n}{2x}$
- d) $A = \frac{x}{2^{n-1}}$

$\sqrt[5]{2x^8y^{11}}$ is equivalent to which of the following?

- a) $2xy^2\sqrt[5]{x^3y}$
- b) $2x^5y^{10}\sqrt[5]{x^3y}$
- c) $2x^3y^6\sqrt[5]{xy}$
- d) $2xy^2\sqrt[5]{2x^3y^2}$

$x > 0$, then $\frac{1}{2x} + \frac{1}{3x}$ is equivalent to which of the following?

- a) $\left(\frac{25}{6x^2}\right)^{\frac{1}{2}}$

b) $\left(\frac{4}{25x^2}\right)^{\frac{1}{2}}$

c) $\left(\frac{5}{6x}\right)^2$

d) $\left(\frac{25}{36x^2}\right)^{\frac{1}{2}}$

$x^2 + ax = b$, where a and b are constants, what are the solutions for x ?

a) $x = -\frac{a}{2} \pm \sqrt{2b + \frac{a^2}{2}}$

b) $x = -\frac{a}{2} \pm \sqrt{b + \frac{a^2}{4}}$

c) $x = -\frac{b}{2} \pm \sqrt{\frac{a^2}{2} - 2b}$

d) $x = -\frac{a}{2} \pm \sqrt{b^2 + \frac{a^2}{4}}$

$x^2 + 15xy + 6y^2 = ?$

a) $3(2x + y)^2$

b) $(3x + y)(x + 3y)$

c) $(3x + 3y)(2x + 2y)$

d) $3(x + 2y)(2x + y)$

What are the solution(s) for x in the equation below?

$$x - 6 = \sqrt{75 - 2x}$$

a) 13

b) 13 and -3

c) -3 and 3

d) No solution

At what points will $f(x) = 8x^2 - 22x + 15$ intersect the x -axis?

a) 15

- b) $\frac{3}{2}$ and $\frac{5}{4}$
- c) $-\frac{5}{4}$ and $-\frac{3}{2}$
- d) $\frac{5}{4}$ and $\frac{15}{2}$

Which value of n will cause the value of $f(x) = xn$ to be consistently positive and increase the most rapidly, given that x is greater than 1 and that n is an even integer?

- a) -2
- b) -1
- c) 1
- d) 2

To see if two sets of data are correlated, one can calculate the correlation coefficient between two populations, r_{xy} , using the formula $r_{xy} = \frac{s_{xy}}{s_x s_y}$, where s_{xy} is the covariance of the population, s_x is the standard deviation of population x , and s_y is the standard deviation of population y . If the dispersion of population x and the dispersion of population y both increase while the covariance between the populations remains the same, what would happen to the correlation coefficient of the two populations?

- a) It would decrease.
- b) It would increase.
- c) It would stay the same.
- d) It cannot be determined.

If the function $f(x) = x^n + 3x^m$ has 5 zeros and if $f(x)$ is multiplied by -1 , how many zeros will the resulting function have?

- a) -5
- b) 4
- c) 5

) 6

Solutions

1. **(B)** The amount after one half-life is $\frac{1}{2}x$. After the second half-life, half of this new amount decays, and we're left with:

$$\frac{1}{2}\left(\frac{1}{2}x\right) = \frac{1}{4}x = \left(\frac{1}{2^2}\right)x$$

After the third half-life, half of that amount left decays. We now have left:

$$\frac{1}{2}\left(\frac{1}{4}x\right) = \frac{1}{8}x = \left(\frac{1}{2^3}\right)x$$

We can start to see a pattern: each time a half-life passes, we multiply the amount we previously had by $\frac{1}{2}$. So after n half-lives, $A = \left(\frac{1}{2^n}\right)x$ remains, or $A = \frac{x}{2^n}$. This matches choice **(B)**.

2. **(A)** $\sqrt[5]{32} = 2$. So go ahead and bring a 2 outside of the radical before dealing with the variables:

$$\sqrt[5]{32x^8y^{11}} = 2\sqrt[5]{x^8y^{11}}$$

Let's deal with the x term under the root next. Notice the following:

$$\sqrt[5]{x^8} = \sqrt[5]{x^5x^3} = \sqrt[5]{x^5}\sqrt[5]{x^3} = x\sqrt[5]{x^3}$$

Similarly, we can rewrite the y term under the root as follows:

$$\sqrt[5]{y^{11}} = \sqrt[5]{y^5y^5y} = \sqrt[5]{y^5}\sqrt[5]{y^5}\sqrt[5]{y} = y \cdot y\sqrt[5]{y} = y^2\sqrt[5]{y}$$

Thus, the entire expression can be rewritten as:

$$\sqrt[5]{32x^8y^{11}} = 2\sqrt[5]{x^8y^{11}} = 2\sqrt[5]{x^8}\sqrt[5]{y^{11}} = 2xy^2\sqrt[5]{x^3y}$$

This is choice **(A)**.

3. **(D)** All of the answer choices are a single fraction. So first find a common denominator and add the two fractions. The least common denominator is $6x$. Multiply the first fraction by $\frac{3}{3}$ and the second by $\frac{2}{2}$.

$$\frac{3}{6x} + \frac{2}{6x} = \frac{5}{6x}$$

We can rule choice (C) out because it's our answer squared, so it will not equal our answer.

The rest of the answers are being raised to the $\frac{1}{2}$ power, which is equivalent to taking the square root. In order to find out what should be inside the parentheses, we must work backward by doing the opposite to our function. Because the answer choices take the square root of an expression, we must square our expression to find what should go inside the parentheses:

$$\left(\frac{5}{6x}\right)^2 = \frac{5^2}{(6x)^2} = \frac{25}{36x^2}$$

Therefore, the answer is choice **(D)**.

Alternatively, we could have taken the square root of choices (A), (B), and (D) to see which one is equivalent to $\frac{5}{6x}$:

$$\left(\frac{25}{6x^2}\right)^{\frac{1}{2}} = \frac{\sqrt{25}}{\sqrt{6x^2}} = \frac{5}{\sqrt{6x}}$$

So, choice (A) is incorrect.

$$\left(\frac{4}{25x^2}\right)^{\frac{1}{2}} = \frac{\sqrt{4}}{\sqrt{25x^2}} = \frac{2}{5x}$$

So, choice (B) is incorrect.

$$\left(\frac{25}{36x^2}\right)^{\frac{1}{2}} = \frac{\sqrt{25}}{\sqrt{36x^2}} = \frac{5}{6x}$$

This matches the expression we obtained, so choice **(D)** is correct.

4. **(B)** In their structure, the answer choices all look like the quadratic formula. So subtract b from both sides:

$$x^2 + ax - b = 0$$

Now you can use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We have to be careful, though. Our equation has a and b coefficients, but they don't match up exactly with the a and b given in the quadratic formula. In the quadratic formula, a is the coefficient in front of the x^2 term, b corresponds to the coefficient in front of the x term, and c represents the constant. In our case, however, the coefficient in front of the x^2 term is 1, the coefficient of the x term is a , and the constant is b . Keep this in mind while using the quadratic formula to get:

$$x = \frac{-a \pm \sqrt{a^2 - 4(1)(-b)}}{2(1)} = \frac{-a \pm \sqrt{a^2 + 4b}}{2}$$

This doesn't match any of the answer choices, so we need to simplify further. All of the answer choices have the leading term over 2, so let's divide this into two fractions:

$$\frac{-a \pm \sqrt{a^2 + 4b}}{2} = -\frac{a}{2} \pm \frac{\sqrt{a^2 + 4b}}{2}$$

From the leading term, you can narrow it down to choices (A), (B), and (D). It looks like the answer choices have pulled the denominator

of the second term into the square root. Because 2 equals $\sqrt{4}$, we can change the denominator to $\sqrt{4}$:

$$-\frac{a}{2} \pm \frac{\sqrt{a^2 + 4b}}{2} = -\frac{a}{2} \pm \frac{\sqrt{a^2 + 4b}}{\sqrt{4}} = -\frac{a}{2} \pm \sqrt{\frac{a^2 + 4b}{4}}$$

In all of the answer choices, the fraction inside the square root appears to be broken up, so do that:

$$-\frac{a}{2} \pm \sqrt{\frac{a^2 + 4b}{4}} = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} + \frac{4b}{4}} = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} + b}$$

Reordering what's inside the root gives you choice **(B)**.

5. **(D)** One easy way to solve this problem is to use FOIL on the answer choices and see which matches the original:

Choice (A):

$$3(2x + y)^2 = 3[(2x + y)(2x + y)] = 3(4x^2 + 4xy + y^2) = 12x^2 + 12xy + 3y^2$$

This does not match our expression, so we can rule out this choice.

Choice (B):

$$(3x + y)(x + 3y) = 3x^2 + 10xy + 3y^2$$

This does not match the original.

Choice (C):

$$(3x + 3y)(2x + 2y) = 6x^2 + 12xy + 6y^2$$

This does not quite match the original.

Choice (D):

$$3(x + 2y)(2x + y) = 3(2x^2 + 5xy + 2y^2) = 6x^2 + 15xy + 6y^2$$

This matches the original expression, so the answer is choice **(D)**.

Another way to solve the problem is to factor. Since each term is divisible by 3, we can factor out a 3 to get:

$$3(2x^2 + 5xy + 2y^2)$$

Next, we factor the expression inside the parenthesis as follows:

$$3(2x + y)(x + 2y)$$

This matches choice **(D)**.

6. **(A)** Look at the answer choices. There are only three possible x values in the answer choices (13, 3, and -3), so it's probably easiest to just plug in these three values for x in our original equation to see which ones work:

$$\begin{aligned} 13 - 6 &= \sqrt{75 - 2(13)} \\ 7 &= \sqrt{49} \end{aligned}$$

The square root of 49 is 7, so 13 works.

$$\begin{aligned} -3 - 6 &= \sqrt{75 - 2(-3)} \\ -9 &= \sqrt{81} \end{aligned}$$

Square roots always give nonnegative answers, so this one doesn't work.

$$\begin{aligned} 3 - 6 &= \sqrt{75 - 2(3)} \\ -3 &= \sqrt{69} \end{aligned}$$

Only 13 works, so the answer is choice **(A)**.

You could also solve this directly by squaring both sides of the equation:

$$(x - 6)^2 = (\sqrt{75 - 2x})^2$$

Squaring gives:

$$x^2 - 12x + 36 = 75 - 2x$$

This is a quadratic equation. So we move everything over to one side, combine like terms, and factor:

$$x^2 - 10x - 39 = 0$$

Factoring gives:

$$(x - 13)(x + 3) = 0$$

So $x = 13$ or $x = -3$. Since we initially squared our equation, we have to check our answers since this procedure can lead to extraneous answers. Indeed, only $x = 13$ works. So 13 is the only solution to the equation.

7. **(B)** Functions intersect the x -axis at their zeros. Zeros can be found by setting the function equal to 0 and either factoring or using the quadratic formula to solve for possible values of x .

The polynomial can be factored to $(4x - 5)(2x - 3) = 0$. Set each factor equal to 0 and solve for x :

$$4x - 5 = 0$$

Adding 5 to both sides and then dividing by 4 gives you $x = \frac{5}{4}$.

$$2x - 3 = 0$$

Adding 3 to both sides then dividing by 2 gives you $x = \frac{3}{2}$.

Choice **(B)** is the answer.

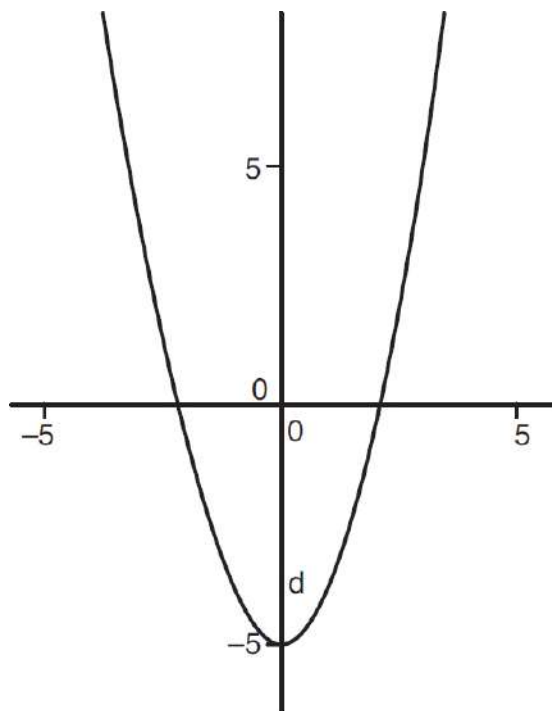
Alternatively, you could have used the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{22 \pm \sqrt{(-22)^2 - 4(8)(15)}}{2(8)} = \frac{22 \pm \sqrt{4}}{16} = \frac{22 \pm 2}{16}$$
$$x = \frac{24}{16} = \frac{3}{2} \text{ or } x = \frac{20}{16} = \frac{5}{4}$$

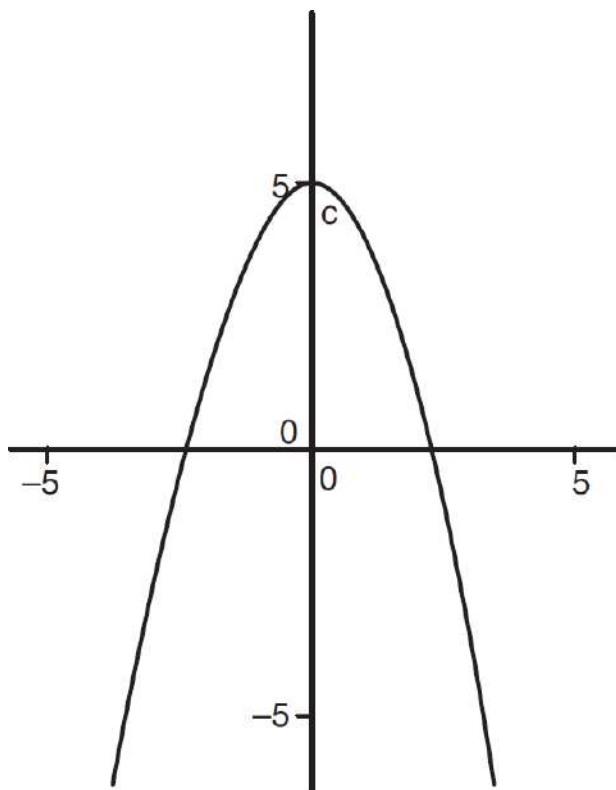
8. **(D)** We are told that x is greater than 1, so the function will always be positive regardless of n . We are also told that n is an even integer, so this narrows down the answer to 2 or -2 . In order for the function to increase most rapidly, you want the exponent n to be the largest even number possible, which is choice **(D)**.
9. **(A)** If the dispersion of population x increases, the standard deviation of x increases. If the dispersion of population y increases, the standard deviation of y increases. Therefore, you'd be holding the numerator constant while increasing the denominator. Therefore, the correlation coefficient would decrease, which is choice **(A)**.
10. **(C)** Multiplying a function by -1 will flip it about the x -axis. The new function will still cross the x -axis the same number of times at the same values of x as the original function. So the zeros will not change. If $f(x)$ has 5 zeros, $-f(x)$ will also have 5 zeros.

Consider this simpler function to see how multiplying it by -1 would affect its graph.

The graph of $y = x^2 - 5$ is shown:



If the function is multiplied by -1 on the right-hand side, it will give the function $y = -x^2 + 5$, which is shown:



So the functions have the same zeros even though they are mirror images of one another.

Advanced Math Drill 2

A particular savings account provides no interest in the first year of a deposit and 3% annual compounded interest on a deposit for each year thereafter. If x dollars are deposited initially, which of the following equations expresses the total amount of money $A(n)$ in the account n years later, where n is an integer greater than 2?

- A) $A(n) = x [(0.03)^n]$
- B) $A(n) = x [(0.97)^n]$
- C) $A(n) = x [(0.03)^{n-1}]$
- D) $A(n) = x [(1.03)^{n-1}]$

What is the value of x in the following equation?

$$x^2 + 9 = -6x$$

- A) -3
- B) 0
- C) 3
- D) No solution

For $y < 0$, which of the following is equivalent to $\frac{3}{x^2 y}$?

- A) $\frac{3x^{-2}}{\sqrt[4]{y^2}}$
- B) $\frac{3x^{-2}}{-\sqrt[4]{y^4}}$
- C) $\frac{3x^{\frac{1}{2}}}{\sqrt[4]{y^4}}$
- D) $\frac{3x^{-2}}{\sqrt[4]{y^4}}$

or positive x and y , $x^{-\frac{3}{4}}y^{\frac{4}{3}}$ is equivalent to

a) $-\frac{x^3y^4}{x^4y^3}$

b) $\frac{\sqrt[4]{y^3}}{\sqrt[3]{x^4}}$

c) $\frac{y^3\sqrt{y}}{\sqrt[4]{x^3}}$

d) $\frac{y^{4^3}\sqrt{y}}{x^{3^4}\sqrt{x}}$

What are the possible values of x in the following equation?

$$3x^2 + 12x + 6 = 0$$

a) $-12 \pm \sqrt{3}$

b) $-2 \pm \sqrt{2}$

c) $-2 \pm \sqrt{3}$

d) $2 \pm \sqrt{2}$

$x^2 + x - 12 = 0$ and if $x < 0$, what is the value of x ?

a) -6

b) -4

c) -3

d) 3

$16x^6 - 16y^8$ is equivalent to which of the following expressions?

a) $16(4x^3 + y^4)(x^3 - y^4)$

b) $16(4x^3 - y^4)(x^3 - y^4)$

c) $16(2x^3 + y^4)(2x^3 - y^4)$

d) $16(2x^6 - y^8)(2x - y)$

Solve the following equation for all possible x values.

- a) 3
- b) 3 and 8
- c) -3 and -8
- d) Infinitely many solutions

The graph of $x - 4 = y^4$ has a minimal x value that compares in what way to the minimal x value of the graph $x = y^4$?

- a) It is 4 less.
- b) It is 4 greater.
- c) They are the same.
- d) The answer cannot be determined.

Which of the following functions represents the reflection across the x axis of

$$y = 3(x - 5)^2 + 4?$$

- a) $y = -3(x + 5)^2 - 4$
- b) $y = -3(x - 5)^2 - 4$
- c) $y = 3(-x - 5)^2 + 4$
- d) $y = 3(-x + 5)^2 + 4$

Solutions

- (D)** If 3% is added annually after the first year is complete, after the second year, 3% will have been added. Thus, after 2 years, the total amount of money in the account can be represented as:

$$x + 0.03x$$

By combining like terms, it can also be expressed as:

$$1.03x = x(1.03)^1$$

After three years, another 3% is added to the new amount:

$$1.03x + 0.03(1.03x) = 1.03(x + 0.03x) = 1.03(1.03x) = x(1.03)^2$$

Continue in this manner. So after the fourth year, the amount of money in the account is $x(1.03)^3$.

Keeping this pattern in mind, the money in the account after n years is $x(1.03)^{n-1}$, which is choice **(D)**.

Note that raising 1.03 to the $n - 1$ power means that you're multiplying x by 1.03 one time less than the number of years that have passed. This is because no interest is added in the first year.

2. **(A)** Add the $6x$ to both sides so that the polynomial is equal to 0:

$$x^2 + 6x + 9 = 0$$

You can either factor this or use the quadratic formula. This is easily factorable:

$$(x + 3)(x + 3) = 0$$

You can set each factor equal to 0 to solve for the possible values of x . However, since they're the same factor, you need to do it only once:

$$x + 3 = 0$$

Subtracting 3 from both sides tells you that $x = -3$, which is choice **(A)**.

3. **(B)** The key here is to notice that in the problem, $y < 0$. In the original expression, x is squared. So regardless of its sign, x^2 will be positive. Therefore, $\frac{3}{x^2y}$ will be negative when y is negative. Thus, the new expression must also be negative.

Simplify the answer choices to see which matches the original expression.

Choice (A):

Negative exponents get sent to the denominator:

$$\frac{3x^{-2}}{\sqrt[4]{y^2}} = \frac{3}{x^2\sqrt[4]{y^2}}$$

Roots can be expressed as fractional exponents:

$$\frac{3}{x^2\sqrt[4]{y^2}} = \frac{3}{x^2y^{\frac{2}{4}}}$$

However, $y^{\frac{2}{4}} \neq y$. So this is different than our original expression. We can rule out choice (A).

Choice (B):

Following the same process as the previous answer choice, we can rewrite this answer as:

$$\frac{3x^{-2}}{-\sqrt[4]{y^4}} = -\frac{3}{x^2\sqrt[4]{y^4}}$$

Now consider $\sqrt[4]{y^4}$. It is tempting to say that this just equals $y^{\frac{4}{4}} = y$, but we have to be careful not to ignore a subtle point. When we take the fourth root of something, or in general the even root of something, we necessarily get a result that is nonnegative. In particular, $\sqrt[4]{y^4}$ must be nonnegative. However, y is negative, so the expression can't equal y . The expression actually equals $-y$, which is positive since y is negative.

(Convince yourself of this. For example, let $y = -2$ and consider $\sqrt[4]{y^4} = \sqrt[4]{(-2)^4}$. This expression is equal to $\sqrt[4]{16} = 2 = -(-2) = -y$ since $y = -2$.)

Thus, plugging in $1/y$ for $\sqrt[4]{y^4}$ in our expression gives the following after canceling out the negative signs:

$$\frac{3x^{-2}}{-\sqrt[4]{y^4}} = -\frac{3}{x^2\sqrt[4]{y^4}} = -\frac{3}{x^2(-y)} = \frac{3}{x^2y}$$

This is our original expression, so choice **(B)** must be the correct answer.

If you wanted to explore the other answer choices, you could.

Choice (C):

By the same logic as before, this simplifies to:

$$\frac{3x^{\frac{1}{2}}}{\sqrt[4]{y^4}} = \frac{3\sqrt{x}}{-y} = -\frac{3\sqrt{x}}{y}$$

This clearly doesn't match the original expression, so it can be eliminated.

Choice (D):

Simplifying gives:

$$\frac{3x^{-2}}{\sqrt[4]{y^4}} = \frac{3}{x^2(-y)} = -\frac{3}{x^2y}$$

This is close to our original expression but has a negative sign in front, so we can eliminate this choice.

4. **(C)** Remember that negative exponents can be made positive by moving whatever is being raised to that exponent to the denominator:

$$x^{-\frac{3}{4}}y^{\frac{4}{3}} = \frac{y^{\frac{4}{3}}}{x^{\frac{3}{4}}}$$

Fractional exponents are the same as roots:

$$\frac{y^{\frac{4}{3}}}{x^{\frac{3}{4}}} = \frac{\sqrt[3]{y^4}}{\sqrt[4]{x^3}}$$

This doesn't match any answer choices, so we need to simplify further. The y is being raised to a power higher than its root, so we should be able to pull something out of the root.

Since y is positive:

$$\sqrt[3]{y^4} = \sqrt[3]{y^3 y} = \sqrt[3]{y^3} \sqrt[3]{y} = y \sqrt[3]{y}$$

Therefore, our whole expression can be rewritten as:

$$\frac{y \sqrt[3]{y}}{\sqrt[4]{x^3}}$$

Choice **(C)** is the correct answer.

5. **(B)** Factor out a 3:

$$3(x^2 + 4x + 2) = 0$$

Divide both sides by 3:

$$x^2 + 4x + 2 = 0$$

This isn't easily factorable, so use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

Choice **(B)** is the answer.

6. **(B)** We need to find all solutions less than 0. This equation is easily factorable:

$$x^2 + x - 12 = (x + 4)(x - 3) = 0$$

If $x + 4$ or $x - 3$ equaled 0, the whole expression would equal 0. Therefore, setting both factors equal to 0 will tell us the two potential values of x :

$$x + 4 = 0 \text{ so } x = -4$$

$$x - 3 = 0 \text{ so } x = 3$$

The question asks for only the value of x that's less than 0, so the answer is $x = -4$, which is choice **(B)**.

7. **(C)** All of the answers have a 16 factored out, so do that first:

$$64x^5 - 16y^8 = 16(4x^5 - y^8)$$

When anything is in the form $(a^2 - b^2)$, it can be factored using the difference of squares formula: $(a + b)(a - b)$. The trick to this problem is figuring out what a and b are. Set $4x^5$ equal to a^2 to find a :

$$4x^5 = a^2$$

$$a = \sqrt{4x^5}$$

Take the square root of both 4 and the x term:

$$a = 2x^{\frac{5}{2}} = 2x^2$$

Next, set y^8 equal to b^2 to solve for b :

$$y^8 = b^2$$
$$b = \sqrt{y^8} = y^{\frac{8}{2}} = y^4$$

Therefore, if you wanted to express $(4x^6 - y^8)$ in the form of $(a + b)(a - b)$, it would be:

$$(2x^3 + y^4)(2x^3 - y^4)$$

Putting the 16 in front gives you choice **(C)**.

8. **(B)** Get rid of the square root by squaring both sides:

$$x^2 = 11x - 24$$

To find the possible values of x , subtract $11x$ and add 24 to both sides, setting the left side equal to 0. Then factor or use the quadratic formula to solve:

$$x^2 - 11x + 24 = 0$$

This equation factors as:

$$(x - 3)(x - 8) = 0$$

This statement would hold true if $x - 3 = 0$ or if $x - 8 = 0$. In other words, the statement would hold true if $x = 3$ or if $x = 8$, which is choice **(B)**.

Be careful, though. When we squared both sides of the equation, we were no longer guaranteed to get the same exact solutions as our original equation. In other words, because we square both sides, it is possible that we get extraneous solutions. So we should get in the habit of checking that both of our solutions are indeed solutions. We can do this by plugging both answer choices into our original equation:

$$\begin{aligned} x &= 3 \\ \sqrt{11(3) - 24} &= \sqrt{9} = 3 = x \end{aligned}$$

This solution checks out.

$$\begin{array}{c} x = 8 \\ \sqrt{11(8) - 24} = \sqrt{64} = 8 = x \end{array}$$

This solution checks out as well.

In this problem, both solutions check out. However, you should be cautious in general when squaring equations.

9. **(B)** The first function can be expressed as $y = \sqrt[4]{x - 4}$. It's an even root. This means that the value within the root must be greater than or equal to 0 since we can find only the even root of a nonnegative number:

$$\begin{array}{c} x - 4 \geq 0 \\ x \geq 4 \end{array}$$

Therefore, the domain of the function is $[4, \infty)$.

The second function can be expressed as $y = \sqrt{x}$. Again, it's an even root. So what's within the root must be greater than or equal to 0:

$$x \geq 0$$

The domain of this function then is $[0, \infty)$.

Therefore, the minimal x value of the first function is 4 more than the minimal x value of the second function. Choice **(B)** is the answer.

10. **(B)** To reflect something across the x -axis, multiply the entire equation by -1 :

$$-y = -[3(x - 5)^2 + 4] = -3(x - 5)^2 - 4$$

Alternatively, notice that our original function gives the equation of a parabola; think about what this parabola looks like. Recall that the vertex form of a parabola is $y = a(x - h)^2 + k$. Our original parabola

will open upward since $a = 3$ is positive. This parabola will also have a vertex at $(h, k) = (5, 4)$. If you want to reflect this across the x -axis, it would need to open downward and have a vertex at $(5, -4)$.

Therefore, the vertex form of this new parabola would be:

$$y = -3(x - 5)^2 - 4$$

The answer is choice **(B)**.

Problem Solving and Data Analysis Drill

there are 4 cars for every 5 trucks in the parking lot (with no other types of vehicles), what is the ratio of cars to the total number of vehicles in the parking lot?

- A) 1 to 5
- B) 4 to 9
- C) 5 to 9
- D) 4 to 5

On Monday, the highest temperature reached was 70 degrees Fahrenheit. On Tuesday, the highest temperature increased by 20%. On Wednesday, the highest temperature decreased by 25% from the previous day. What was the difference between Monday's and Wednesday's highest temperatures in degrees Fahrenheit?

- A) 5
- B) 7
- C) 14
- D) 21

A recipe calls for 3 cups of sugar. There are 16 tablespoons in a cup and 3 teaspoons in a tablespoon. If a cook has 1.5 cups of sugar available in the pantry, how many teaspoons of sugar must the cook obtain from other sources to follow the recipe?

- a) 72
- b) 104
- c) 144
- d) 216

Price per Gallon of Milk	Number of Gallons Sold
\$1.50	650
\$0.90	780
\$1.95	530
\$2.80	330
\$3.40	190

Which of these functions best models the relationship between the number of gallons of milk sold, $N(g)$, and the price per gallon of milk, g ?

- a) $N(g) = 530 + 590(g - 1.95)^2$
- b) $N(g) = 1,000 - 240g$
- c) $N(g) = 1,000 + 240g$
- d) $N(g) = 780 - 360(g - 0.9)^2$

	Employed	Unemployed
Population X	890	112
Population Y	748	205

What is the difference between the unemployment percentage in Population Y and the unemployment percentage in Populations X and Y combined, calculated to the nearest tenth?

- a) 5.0%

- b) 5.3%
- c) 5.8%
- d) 6.2%

Questions 6–7 use the following table.

Election Results

	Candidate A	Candidate B	Total
Columbus	350,000	270,000	620,000
Cleveland	180,000	195,000	375,000
Total	530,000	465,000	995,000

If all eligible voters in Columbus, 40% actually voted in the election. How many total eligible voters did Columbus have?

- a) 248,000
- b) 875,000
- c) 1,550,000
- d) 2,487,500

Suppose that a survey of total 200 randomly selected voters from both cities accurately predicted the results of the election. How many of the people surveyed would have been supporters of Candidate A from Columbus?

- a) 50
- b) 70
- c) 113
- d) 132

Questions 8–9 use the following table.

Grade	Test 1	Project 1	Test 2	Project 2	Total
A	5	8	9	7	29
B	7	6	5	10	28
C	6	7	4	5	22
D	4	1	3	2	10
F	1	1	3	0	5
Total	23	23	24	24	94

What is the median letter grade for assignments in the class is:

- a) A
- b) B
- c) C
- d) D

For which assignment is the standard deviation of the grade results the least?

- a) Test 1
- b) Project 1
- c) Test 2
- d) Project 2

An online shopping site allows customers to post 1-star, 2-star, 3-star, 4-star, and 5-star reviews for products. If an item currently has an average star rating of 2.3 based on a total of 10 reviews, what is the minimum number of reviews that could bring up the overall average rating to at least a 3.0?

- a) 2
- b) 3
- c) 4
- d) 5

Solutions

b) Let's imagine the simplest version of this ratio: there are only 4 cars and 5 trucks in the parking lot. So how many total vehicles are there?

$$4 \text{ cars} + 5 \text{ trucks} = 9 \text{ vehicles}$$

c) the ratio of cars to vehicles is 4 cars to 9 vehicles, choice **(B)**.

b) If the highest temperature on Tuesday increased 20% from the highest temperature on Monday, then the highest temperature on Tuesday was 120%, or 1.2, of Monday's temperature. Find 120% of 70 degrees:

$$1.2(70) = 84$$

c) Tuesday's high temperature was 84 degrees Fahrenheit. Wednesday's high was 25% lower than Tuesday's temperature, so it was only 75%, or 0.75, of Tuesday's temperature. Find 75% of 84 degrees:

$$0.75(84) = 63$$

d) The highest temperature on Wednesday was 63 degrees Fahrenheit. We want to know the difference between Monday's high temperature and Wednesday's high. The difference is given by $70 - 63 = 7$, which is choice **(B)**.

a) If a cook has 1.5 cups of sugar in the pantry but needs 3 cups, he needs $3 - 1.5 = 1.5$ more cups from other sources. We want to convert this to teaspoons:

$$1.5 \text{ cups} \times \frac{16 \text{ tablespoons}}{\text{cup}} \times \frac{3 \text{ teaspoons}}{\text{tablespoon}} = 72 \text{ teaspoons}$$

e) the answer is choice **(A)**.

b) Test the answer choices.

choice (A):

$$N(1.50) = 530 + 590(1.50 - 1.95)^2 = 649.48$$

This is a fairly good estimate, so let's try another value:

$$N(0.90) = 530 + 590(0.90 - 1.95)^2 = 1,180.48$$

We can rule out choice (A).

choice (B):

$$N(1.50) = 1,000 - 240(1.50) = 640$$

This is also a fairly good estimate, so let's try the other values:

$$N(0.90) = 1,000 - 240(0.90) = 784$$

$$N(1.95) = 1,000 - 240(1.95) = 532$$

$$N(2.80) = 1,000 - 240(2.80) = 328$$

$$N(3.40) = 1,000 - 240(3.40) = 184$$

All of these values are pretty close to the actual values, so this answer choice may be correct. However, the question asks for the best model, so we need to make sure that there are no better models.

choice (C):

$$N(1.50) = 1,000 + 240(1.50) = 1,360$$

We can rule out choice (C).

choice (D):

$$N(1.50) = 780 - 360(1.50 - 0.9)^2 = 650.4$$

$$N(0.90) = 780 - 360(0.90 - 0.9)^2 = 780$$

$$N(1.95) = 780 - 360(1.95 - 0.9)^2 = 383.1$$

We can rule out choice (D).

choice **(B)** is the best model of the relationship.

b) In Population Y, 205 of the $748 + 205 = 953$ people are unemployed. Therefore, the unemployment percentage is:

$$\frac{205}{953} \times 100\% = 21.51\%$$

both populations combined, there are a total of $112 + 205 = 317$ unemployed people, and a total population of $890 + 112 + 748 + 205 = 1,955$. Therefore, the unemployment rate is:

$$\frac{317}{1,955} \times 100\% = 16.21\%$$

The difference between the two unemployment rates is $21.51\% - 16.21\% = 5.3\%$, which is choice **(B)**.

2) The number of people who voted in Columbus was 620,000. If this number represents only 40% of eligible voters, you can set up a proportion to solve for 100% of the number of eligible voters:

$$\frac{620,000}{40} = \frac{x}{100}$$

In the proportion, x represents the number of eligible voters in Columbus. Cross multiply to get:

$$\begin{aligned} 620,000(100) &= 40x \\ 62,000,000 &= 40x \end{aligned}$$

Dividing by 40 tells you that $x = 1,550,000$, which is choice **(C)**.

Another way we can solve this problem is as follows. We know that 620,000 is 40% of the eligible voters, x , in Columbus. In other words, $620,000 = 0.4x$. Dividing by 0.4 gives 1,550,000, which is choice **(C)**.

3) Since there are 350,000 Columbus voters who support Candidate A and 15,000 voters in the two cities combined, we know that $\frac{350,000}{995,000} = 0.352$ of the voters in the table were supporters of Candidate A from Columbus. We would expect the same proportion of the 200 randomly surveyed voters to be supporters of Candidate A from Columbus. This can be found by taking $0.352(200) = 70.4$. Thus, we could expect about 70 of the randomly surveyed fall into this category. So the correct answer is choice **(B)**.

b) There are 94 total grades. To find the median term in a series with an even number of terms, you have to take the mean of the two middle terms. To find these two middle terms, first divide the total number of terms by 2. This will tell you the number of the 1st term you'll use in your average. Then add 1 to that number to find the number of the 2nd term you'll use in your average.

If we listed all 94 grades starting with the lowest grades, the F's, and ending with the highest grades, the A's, the median terms would be the average of the $\frac{94}{2} = 47\text{th}$ term and the $\frac{94}{2} + 1 = 48\text{th}$ term.

Let's take us through the first 5 terms.

Let's take us through 10 more, to the 15th term.

Let's take us through another 22, to the $15 + 22 = 37\text{th}$ term.

Let's take us through another 28 to the $37 + 28 = 65\text{th}$ term. The 47th and 48th terms will then both be B's, so the median grade is a B, which is choice **(B)**.

d) Standard deviation is usually lowest when range is lowest because it means that more values are centered near the mean. Because no one received an F on project 2, the range was only between an A and a D, with only two people receiving D's. Because this project has a rather small range, it will have a rather small standard deviation. In all of the other assignments, at least one student earned an A and at least one earned an F, so the ranges will be higher for all other answer choices. Thus, choice **(D)** is the best choice.

c) An average (in other words, a mean) is given by the following expression:

$$\text{Mean} = \frac{\text{Sum}}{n}$$

In the expression, "sum" is the total of the terms you're averaging and n is the number of terms you're averaging. In order to calculate the minimum number of reviews needed to raise the mean to a 3.0, we'll need to know the sum of the current ratings. Plugging the numbers into the equation gives:

$$2.3 = \frac{\text{Sum}}{10}$$

Therefore, the sum of the current ratings is $2.3(10) = 23$.

In order to raise the average with the minimum number of reviews, the reviews need to all be as high as possible, so they must all be 5-star reviews.

Let's test our answer choices. When plugging in answer choices, start with one of the middle answers since answers tend to be arranged from smallest to largest or from largest to smallest.

Choice (B):

If the product gets 3 new 5-star reviews, this will add $3(5) = 15$ to our current sum. This will also add 3 to our current n :

$$\text{Mean} = \frac{23 + 15}{10 + 3} = \frac{38}{13} = 2.92$$

The mean is not high enough yet, so we need more reviews.

Choice (C):

If the product gets 4 new 5-star reviews, this will add $4(5) = 20$ to our current sum and 4 to our current n :

$$\text{Mean} = \frac{23 + 20}{10 + 4} = \frac{43}{14} = 3.07$$

This mean is higher than our desired mean, so 4 is the minimum number of reviews needed, choice (C).

Alternatively, you could have solved this algebraically. Adding x more 5-star reviews will add $5x$ to the sum and x to n , the total number of reviews.

Therefore, the mean can be represented with the following equation:

$$\text{Mean} = \frac{23 + 5x}{10 + x}$$

Since we know we want the mean to be 3.0, we can set the right side of our equation equal to 3.0:

$$3.0 = \frac{23 + 5x}{10 + x}$$

Let's get rid of the denominator by multiplying both sides by $(10 + x)$:

$$30 + 3.0x = 23 + 5x$$

Combine like terms by subtracting $3.0x$ and 23 from both sides:

$$7 = 2.0x$$

Dividing both sides by 2.0 tells you that the minimum number of new reviews needed is 3.5 . Recall that we want the minimum number of reviews needed to raise the average to a 3.0 . Since 3 reviews would not be enough and a person can't give 0.5 of a review, we must round up to the nearest integer, 4 . Therefore, to raise the average to a 3.0 , you need 4 new reviews.

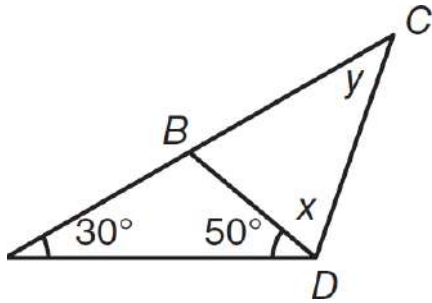
Geometry and Trigonometry Drill

A box in the shape of a rectangular prism has dimensions 20 inches by 30 inches by 12 inches. Inside the box are four solid cubes, each with edge lengths of 4 inches. If the inside of the larger box is empty except for the solid cubes, what is the volume of empty space in the box?

- A) 248 cubic inches
- B) $1,800$ cubic inches
- C) $6,944$ cubic inches
- D) $7,200$ cubic inches

A right triangle has a hypotenuse of 10 inches and legs that measure 8 inches and 6 inches. What is the area of the triangle in square inches?

- A) 12
- B) 18
- C) 24
- D) 36



In triangle ACD above, point B is on $\overline{AC}$, and B and D form a line. What is the sum of $\angle x$ and $\angle y$ in degrees?

- a) 100
- b) 110
- c) 120
- d) 140

A circle has the equation $(x - 4)^2 + (y - 6)^2 = 9$, what is the equation of the circle if it is reflected directly across the x -axis?

- a) $(x + 4)^2 + (y + 6)^2 = -9$
- b) $(x - 4)^2 - (y + 6)^2 = -9$
- c) $(x + 4)^2 - (y + 6)^2 = 9$
- d) $(x - 4)^2 + (y + 6)^2 = 9$

The area of an equilateral triangle is $2\sqrt{3}$ square units, what is the perimeter of the triangle?

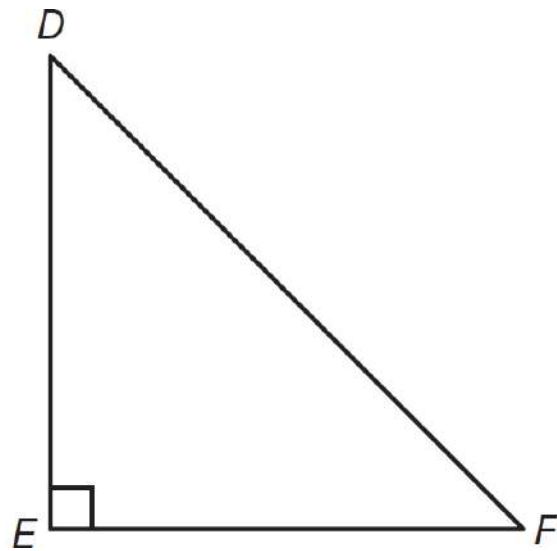
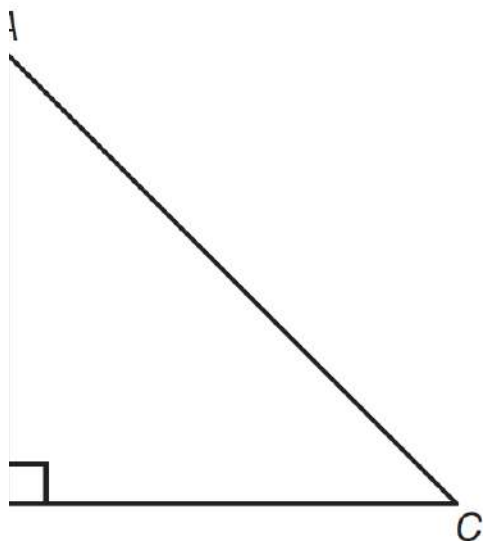
- a) 6
- b) $4\sqrt{3}$
- c) $6\sqrt{2}$
- d) $6\sqrt{3}$

Given that $\angle X$ is between 0 and 90 degrees, and that $\cos X = Y$, what is the measure of an angle with the measure $(90 - X)$ degrees?

- a) $90 - Y$
- b) Y
- c) $\frac{2}{3}$
- d) 90

An angle measuring 3π radians would be equivalent to how many angles with measure of 45° ?

- a) 4
- b) 8
- c) 9
- d) 12



Triangles ABC and DEF are similar to one another. If the cosine of $\angle BAC$ is $\frac{4}{5}$, and the length of $\overline{DE}$ is 24 units, what is the length of $\overline{DF}$?

- a) 14
- b) 20
- c) 26
- d) 30

ver a 20-second time period, Jeff runs in a circle of 8 feet in radius around a pole. Assuming that Jeff starts the time period at a radius of 8 feet from the pole, the graph during this interval of his time (as the x -coordinates) and his stance from the pole (as the y -coordinates) would have a slope of what?

- a) -1
- b) 0
- c) 1
- d) 2

rectangular dog park is x feet wide and y feet long. The park is to be fenced along its sides with wooden material, except for part of one side where there will be a metal gate that is g feet long. How long must the fence material be in order to meet these conditions?

- a) $2x + 2y - g$
- b) $2x + 2y + g$
- c) $xy - g$
- d) xyg

Solutions

c) Calculate the volume of the larger box, and then subtract the volume of the solid cubes to find the volume of empty space. Use $V = lwh$ to calculate the volume of each rectangular prism.

$$\begin{aligned} \text{Volume of larger prism} - \text{Volume of 4 smaller cubes} &\rightarrow \\ (20 \times 30 \times 12) - 4(4 \times 4 \times 4) &= \\ 7,200 - 256 &= 6,944 \end{aligned}$$

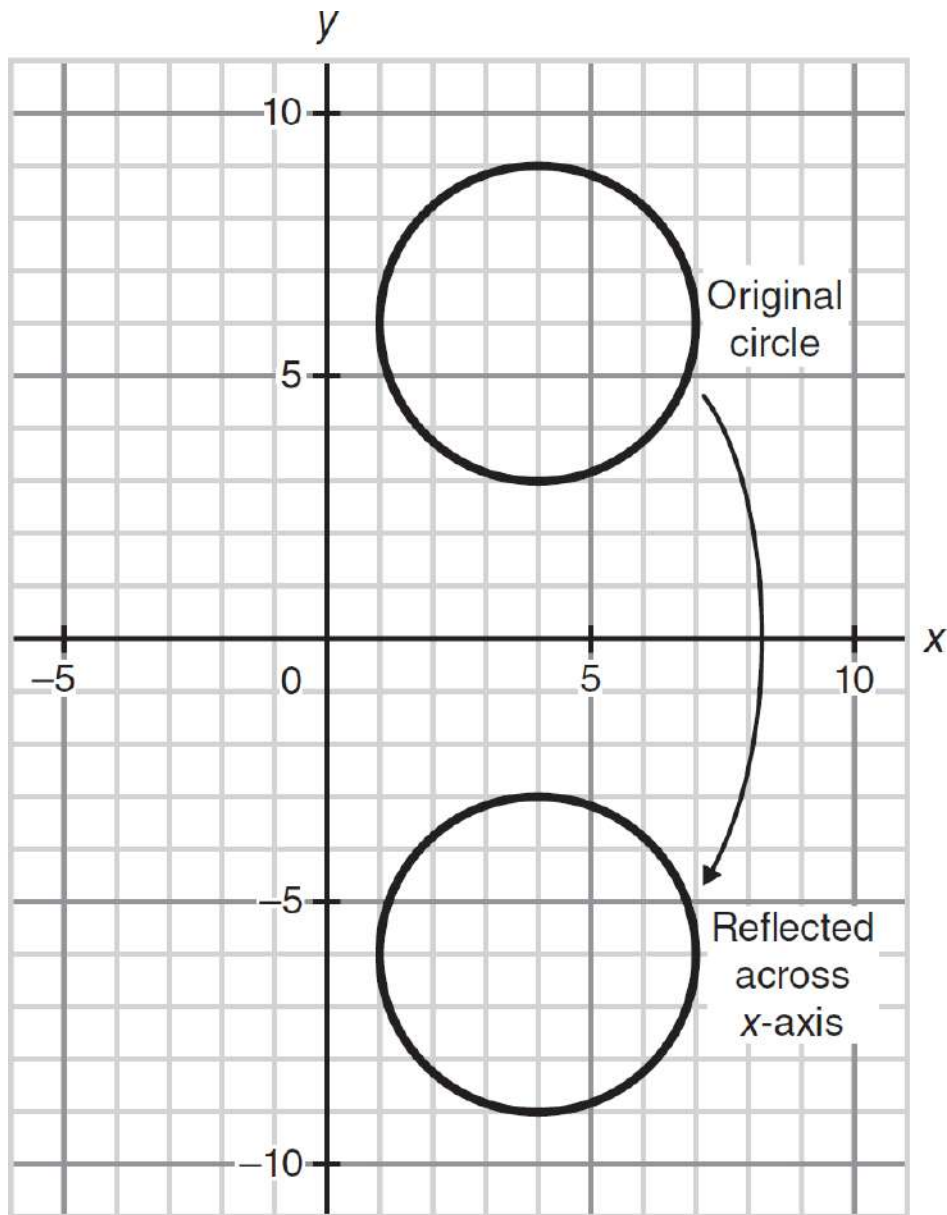
c) Since the triangle has a right angle, the two legs would be the height and the base of the triangle. So, we can consider 8 as the base and 6 as the height. The area formula for a triangle is $\frac{1}{2} \times B \times H$. Plug in 8 and 6 to this formula to get your answer:

$$\frac{1}{2} \times 6 \times 8 = 24$$

k) The angles with measures 30° , 50° , x , and y all add up to form 180° since they are angles within the triangle ACD . Therefore, you can solve for the sum of x and y by subtracting 30 and 50 from 180:

$$180 - 30 - 50 = 100$$

l) Based on the circle equation, in which $(x - h)^2 + (y - k)^2 = r^2$, with (h, k) the center and r as the radius, the original circle will have a center at $(4, 6)$ and radius of 3. To reflect this circle, which is entirely in the first quadrant, across the x -axis, just change the y -coordinate of its center to -6 . The reflected circle will have an equation of $(x - 4)^2 + (y + 6)^2 = 9$ and the reflection is shown below:



c) To find the perimeter of the triangle, we must add up the three side lengths of the triangle. The formula for the area of an equilateral triangle is $= \frac{\sqrt{3}}{4} a^2$, in which a is the side length. So, solve for the side length of the triangle given that we know its area:

$$2\sqrt{3} = \frac{\sqrt{3}}{4} a^2 \rightarrow$$

$$2 = \frac{a^2}{4} \rightarrow$$

$$8 = a^2 \rightarrow$$

$$2\sqrt{2} = a$$

ow, just triple the side length to find the perimeter:

$$3 \times (2\sqrt{2}) = 6\sqrt{2}$$

b) Since X and $(90 - X)$ are complementary, the sine of one will equal the cosine of the other. Therefore, the sine of $(90 - X)$ is simply Y .

d) There are π radians in 180° , so set up a proportion to make a conversion to the total number of degrees in an angle measuring 3π radians:

$$\frac{180}{\pi} = \frac{x}{3\pi} \rightarrow$$

$$\frac{180}{\pi} \times (3\pi) = x \rightarrow$$

$$180 \times 3 = x \rightarrow$$

$$540 = x$$

ow, divide 540 by 45 to see how many angles of 45° would go into 540:

$$\frac{540}{45} = 12$$

c) Set up a proportion to solve for the length of $\overline{DF}$. The cosine of $\angle BAC$ is the same as the cosine of $\angle EDF$ since the triangles are similar to one another. Set the cosine of $\angle BAC$ equal to the cosine of $\angle EDF$, using 24 as the "adjacent" side and the unknown $\overline{DF}$ as the hypotenuse:

$$\frac{12}{13} = \frac{24}{x} \rightarrow$$

$$x = \frac{24 \times 13}{12} = 26$$

b) Since the distance that Jeff is from the center remains 8 over the time period, the slope of the line formed by these values would simply be zero since the line would be horizontal.

a) Find the perimeter of the rectangular dog park by adding the width twice and the length twice:

$$2x + 2y$$

Then, subtract the length of the metal gate from this total to find the total amount of wooden material required:

$$2x + 2y - g$$

Mixed Drill

Which of the following lines is perpendicular to $5y - 2.5x = -10$?

- a) $y = -2x + 8$
- b) $y = 0.5x + 2$
- c) $y = 2x - 7$
- d) $y = 0.4x + 10$

Questions 2–3 use the following table.

Hours of Sleep per Night

	More Than 8	6–8	Less Than 6	Total
Under Age 13	15	8	1	24
Ages 13–18	13	17	14	44
Ages 19–22	18	12	20	50
Total	46	37	35	118

The least possible median age of those surveyed would be which of the following?

- a) 13

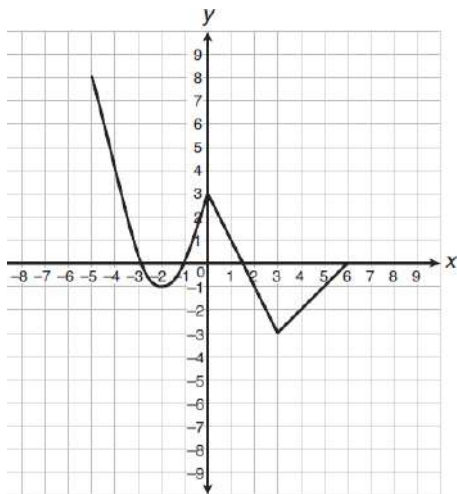
- b) 15
- c) 19
- d) Cannot be determined from the given information

the mean number of hours of sleep of all those surveyed is

- a) 6.5
- b) 7.5
- c) 8.5
- d) Cannot be determined from the given information

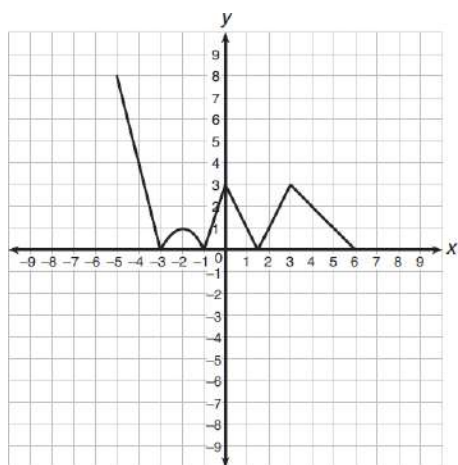
for values of a not equal to zero, $\left(\frac{2}{\sqrt[3]{a}}\right)^6$ equals

- a) $2a-2$
- b) $12a-2$
- c) $64a\frac{1}{2}$
- d) $64a^{-2}$

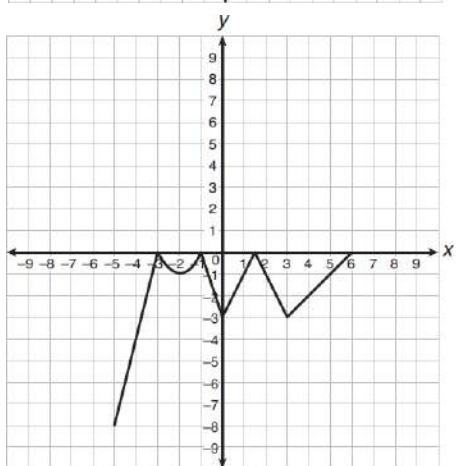


The graph of $y = f(x)$ is shown above. Which of the following graphs best presents the graph of $y = |f(x)|$?

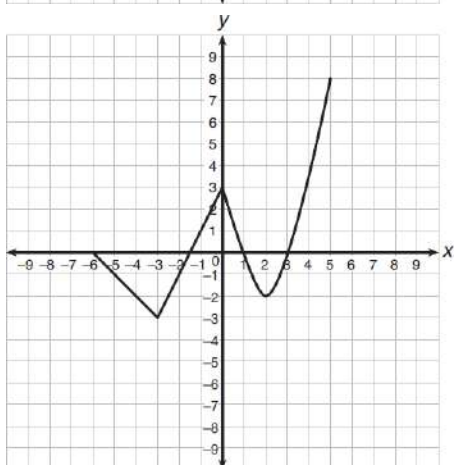
a)



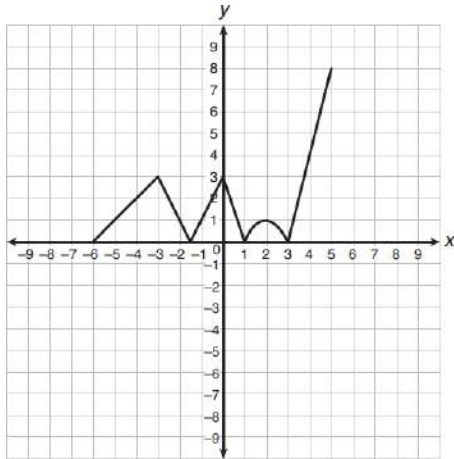
b)



c)



1)



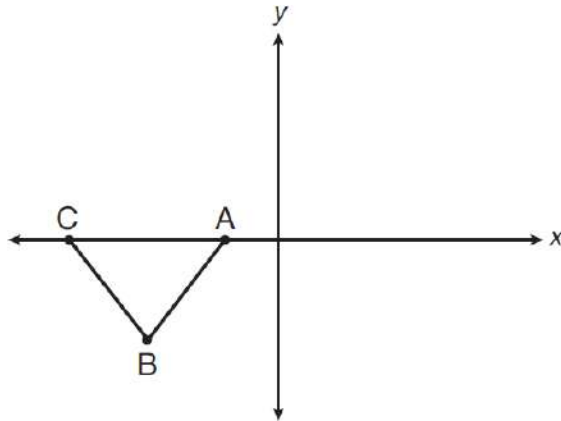
Which of the following is equivalent to $(6x^3 + 3x^2 - 1) + (4x^3 - 4x^2 + 2x + 3)$?

- A) $11x^6 + 2$
- B) $10x^3 - 7x^2 + 2x + 2$
- C) $10x^3 - x^2 + x + 3$
- D) $10x^3 - x^2 + 2x + 2$

A worker makes \$15 per hour when he works 40 hours a week. For each hour exceeding 40, he is paid 50% more than his usual hourly rate. Assuming that a worker has worked at least 40 hours in a particular week, which inequality properly expresses the range of hours, h , he must work to make over \$800 in a week?

- A) $800 < 600 + 22.5 \times (h - 40)$
- B) $800 < 600 + 22.5h$
- C) $800 < 600 + 7.5 \times (h - 40)$
- D) $800 < 40 + 15h$

In the xy -plane below, ABC is an equilateral triangle with sides of length 2. If point A has coordinates $(-2, 0)$, what are the coordinates of point B ?



- a) $(-\sqrt{3}, -3)$
- b) $(-2, -\sqrt{3})$
- c) $(-3, \sqrt{2})$
- d) $(-3, -\sqrt{3})$

A pet store with 30 customers, 60% of the customers like dogs and 11 customers like cats. What is the minimum number of customers who like both cats and dogs?

- a) 0
- b) 1
- c) 4
- d) 7

The function f is defined below. If $f(n)$ and n are both integers, what is the largest value that n can be?

$$f(n) = \frac{2}{\sqrt[4]{n-300}}$$

- a) 0
- b) 301
- c) 316
- d) 426

Solutions

1) We first need to get the given line into slope-intercept form, i.e., $y = mx + b$ so that we can easily see the slope. Begin by adding $2.5x$ to both sides:

$$5y = 2.5x - 10$$

Now divide by 5:

$$y = \frac{1}{2}x - 2$$

The slope of this line is $\frac{1}{2}$. The slope of a line perpendicular to this will have a slope that is the negative reciprocal of $\frac{1}{2}$. The negative reciprocal of $\frac{1}{2}$ is -2 , and only choice (A) has a slope of -2 .

1) There were 118 people surveyed. If you lined up the people by age, the median age would be the average of the 59th and 60th persons. (Prove this to yourself. If you have a series of four terms, the median is between your second and third term. Divide your even number by 2 to get the first term, and add 1 to get the next.)

The first 24 people are younger than 13.

People numbered 25–68 are between ages 13 and 18. Therefore, the 59th and 60th people are both within this age category. If this whole age group were 13 or even if all but 8 of them were), then the median age would be 13. This is the least possible age in this category, so it is the least possible median age.

1) Without knowing everyone's exact response or the average number of hours slept per group, we can't calculate the mean. There's no way to add up the responses without knowing the responses or averages of the responses.

1) Raising a fraction to an exponent raises both the numerator and the denominator to that exponent:

$$\left(\frac{2}{\sqrt[3]{a}}\right)^6 = \frac{2^6}{\sqrt[3]{a^6}}$$

oots can be written as fractional exponents. So the expression can be written as:

$$\frac{64}{a^{\frac{6}{3}}}$$

$\frac{6}{3}$ simplifies to a^2 :

$$\frac{64}{a^{\frac{6}{3}}} = \frac{64}{a^2}$$

n exponent in the denominator of a fraction can be expressed as a negative exponent in the numerator:

$$\frac{64}{a^2} = 64 a^{-2}$$

hoice **(D)** is the answer.

A) If the new function is the absolute value of the old function, the graphs could be the same in all of the places where y is already 0 or positive. This includes the intervals from $-5 \leq x \leq -3$ and from $-1 \leq x \leq 1.5$. The values for when the function is below the x -axis, $-3 < x < -1$ and $1.5 < x < 6$, will simply be their positive counterparts; they will be reflected above the x -axis.

he only graph that shows this relationship is choice **(A)**.

D) There's nothing to distribute, so you can just get rid of the parentheses and combine like terms:

$$6x^3 + 3x^2 - 1 + 4x^3 - 4x^2 + 2x + 3$$

's probably easiest if you start with the highest degree of x and move downward:

here's a $6x^3$ term and a $4x^3$ term. Combine these:

$$10x^3 + 3x^2 - 1 - 4x^2 + 2x + 3$$

ext, there's $3x^2$ and $-4x^2$:

$$10x^3 - x^2 - 1 + 2x + 3$$

here's only one term with an x in it, so move on to the constant terms: -1 and

$$10x^3 - x^2 + 2x + 2$$

his corresponds to choice **(D)**.

1) If Peter wants to make at least \$800, we can come up with an expression for the amount of money that he'll make working h number of hours and set that expression greater than 800:

$$\text{Amount of money Peter makes weekly} > 800$$

The problem states that he makes \$15/hour for his first 40 hours, and we assume that he's already worked 40 hours this week. Thus, Peter will make $15(40) = \$600$ for those 40 hours. This is a fixed constant.

For every hour after his 40th hour, he makes 50% more than his original hourly wage, for a total of 150% of his hourly wage. Thus, every hour Peter works after his 40th hour, he makes $1.5(\$15) = \22.50 .

However, we can't just express this as $22.5h$, because that would imply that Peter makes \$22.50 for every single hour that he works rather than for just the hours past 40. Thus, it must be expressed as $22.5(h - 40)$.

To see that this is true, we can plug 41 in for h to see that for the 41st hour he works, Peter makes an extra:

$$22.5(41 - 40) = 22.5(1) = 22.5$$

This is what we would expect.

Our total expression for the amount of money Peter makes for working h hours, then, is:

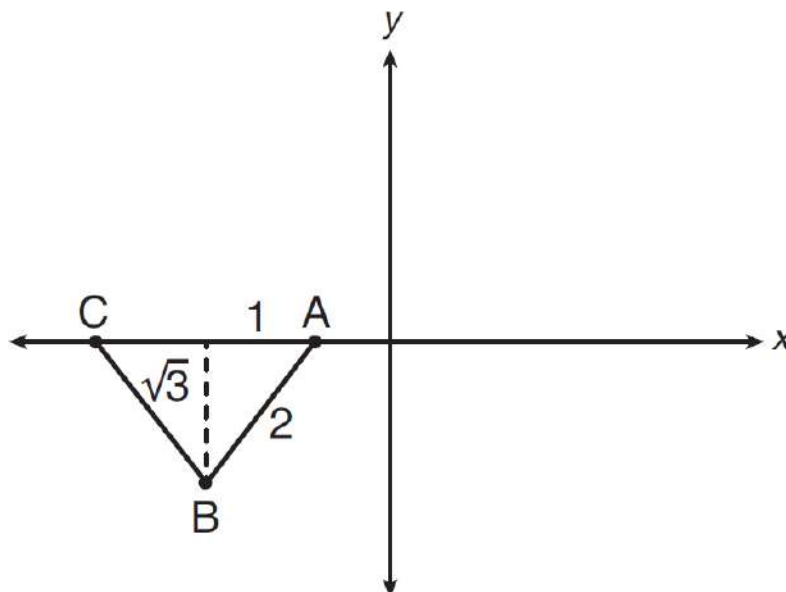
$$600 + 22.5(h - 40)$$

plug this into our original inequality to get:

$$600 + 22.5(h - 40) > 800$$

this is the same as choice (A).

))



We are told that this triangle is equilateral, so each of the three angles is 60° . We are also told that point A has coordinates $(-2, 0)$. Since point C is on the x -axis and is 2 units to the left of point A , point C must have coordinates $(-4, 0)$. By drawing a line that starts halfway between points A and C and bisects the angle at point B , as shown in the drawing, we can split this equilateral triangle into two 30-60-90 triangles. Because the equilateral triangle has side lengths of 2 and the line splits the side along the x -axis in half, the new triangles have base lengths of 1 along the x -axis.

You can see, then, that point B has an x value 1 unit from both point A 's and point C 's x values. Because points A and C have x values of -2 and -4 , respectively, point B will have an x value of -3 .

You may remember that a 30-60-90 triangle is a special right triangle that has opposite side lengths of 1, $\sqrt{3}$, and 2, respectively. Since you've already found the base lengths of 1 and 2, the dotted line will have a length of $\sqrt{3}$. Because it's in

the negative y direction, it will have a y -coordinate of $-\sqrt{3}$. Therefore, the coordinates for point B are $(-3, -\sqrt{3})$.

Alternatively, if you didn't remember the 30-60-90 triangle, you could have used the

Pythagorean theorem with the two known side lengths to find the third length:

$$\begin{aligned}a^2 + b^2 &= c^2 \\(1)^2 + b^2 &= (2)^2 \\1 + b^2 &= 4\end{aligned}$$

Subtract 1 from both sides:

$$b^2 = 3$$

To find b , take the square root of both sides:

$$b = \sqrt{3}$$

So the third side has a length of $\sqrt{3}$.

1) We are told that 60% of the customers like dogs. Since there are 30 customers, $0.6(30) = 18$ customers like dogs. We are also told that 11 people like cats. It is possible that 18 customers like only dogs, 11 people like only cats, and $30 - 18 - 11 = 1$ person likes neither. Thus, the minimum number of customers who necessarily like both is 0, choice (A).

2) If $f(n)$ needs to be an integer and we want a number in the denominator to be as large as possible, that means we're looking for the smallest possible integer for $f(n)$. This integer must be positive. Because the numerator is positive and the denominator is being raised to an even root, the denominator must be positive as well. The smallest possible positive integer is 1, so set $f(n)$ equal to 1. If $f(n)$ is 1, the denominator must be 2 since $\frac{2}{2}$ is the only fraction with a numerator of 2 that equals 1. Thus, set the denominator equal to 2:

$$2 = \sqrt[4]{n - 300}$$

aise both sides to the fourth power:

$$16 = n - 300$$

dd 300 to both sides:

$$n = 316$$

ou can plug this number back in to the original expression to verify.

PART 6

Practice Tests 1 and 2

SAT Test Overview

This SAT Practice Test is made up of a Reading and Writing section and a Math section. On these two non-adaptive practice tests, the second module of each section is more challenging than the first module of each section.

Note: In this book, there is an answer sheet you can use to write down your letter choices and math answers. Feel free to use the sheet to record your answers or simply circle and write down your answers in the test as you go.

Section 1: Reading and Writing (54 Questions)

There are two 27-question modules in the Reading and Writing section.

Section 2: Math (44 Questions)

There are two 22-question modules in the Math section. You can use a calculator throughout the math section.

Modules

The modules in each section are timed separately. You can review your answers in each module before time expires. When the time reaches zero, you will automatically move on to the next section. You are unable to return to a completed module.

Directions

At the beginning of each section, there are directions for answering the questions.

Practice Test 1

Section 1, Module 1: Reading and Writing

32 MINUTES, 27 QUESTIONS

DIRECTIONS ▼

You will be tested on a variety of important reading and writing skills. Each question has one or more passages, possibly including a graph or table. Carefully read each passage and question and choose the best answer to the question based on the passage(s).

Every question in this section is multiple-choice, with four possible answers. Each question has only one best answer.

1

MODULE 1

1

The boss said to _____. The rain made the work impossible, and half the materials hadn't been delivered on time. He was too frustrated to think!

1

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ do what you can
- Ⓑ start the work
- Ⓒ appreciate the sunlight
- Ⓓ call it a day

Emily had _____ about being invited to Kayla's house. The last time they hung out it hadn't gone very well. In fact, Emily had stormed out angrily after Kayla made some rude comments.

2

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ detainments
- Ⓑ territories
- Ⓒ reservations
- Ⓓ anticipation

The basic engineering degrees—mechanical, electrical, chemical, and civil—are consistently ranked in the top ten degrees in terms of _____ after graduation. As such, someone who wants to earn a high salary in a job after graduation may find an engineering degree to be a good fit.

3

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ expenditure
- Ⓑ compensation
- Ⓒ enterprises
- Ⓓ reimbursement

The issue is that car testers just don't use female dummies. A female dummy was first requested by testers in the 1980s and wasn't incorporated into tests by the National Highway Safety and Traffic Administration (NHSTA) until 2003. That female dummy is not adjusted in any way to make up for the difference in shape and composition between men and women—it is just a miniaturized model of the male dummy. In fact, it is so _____ that it only represents the smallest 5 percent of women. In addition, it is only used as a passenger in testing, never as a driver.

4

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ augmented
- Ⓑ built up
- Ⓒ scaled down
- Ⓓ abbreviated

Psychological researchers should be more modest about the certainty of data that involves human behavior. After all, human behavior is more difficult to _____ measure than criteria that other scientific fields assess, like temperature or velocity.

5

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ impulsively
- Ⓑ subjectively
- Ⓒ irrationally
- Ⓓ objectively

Nobody wants to feel easily influenced. Yet, I beg you to hear me out: advertising is everywhere because it works. U.S. companies spend an annual \$70 billion in television ads, and this is before we take a look at other mediums of advertising like radios, magazines, website cookies, and even social media “sponsored” ads.

6

Which choice best describes the function of the underlined sentence in the text as a whole?

- Ⓐ To explain the process by which advertising is created

- Ⓐ To critique corporations for misleading, unethical practices
- Ⓑ To give concrete evidence to illustrate the expanse of advertising
- Ⓒ To demonstrate instances when consumers feel consciously manipulated

The following text is from Walt Whitman's 1891 poem "A Promise to California." The poem was written in the aftermath of the U.S. Civil War when the country was trying to heal its emotional wounds.

A promise to California,
Also to the great Pastoral Plains, and for Oregon:
Sojourning east a while longer, soon I travel toward you, to remain, to
teach robust American love;
For I know very well that I and robust love belong among you, inland,
and along the Western Sea;
For These States tend inland, and toward the Western Sea—and I will
also.

7

Which choice best states the main purpose of the text?

- Ⓐ It outlines how with the skilled narrator's leadership the United States will gain new territory.
- Ⓑ It draws a comparison between how just as the United States has spread, so will the narrator's love spread.
- Ⓒ It compares the geographic and economic features of several U.S. states.
- Ⓓ It sketches a plan for national harmony amidst an ongoing bloody conflict.

8. The following text is adapted from F. Scott Fitzgerald's 1920 novel *This Side of Paradise*. The novel opens with the following character introduction of Fitzgerald's semi-autobiographical protagonist, Amory Blaine.

“Amory.”

“Yes, Beatrice.” (Such a quaint name for his mother; she encouraged it.)

“Dear, don’t think of getting out of bed yet. I’ve always suspected that early rising in early life makes one nervous. Clothilde is having your breakfast brought up.”

“All right.”

“I am feeling very old to-day, Amory,” she would sigh, her face a rare cameo of pathos, her voice exquisitely modulated, her hands as facile as Bernhardt’s. “My nerves are on edge—on edge. We must leave this terrifying place to-morrow and go searching for sunshine.”

8

Amory’s relationship with his mother is best described as

- Ⓐ traditionally pious.
- Ⓑ unusually friendly.
- Ⓒ blatantly disrespectful.
- Ⓓ cold and distant.

An English professor analyzes the 1917 poem titled “The River Merchant’s Life: A Letter” by Li Bai, translated by Ezra Pound. The professor claims that the poem emphasizes the narrator’s desire for reunification with her departed husband: _____

9

Which quotation from the poem most effectively illustrates the claim?

- Ⓐ “While my hair was still cut straight across my forehead / I played about the front gate, pulling flowers.”
- Ⓑ “Lowering my head, I looked at the wall. / Called to, a thousand times, I never looked back.”

- Ⓒ “You went into far Ku-to-en, by the river of swirling eddies, / And you have been gone five months.”
- Ⓓ “If you are coming down through the narrows of the river, / Please let me know beforehand, / And I will come out to meet you.”

Table 1

Crop	Yield (Unit/Acre)	Nitrogen Removal (Pounds/Acre)
Barley	40	30
Corn	50	35
Soybeans	30	103
Wheat	40	42
Corn Silage	15	116
Grass-Legume Hay	2	87

All data is sourced from the University of Delaware, *Nitrogen Removal by Delaware Crops*.

In general, crop rotation is fairly simple. In one year, a nitrogen-fixing plant, like a legume, will be planted. These crops deposit nitrogen into the soil. The following year a nitrogen-depleting crop should be planted and will thrive. This process can be repeated with different crops, all of which will either fix or remove nitrogen. For example, after planting a legume in a field for a growing season, a farmer would be well advised to plant _____.

10

Which choice most effectively uses data from the graph to complete the example?

- Ⓐ barley, because it has a relatively high rate of nitrogen removal
- Ⓑ corn silage, because it has a relatively high rate of nitrogen removal
- Ⓒ corn, because it has a relatively low rate of nitrogen removal

- Ⓓ wheat, because it has a relatively low rate of nitrogen removal

The following is an adaptation of remarks given by Don Evans (the secretary of commerce under George W. Bush) as part of a panel at an economics conference in December of 2004. Note: A tort is a type of lawsuit.

I was in Missouri this last year, and I had a chance to really see up close and personal how [increased costs from healthcare lawsuits are] impacting the health care industry. I talked to a David Carpenter, who is the CEO of North Kansas City Hospital, and what he told me was that there had been 30 doctors that had moved from Missouri to Kansas because Kansas had, indeed, passed tort reform and had put some caps in place. So, you see it happening all across America, where doctors are moving around and trying to find a more friendly environment.

11

Which of the following findings, if true, would best support Evans's overall argument as presented in the text?

- Ⓐ A survey of medical doctors indicated that 85% of them prefer to work close to the community of patients they treat.
- Ⓑ Cities with unlimited caps on business lawsuit payouts had 15% overall greater life satisfaction among residents than cities that limited such payouts.
- Ⓒ States with tort reforms in place saw 20% more businesses relocate to their territories than states without such reforms.
- Ⓓ Medical practices are 30% more likely to move to states with populations in excess of 10,000,000 residents.

“An Encounter” is part of James Joyce’s 1914 short story collection, *Dubliners*. A young, unnamed narrator searches for release from the constraints of daily routine. An English student claims that the narrator’s

willingness to follow the intellectual requirements of his school could best be described as “half-hearted: _____”

12

Which quotation from “An Encounter” most effectively illustrates this claim?

- Ⓐ “Everyone was incredulous when it was reported that he had a vocation for the priesthood. Nevertheless it was true.”
- Ⓑ “This page or this page? This page? Now, Dillon, up! ‘*Hardly had the day*’ . . . Go on! What day? ‘*Hardly had the day dawned*’ . . . Have you studied it? What have you there in your pocket?”
- Ⓒ “I’m surprised at boys like you, educated, reading such stuff. I could understand it if you were . . . National School boys. Now, Dillon, I advise you strongly, get at your work or . . .”
- Ⓓ “But when the restraining influence of the school was at a distance I began to hunger again for wild sensations, for the escape which those chronicles of disorder alone seemed to offer me.”

Many studies suggest that while exercising and having a healthy diet can drastically reduce the concentration of cholesterol in our blood, genes do play a crucial role in an individual’s risk of cardiovascular disease. The strong role genes play in cholesterol processing is seen in bears, especially polar bears. Polar bears are among the most fat-obsessed members of the animal kingdom, yet they do not experience any cardiovascular complications. About half of a polar bear’s weight is fat and its blood cholesterol levels are high enough that they would cause serious cardiovascular disease in humans; even so, _____

13

Which choice most logically completes the text?

- Ⓐ polar bears have a varied diet despite the harsh environment in which they live.

- Ⓐ heart attacks are not a concern for polar bears.
- Ⓑ polar bears are equally susceptible to cardiac issues.
- Ⓒ cardiovascular disease remains a leading cause of death.

As Mendeleev assembled the periodic table, he had noticed several gaps in the pattern of properties which—cleverly—he hypothesized to be areas held by yet undiscovered elements. Apart from reserving space on the table for these elements, he went so far as to predict not only their existence, but their chemical properties as well. Several years later, the spectroscopic discovery of one of these elements—specifically gallium—and the confirmation of Mendeleev’s predictions caused the popularity of his theory to skyrocket, and _____

14

Which choice most logically completes the text?

- Ⓐ his contemporaries allowed Mendeleev a limited degree of credit for his work.
- Ⓑ applications to anatomical and zoological research naturally followed.
- Ⓒ Mendeleev’s scientific curiosity finally found an outlet.
- Ⓓ the periodic table quickly became a standard fixture in the study of chemistry.

Birdfeeders attract birds, but they also attract foragers like squirrels and chipmunks. There are several methods for keeping them away from feeders; a common one is putting the feeder on a tall pole at least five feet away from bushes and trees and adding a _____

15

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ baffle; a cone-shaped device that prevents squirrels from climbing the pole.
- Ⓑ baffle: a cone-shaped device that prevents—squirrels from climbing the pole.
- Ⓒ baffle (a cone-shaped device that prevents squirrels from climbing the pole).
- Ⓓ baffle, a cone-shaped device, that prevents squirrels from climbing the pole.

In a situation where CPR is being administered in a public space, oftentimes no one calls medics because they assume someone else already _____

16

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ has.
- Ⓑ have had.
- Ⓒ have.
- Ⓓ are.

Today, the concept of the high school rock band _____ yet students still have lofty dreams of making a career out of something equally improbable: social media.

17

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ is out of a bit of style,
- Ⓑ is of style out a bit,
- Ⓒ style is a bit out of,

Ⓓ is a bit out of style,

The possibility of becoming a physician assistant or a nurse practitioner allows students to become health care professionals who _____ people without the rigors of medical school and the associated student debt.

18

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ helps
- Ⓑ helping
- Ⓒ helped
- Ⓓ help

Rodents only make up a portion of a _____. Servals will also use their large ears to listen carefully for the flutter of bird wings overhead or the buzz of an insect nearby.

19

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ serval's diet.
- Ⓑ servals diets.
- Ⓒ servals' diet.
- Ⓓ serval's diets'.

Schrödinger posited the issue of when superposition collapses into reality—when multiple occupying states simultaneously _____ one or the other.

20

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ becomes
- Ⓑ become
- Ⓒ have becoming
- Ⓓ had becoming

Even in his short life, Cannon managed to make an enduring difference.

_____ In his wake, Native American art was refreshed, moving away from strict rules that had governed traditional forms and into a new space where artists could be more expressive by using traditional styles and concepts while incorporating modern ideas and inspirations.

21

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ Art historians trace the resurgence and change; in Native American art to a movement that started with Cannon.
- Ⓑ Art historians trace the resurgence, and change in Native American art to a movement that started with Cannon.
- Ⓒ Art historians trace the resurgence and change in Native American art to a movement that started with Cannon.
- Ⓓ Art historians trace the resurgence and change in Native American art—to a movement—that started with Cannon.

Melodies are so loved that people will hum them even when they are trying to concentrate on other things. Beats are so universally loved that babies will dance to them, and hard-of-hearing people will seek out their vibrations. _____ in music. Humans, since the very beginning of our species, have used them to express joy, sadness, heartache, love, and pain.

22

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ These two things (the melodical notes and the beats), come together
- Ⓑ These two things the melodical notes and the beats, come together
- Ⓒ These two things—the melodical notes and the beats—come together
- Ⓓ These two things—the melodical notes and the beats, come together

Technology has transformed how job seekers should present their qualifications to prospective employers. _____ the old paper resume needed to be concise and fit within the scope of one or two pages, your online career profile can be much more thorough.

23

Which choice completes the text with the most logical transition?

- Ⓐ Whereas
- Ⓑ Since
- Ⓒ Because
- Ⓓ Moreover

Working class families will continue to buy only in neighborhoods where they can afford homes, and where home values are unpredictable. _____ they will face further financial instability; meanwhile, the privileged few who can afford to purchase real estate in New York, Chicago, London, and so on, will see their returns peak.

24

Which choice completes the text with the most logical transition?

- Ⓐ Unexpectedly,
- Ⓑ Paradoxically,
- Ⓒ In contrast,
- Ⓓ Hence,

While conducting research, a student took these notes:

- Gyoja monks, also known as Marathon monks, are a Buddhist group in Japan.
- Each monk, over seven years, walks around 27,000 miles, generally at a rate of 25 miles per day for months on end.
- Toward the end of the seven years, the monks will walk up to 52 miles per day.
- The grueling physical demands of completing this task on little food or sleep often proves to be fatal—only around 50 monks have completed the task in the past 135 years.
- This challenge of prayer, meditation, and physical discipline, when completed, leads to the monk being known as a living saint.

25

The student wants to emphasize the physical demands of being a Gyoja monk. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ The Gyoja monks walk up to an incredible 52 miles per day on very little food and sleep.
- Ⓑ Gyoja monks sometimes walk about 25 miles per day, a task that many people might find challenging.
- Ⓒ Those who want to become Gyoja monks must dedicate seven straight years of their lives to this challenge.
- Ⓓ While the spiritual introspection might be psychologically burdensome, the public recognition makes the work worthwhile.

While researching a topic, a student has taken the following notes:

- The type of springs used in the seating surface of furniture impacts comfort and durability.
- Furniture constructed with eight-way hand-tied coil springs can be expected to be extremely durable and very comfortable to sit on.
- Eight-way hand-tied coil springs involve a labor-intensive fabrication process.
- Sinuous springs are more common than eight-way hand-tied coil springs.
- Sinuous springs have a wide range of costs due to flexibility in the quality of the materials used.

26

The student wants to make a generalization about the makeup of typical furniture. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Since most consumers seek to maximize comfort, it is more likely that coil springs would be found in a typical piece of furniture than would sinuous springs.
- Ⓑ Most pieces of furniture are made using a labor-intensive crafting process.
- Ⓒ It is more likely that the springs in an everyday piece of furniture are sinuous than coil.
- Ⓓ Both sinuous and coil springs are found throughout the components of typical furniture, from the backs to the seats.

While researching a topic, a student has taken the following notes:

- Offshore wind farms are an increasingly viable source of renewable energy.

- These wind farms are typically located on the surface of the ocean near coastal cities.
- The average wind speed offshore tends to be higher than on land since there are fewer geographical obstacles to wind gusts.
- Wind farms can often be placed relatively close to major coastal urban areas, thereby reducing the distance needed for electricity transmission.
- While wind farms offshore are promising, increased research should be done to minimize the maintenance costs for these facilities.

27

The student wants to point out a reason why an offshore wind farm would be better than a land-based wind farm. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Since offshore wind farms provide the possibility of renewable energy, builders should utilize them instead of land-based wind farms.
- Ⓑ Given the higher typical wind speeds offshore, offshore wind farms are more sensible than land-based ones.
- Ⓒ The convenience of having the source of electricity closer to major urban areas makes land-based wind farms more promising than offshore ones.
- Ⓓ The higher maintenance costs associated with offshore wind farms make them less viable than land-based ones.

Section 1, Module 2, Reading and Writing

32 MINUTES, 27 QUESTIONS

DIRECTIONS ▼

You will be tested on a variety of important reading and writing skills. Each question has one or more passages, possibly including a graph or table. Carefully read each passage and question and choose the best answer to the question based on the passage(s).

Every question in this section is multiple-choice, with four possible answers. Each question has only one best answer.

1

MODULE 2

1

We rarely pause mid-recipe to consider the chemistry of cooking, but understanding the chemical reactions occurring in our food will help us to become better cooks. Isn't that some food for thought?

1

As used in the text, what does the phrase "some food for thought" most nearly mean?

- Ⓐ An appetizing situation
- Ⓑ A next step for researchers
- Ⓒ A plan for actionable change
- Ⓓ Something worth considering

Despite the wide applicability of the ABC-X model, like any other theory, it has faced critiques of its usefulness. One of the major critiques is its focus

on serious stressor events and inaccessibility to smaller, everyday stressors. Another _____ is its inability to consider how multiple stressors may affect the current stressor situation.

2

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ drawback
- Ⓑ assessment
- Ⓒ trauma
- Ⓓ rumination

Tests should operate as an assessment of what is practiced daily inside the classroom. Classroom teachers who teach to well-written standards _____ that students are prepared for the next grade level.

3

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ assure
- Ⓑ insure
- Ⓒ ensure
- Ⓓ reassure

A folklorist may enter a city or village or subculture and begin to participate in that population's day-to-day life. Possibly, they may end up studying a story, a joke, a dance, a dish, or even a child's game—an apparently trivial practice that, when looked at closely, turns out to be quite _____.

4

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ insignificant
- Ⓑ substantive
- Ⓒ imbecilic
- Ⓓ exuberant

In the 1990s, amidst concern for its effect on wildlife, DDT was largely banned; bedbugs slowly began their comeback. By 2010 their numbers were spiking. Because of a huge increase in personal travel since the last time the bugs were an issue, they spread faster than ever. Laundromats, hotels, and libraries became _____ for transmission. Bug eggs can get tucked into suitcases when traveling, or into books that you check out from the library. In communal living areas (like apartment buildings), they can crawl from one apartment to another as they look for human hosts on which to feed.

5

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ agents
- Ⓑ excursions
- Ⓒ hygienics
- Ⓓ meanderings

Studies show that younger children are much better suited to learn multiple languages at one time, with the effects reaching from elevated attentiveness to enhanced musical abilities. According to the Cornell Language Acquisition Lab, elementary language learners are 70% more likely to reach an intermediate level of communication compared to those who begin in high school. The incorporation of multiple language study at an early age

creates more culturally aware students, but it results in more measurable impacts as well: students who study language before the age of 10—when brain capacity is at its highest—test better in all core subjects, including English, Reading, and Math. Greater cognitive development encourages exceptional creativity and divergent thinking.

6

Which choice best states the main purpose of the text?

- Ⓐ To convince the reader to study a specific world language to become more culturally aware
- Ⓑ To provide an evidence-based argument in favor of early learning of multiple languages
- Ⓒ To suggest a curriculum for graduate-level studies in multiple language acquisition
- Ⓓ To argue that the biological obstacles to learning language at an early age are difficult to overcome

Text 1

Film star Charlie Chaplain was much more than a mere entertainer. In his 1939 film *The Great Dictator*, Chaplin delivers an impassioned speech in which he condemns dictators, prejudice, and xenophobia. Knowing the historical context, Chaplin was clearly making a commentary on Hitler, Mussolini, and fascism. This speech is widely regarded as one of the most powerful performances in film history, which is rather ironic—Chaplin was famous as a silent film star, after all.

Text 2

Stephen Spielberg is a master of the close-up—nearly every Spielberg film features an extended close-up on a wide-eyed, shocked face experiencing awe or terror. Some critics find this technique manipulative, alleging that this technique suggests that the audience should likewise respond with shock or awe at the events onscreen, even if such a response has not been

emotionally “earned” by the scene. But what some call a cheap trick, others call engaging cinema.

7

Based on the texts, what would the “critics” in Text 2 most likely say about Chaplin’s impassioned speech as described in Text 1?

- Ⓐ The emotional response it evokes is well earned.
- Ⓑ It is overly focused on political activism.
- Ⓒ Without a vocal accompaniment, a silent film cannot be poignant.
- Ⓓ Attempts to prompt a visceral reaction to what is on the screen are unwarranted.

The following text is from Willa Cather’s 1918 novel *My Ántonia*.

When Jim was still an obscure young lawyer, struggling to make his way in New York, his career was suddenly advanced by a brilliant marriage. Genevieve Whitney was the only daughter of a distinguished man. Her marriage with young Burden was the subject of sharp comment at the time. It was said she had been brutally jilted by her cousin, Rutland Whitney, and that she married this unknown man from the West out of bravado. She was a restless, headstrong girl, even then, who liked to astonish her friends.

8

Based on the text, the narrator likely believes that Jim married his wife

- Ⓐ out of a love for her intellect.
- Ⓑ because of professional ambition.
- Ⓒ due to his sincere love for her.
- Ⓓ given her sympathy for his poverty.

The following text is adapted from Octave Uzanne’s 1894 work “The End of Books.” In this story, a group of gentlemen gathers after dinner to discuss what they think the future holds. Eventually they come to the topic of the future of books and the following monologue is given by one of the group.

If by books you are to be understood as referring to our innumerable collections of paper, printed, sewed, and bound in a cover announcing the title of the work, I own to you frankly that I do not believe (and the progress of electricity and modern mechanism forbids me to believe) that Gutenberg’s invention can do otherwise than sooner or later fall into desuetude as a means of current interpretation of our mental products.

Printing . . . which has changed the destiny of Europe, and which, especially during the last two centuries, has governed opinion through the book, the pamphlet, and the newspaper—printing, which since 1436 has reigned despotically over the mind of man, is, in my opinion, threatened with death by the various devices for registering sound which have lately been invented, and which little by little will go to perfection.

9

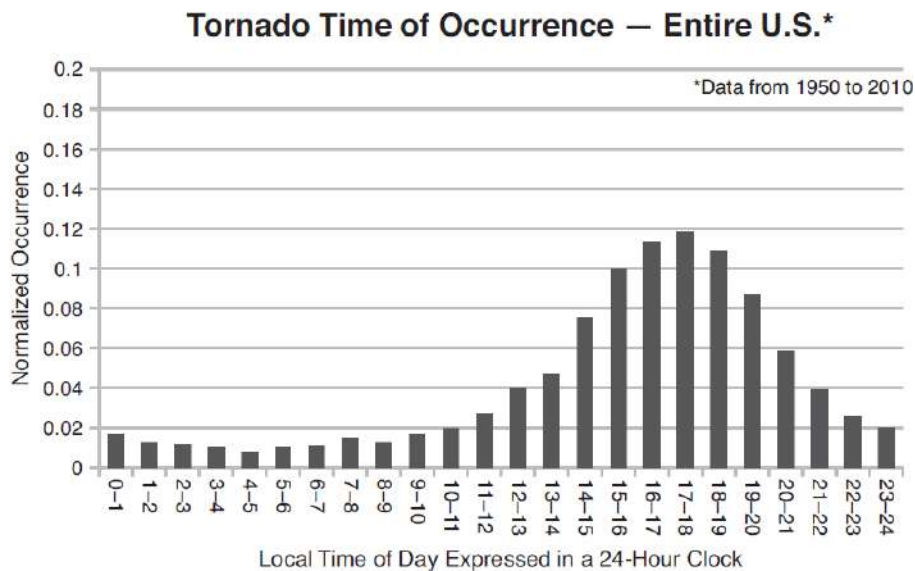
Based on the text, the narrator believes that the impact of printing on history up to the time period when the article was written could be described as

- Ⓐ extremely significant.
- Ⓑ largely irrelevant.
- Ⓒ indisputably oppressive.
- Ⓓ uniformly militaristic.

Author Mary Shelly composed a novel called *The Last Man* in 1826. The narrator of this story suggests that the greatest obstacle to his father’s success is an inability to follow through on his commitments:

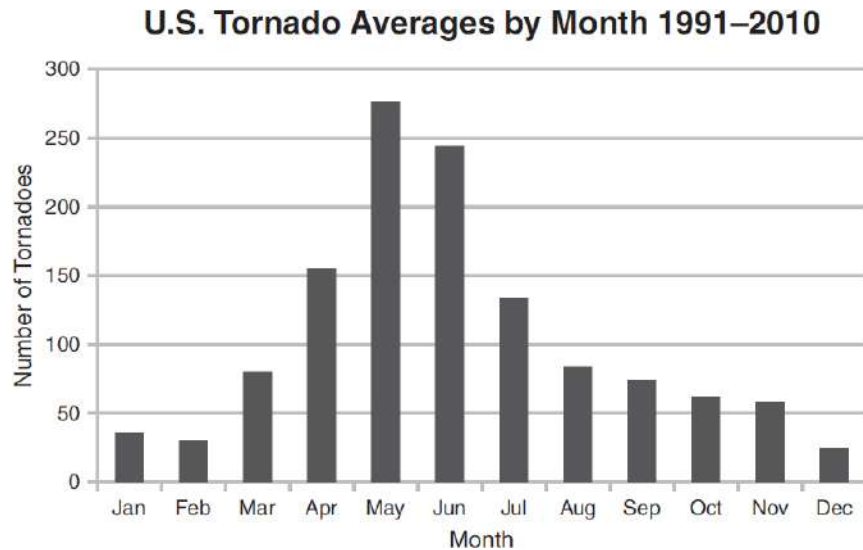
Which quotation from *The Last Man* most effectively illustrates the claim?

- (A) “During the short years of thoughtless youth, he was adored by the high-bred triflers of the day, nor least by the youthful sovereign.”
- (B) “The fiend of gambling, which fully possessed [my father], made his good resolutions transient, his promises vain.”
- (C) “With the quick sensibility peculiar to his temperament, he perceived his power in the brilliant circle to be on the wane.”
- (D) “The king married; and the haughty princess of Austria, who became, as queen of England, the head of fashion, looked with harsh eyes on his defects, and with contempt on the affection her royal husband entertained for him.”



Source: www1.ncdc.noaa.gov/pub/data/cmb/images/tornado/clim/US_nationa_timeofday.png

Figure 1



Source: www1.ncdc.noaa.gov/pub/data/cmb/images/tornado/clim/tornadoes_bymonth.png

Figure 2

Meteorologists in the Northern Hemisphere hypothesize that by observing the most common conditions in which a tornado might form, they can better predict the likelihood of tornado creation. With this knowledge in place, meteorologists can build public awareness about the increased possibility of tornadoes during certain conditions, helping citizens become more aware of potential danger. Given the meteorologists' goal, they would be well-served to inform the public that there is an increased likelihood of tornado formation at _____

11

Which choice most effectively uses data from the table to complete the statement?

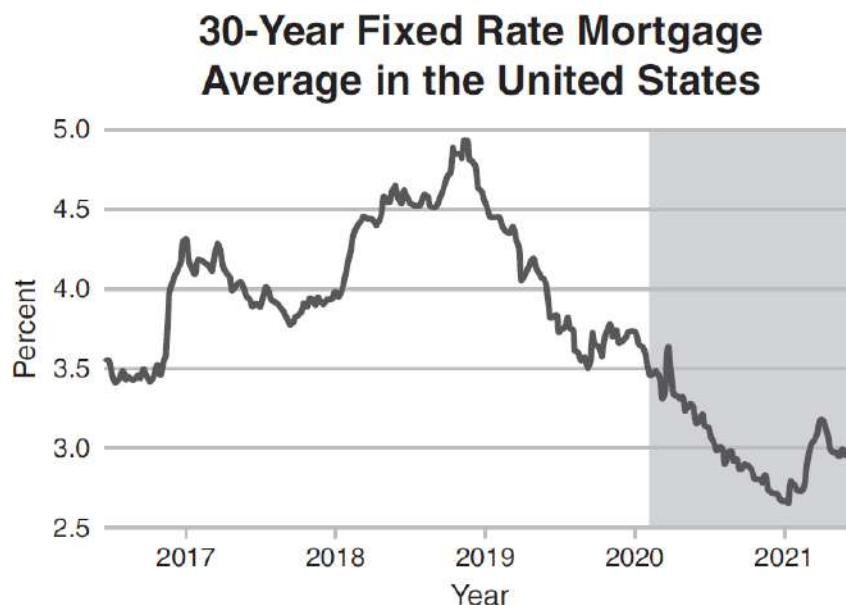
- Ⓐ midday in the summer.
- Ⓑ nighttime in the fall.
- Ⓒ sunset in the springtime.
- Ⓓ sunrise in the winter.

“The Social Contract” is a 1763 treatise of political philosophy by Jean-Jacques Rousseau. During the unsettled time in which the work was written, Rousseau advocated social cohesion. In fact, the central tenet of the social contract is that man must give himself entirely to the state to ensure social order: _____

12

Which quotation from “The Social Contract” most effectively illustrates the claim?

- Ⓐ “Man was born free, but everywhere he is in chains. This man believes that he is the master of others, and still he is more of a slave than they are.”
- Ⓑ “This is the fundamental problem; the social contract offers a solution to it.”
- Ⓒ “And so it follows that each individual immediately recovers his primitive rights and natural liberties whenever any violation of the social contract occurs and thereby loses the contractual freedom for which he renounced them.”
- Ⓓ “The individual member alienates himself totally to the whole community together with all his rights.”



Freddie Mac, 30-Year Fixed Rate Mortgage Average in the United States [MORTGAGE30US], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/MORTGAGE30US>, June 22, 2021.

Economists have observed that during times of economic recession, central banks tend to lower interest rates to help make the cost of having loans less expensive. Since interest payments comprise a major component of any loan, lowering interest rates will help consumers save money. If consumers can save money on interest, they have more money available to purchase goods and services, thereby stimulating the economy and helping the country emerge successfully from a recession. Economists hypothesize that in the relatively recent recession that started in early 2020, interest rates on a variety of loan products would decrease:

13

Which choice most effectively uses data from the chart to support the economists' hypothesis?

- Ⓐ While the U.S. mortgage rate is relatively low by historical standards, it stayed fairly constant during the duration of the recession.
- Ⓑ The interest rate on U.S. 30-year fixed mortgages rose to as high as 3.7% in the aftermath of the recession.
- Ⓒ U.S. 30-year fixed mortgage rates, for example, generally decreased throughout 2020 to a five year low of less than 3 percent.
- Ⓓ Mortgage payments for the average American homebuyer increased in the immediate aftermath of the recent recession.

An area in southern Ohio has been undergoing construction for the past two decades. New apartment complexes are replacing the forests that used to cover that area. Surprisingly, researchers have found that a type of bird, Species X, has been growing in population over that same period. Group 1 hypothesizes that the growth is due to detrimental environmental changes in

another area, causing Species X to move from their preferred area into southern Ohio. Group 2 hypothesizes that the new landscape of southern Ohio is more suitable for Species X, leading more of them to move into the new area.

14

Which finding, if true, would most directly support Group 2's hypothesis?

- Ⓐ Areas in northern Ohio have undergone widespread tree clearing over the past two decades, making it necessary for Species X to find a new habitat.
- Ⓑ Species X preferred habitat is along the edges of tree lines, so with all the fragmentation of land, there are more edges.
- Ⓒ The number of natural predators of Species X in southern Ohio has increased in recent years due to reintroduction of some predators to the wild.
- Ⓓ Over the past two decades, there has been an increase in river-based pollutants throughout the southern Ohio region.

The mite reproductive process causes significant distress to bee colonies since the mites themselves reproduce within a bee colony. The female mite plants herself into the brood cells of the honeycomb where the queen has laid an egg. The cell is then sealed as the egg hatches and the larva grows. During that time, the female mite lays her eggs on the larva so that when the new bee emerges from the cell to join the workforce, it already has mites and is in a weakened state. Since worker bees and beekeepers alike can't see into the capped cell, no one knows how many of the capped cells are infested, and once the cell is uncapped, several more mites can harm the colony. This reproduction method makes the mites very difficult to get rid of as, at any given time, _____

15

Which choice most logically completes the text?

- Ⓐ the queen bee will have the resilience to overcome any infestation that manifests.
- Ⓑ the number of worker bees in a colony exceeds the number of drone bees.
- Ⓒ a beekeeper can utilize thermal imagery to detect the presence of mites in the cells.
- Ⓓ there are more mites breeding in capped cells that mite treatments may not reach.

Originating in 13th-century Japan, haikus were originally used as openings for longer poems called renga poems. But it wasn't until Matsuo Bashō redefined haiku poetry that it _____ what it is today.

16

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ becoming
- Ⓑ becomed
- Ⓒ became
- Ⓓ becomes

When taking out a loan, the fine print is key. It is important to understand the differences between fixed _____ throughout the term of the loan, and adjustable interest rates, which can change over time.

17

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ interest rates, that remain constant
- Ⓑ interest rates, which remain constant
- Ⓒ interest rates which remain constant

- Ⓓ interest rates; remain constant

After the varnish is removed, repair can finally begin. New varnish can be applied and, before it dries, new paint can be mixed into the varnish on damaged areas to replace damaged paint. This method _____ for the old paint to be preserved under the varnish, but for the painting to have a “face lift” and look like new on the surface.

18

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ allowed
Ⓑ would of allowed
Ⓒ will have allowed
Ⓓ allows

For most students, social media is just a part of life. _____ inundated with pictures and videos from “influencers” who appear to live lavish lifestyles and pay for it through sponsorships and ad revenues that come from having a large social media following.

19

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ Regardless of what platform they’re on; they are
Ⓑ Regardless of what platform their on they are
Ⓒ Regardless of what platform they’re on, they are
Ⓓ Regardless, of what platform there on their

The layperson merely associates Newton with gravity and laws of motion, and Einstein with relativity and mass-energy equivalence, failing to

understand the magnitude of _____ contribution.

20

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ either scientist's
- Ⓑ either scientists'
- Ⓒ each scientists
- Ⓓ each scientists'

For the duration of the expedition, Commerson and Baret were aboard two different ships called the *Boudeuse* and the *Étoile*. During their trip, they went to the city of Montevideo, Uruguay; _____ and New Ireland, in the present-day Papua New Guinea.

21

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ the city of Rio de Janeiro, Brazil
- Ⓑ the city of Rio de Janeiro: Brazil
- Ⓒ the city of Rio de Janeiro, Brazil;
- Ⓓ the city, of Rio de Janeiro Brazil,

Sweden's aggressive new policies during the Great Depression came with a price tag, particularly from its massive investment in public works projects. _____ creating a large federal deficit, in 1934 Sweden became the first country to fully emerge from the Depression, and foreign creditors were quickly recompensed.

22

Which choice completes the text with the most logical transition?

- Ⓐ Despite
- Ⓑ For
- Ⓒ By
- Ⓓ With

Humorism held medical discovery back for centuries at a time when the pure sciences were preparing for a renaissance; it established a systemic insularity in the field that cut medicine off from discoveries in biology, chemistry, and physics, and generated a remarkably long-lived illusion of comprehensiveness that categorically rejected revision and innovation. _____ humorism was a far more persistent enemy of medical science than was superstition because it wore the guise of naturalism.

23

Which choice completes the text with the most logical transition?

- Ⓐ Somewhat
- Ⓑ In contrast,
- Ⓒ Further,
- Ⓓ Because of this,

When people understand the idea of time and diversification, they need one final thing to help them be successful in the marketplace: a sense of calm. Panic buying and selling is never a good idea. If an account was started early and properly diversified, _____ severe market downturns should not incite panic. Instead, clear-thinking investors will see that the market will correct; rather than “selling low,” investors wait for the upswing (and maybe longer) in order to see the payout on their investments.

24

Which choice completes the text with the most logical transition?

- Ⓐ thus

- Ⓐ even
- Ⓑ so
- Ⓒ while

While conducting research, a student took these notes:

- In addition to the well-known IQ (Intelligence Quotient), people also have varying levels of EQ (Emotional Quotient), SQ (Social Quotient), and AQ (Adversity Quotient).
- People who develop IQ but not any emotional or social skills may struggle to succeed even if they are incredibly intelligent.
- AQ is a measure of how well people bounce back from adversarial events.
- A person without a degree of AQ may quickly give up in the face of adversity and may see failures as permanent situations instead of learning opportunities.
- Parents should work to ensure that a variety of skills, not just academic prowess, are developed in early childhood.

25

The student wants to introduce the concept of AQ to an audience already familiar with the concept of IQ. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ In contrast to IQ, AQ examines the personal characteristic most important to personal success.
- Ⓑ Unlike IQ, AQ is less focused on intelligence and more on personal grit when faced with adversity.
- Ⓒ While people can succeed without AQ, it is impossible to do so without IQ.
- Ⓓ Between IQ and AQ, AQ is more important to academic achievement.

While researching a topic, a student has taken the following notes:

- There are many high- and low-tech ways to implement sustainable agriculture.
- Stover mulching uses leftover plant material to regenerate the soil and reduce water run-off.
- Genome editing of plants like rice can increase the efficiency with which the plant uses nutrients in the soil.
- Silt traps prevent agricultural run-off from reaching vulnerable waterways.
- Companion planting involves planting mutually beneficial crops closely to increase their success.

26

The student wants to suggest a low-tech way of implementing a sustainable agricultural practice. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Factory manufacturing of silt traps can prevent river pollution.
- Ⓑ When two companion plants are grown near one another, they both benefit.
- Ⓒ Scientists should utilize gene editing to decrease fertilizer waste.
- Ⓓ Stover mulch can potentially be turned into renewable biofuels for transportation.

While researching a topic, a student has taken the following notes:

- There are different ethical theories about how to determine the morality of different actions.
- Utilitarianism considers if the benefits of an action outweigh its costs—if they do, then the action is considered ethical.
- Some summarize the idea behind utilitarianism as “the ends justify the means.”

- Deontology is a competing ethical theory that considers actions to be ethical if they follow universal moral laws.
- Deontologists argue that people should follow ethical rules like “do not kill” or “do not steal,” even if the costs to humanity outweigh the benefits; they believe following duty is more important than a cost-benefit analysis.

27

The student wants to give an example of an action that would be approved by a utilitarian but disapproved by a deontologist. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ A politician lies to secure higher office but implements helpful policies when elected.
- Ⓑ A teacher strongly penalizes cheating, making students less likely to collaborate.
- Ⓒ A mathematician makes a calculator error that results in a bridge collapsing.
- Ⓓ A computer programmer creates a new programming language to enable improved virtual reality.

Section 2, Module 1: Math

35 MINUTES, 22 QUESTIONS

DIRECTIONS ▼

- All expressions and variables use real numbers.
- All figures are drawn to scale.
- Every figure lies in a plane.
- The domain of given functions is the set of all real numbers for which the corresponding value of the function is real.

For **multiple-choice questions**, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer.

For **student-produced response questions**, solve each problem and record your answer following these guidelines:

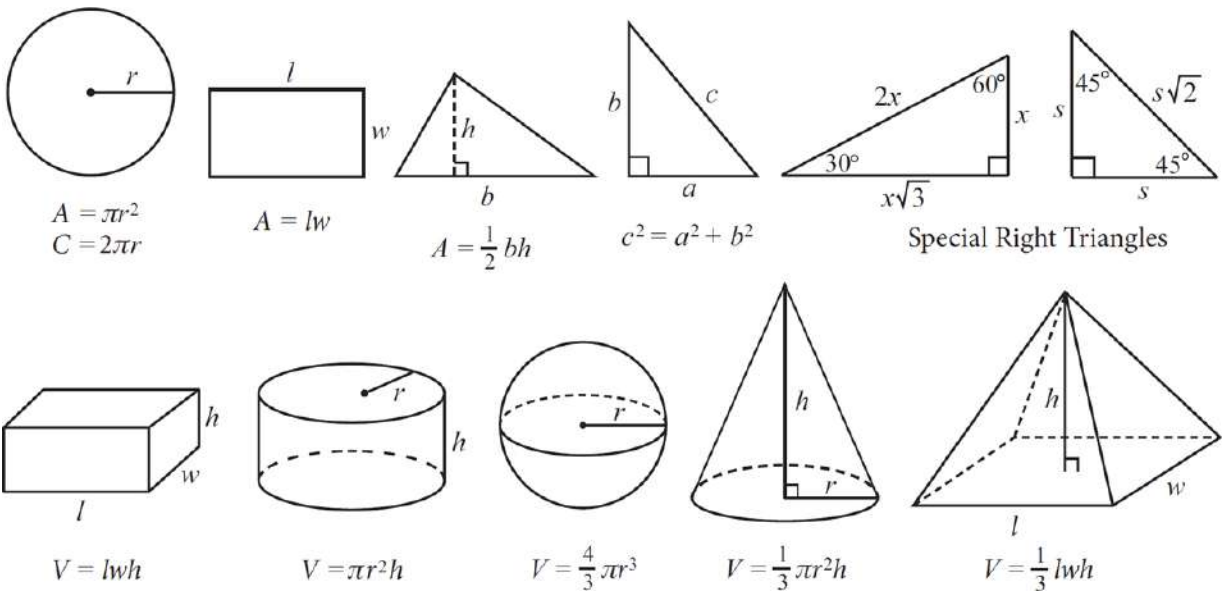
- If you find **more than one correct answer**, enter just one answer.
 - You can enter up to five characters for a **positive** answer and up to six characters (this includes the negative sign) for a **negative** answer.
 - If your answer is a **fraction** that does not fit in the given space, enter the decimal equivalent instead.
 - If your answer is a **decimal** that does not fit in the given space, enter it by stopping at or rounding up at the fourth digit.
 - If your answer is a **mixed number** (like $4\frac{1}{2}$), enter it as an improper fraction ($\frac{9}{2}$) or its decimal equivalent (4.5).
 - Do not enter **symbols** like a comma, dollar sign, or percent sign.
-

Examples

Answer	Acceptable Entries	Unacceptable Entries That Will Receive Zero Credit
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4.5	4.5 4.50 9/2	$4\frac{1}{2}$ 41/2
$\frac{8}{9}$	8/9 .8888 .8889 0.888 0.889	0.88 .88 .89 0.89
$-\frac{1}{9}$	-1/9 -.1111 -0.111	-.11 -0.11

REFERENCE ▾



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

1

If $x = 5$, what is $3(x + 2) - 2x$?

- Ⓐ 5
- Ⓑ 9
- Ⓒ 11
- Ⓓ 13

2

$-3(x^2 - 2x + 4)$ is equivalent to which of the following expressions?

- Ⓐ $-3x^2 - 6x + 12$
- Ⓑ $-3x^2 - 6x - 12$
- Ⓒ $-3x^2 - 6x + 12$
- Ⓓ $-3x^2 + 6x - 12$

3

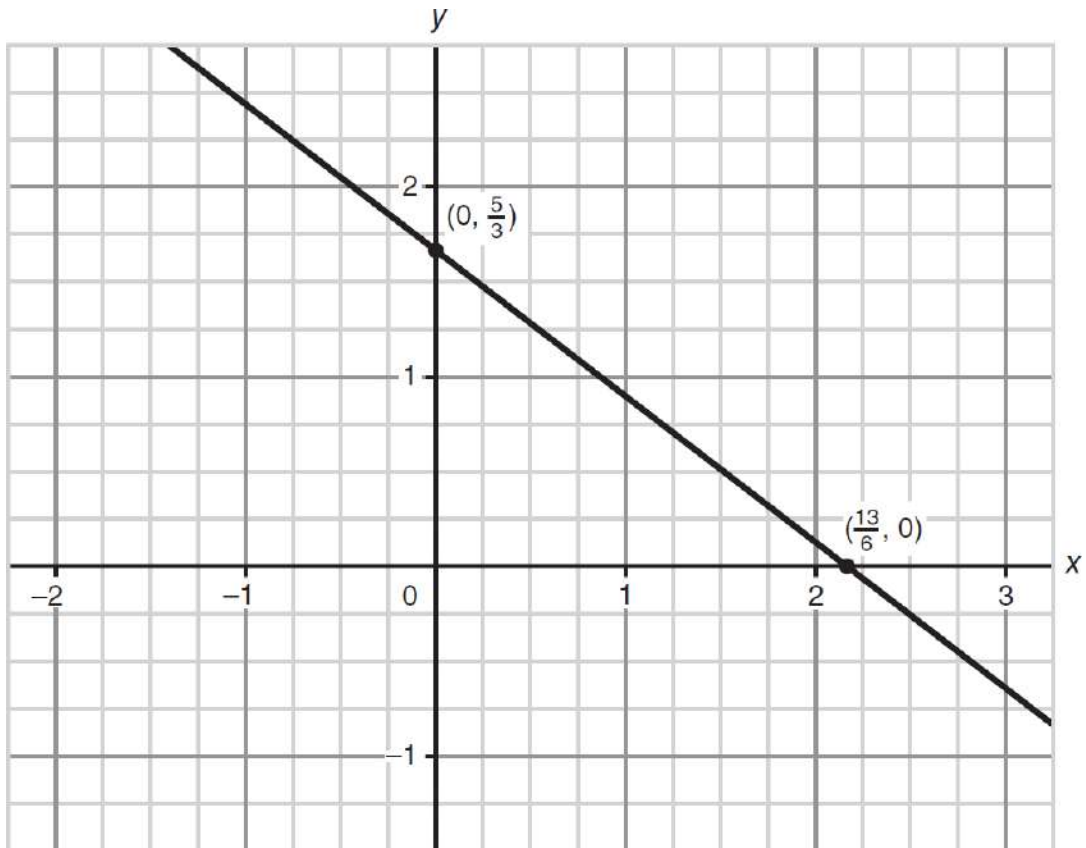
Letter Grade	Percentage of Students with That Grade
A (90–100%)	26
B (80–99%)	33
C (70–79%)	21
D (60–69%)	13
F (<60%)	7

The table above presents the percentage of students in a class with particular letter grades. What letter grade is the median grade for the class?

- Ⓐ A
- Ⓑ B

- Ⓒ C
- Ⓓ D

4



What is the slope of the line in the above graph in the xy -coordinate plane?

- Ⓐ $-\frac{13}{4}$
- Ⓑ $-\frac{10}{13}$
- Ⓒ $-\frac{1}{3}$
- Ⓓ $\frac{7}{4}$

5

What is the product of the solutions to the equation $0 = \left(x + \frac{1}{4}\right)\left(x - \frac{2}{3}\right)$?

- Ⓐ $-\frac{1}{12}$
- Ⓑ $-\frac{1}{6}$
- Ⓒ $\frac{1}{4}$
- Ⓓ $\frac{5}{12}$

6

At a certain college, 27% of all students majored in science or mathematics. If there were a total of 4,000 students at the college, how many of the students would have a science or mathematics major?

- Ⓐ 1,080
- Ⓑ 2,920
- Ⓒ 14,815
- Ⓓ 108,000

7

Mark consistently sleeps for 8 hours per day. Which expression gives the number of hours that Mark would sleep in W weeks?

- Ⓐ $56 \times W$
- Ⓑ $8 \times W$
- Ⓒ $8 + W$
- Ⓓ $56 + W$

8

If $x^3 = \frac{27}{8}$ what is the value of x ?

- Ⓐ $\frac{1}{3}$
- Ⓑ $\frac{3}{4}$
- Ⓒ $\frac{3}{2}$

Ⓓ $\frac{9}{4}$

9

$$B = 2P + 3E$$

A bagel store charges \$2 for each plain bagel, P , and \$3 for each everything bagel, E . The equation above represents the total amount, B , that Samantha paid for a box of bagels. If the box cost a total of \$40 and Samantha purchased 4 everything bagels, how many plain bagels did she purchase?

10

A circle with an original area of 16π units increases in area by 56.25%. What is the radius of the new circle?

- Ⓐ 5 units
- Ⓑ 6 units
- Ⓒ 10 units
- Ⓓ 25 units

11

$$\begin{aligned} 3x - 4y &= -2 \\ x + y &= 11 \end{aligned}$$

Given that (x, y) is a solution to the above system of equations, what is the value of x ?

12

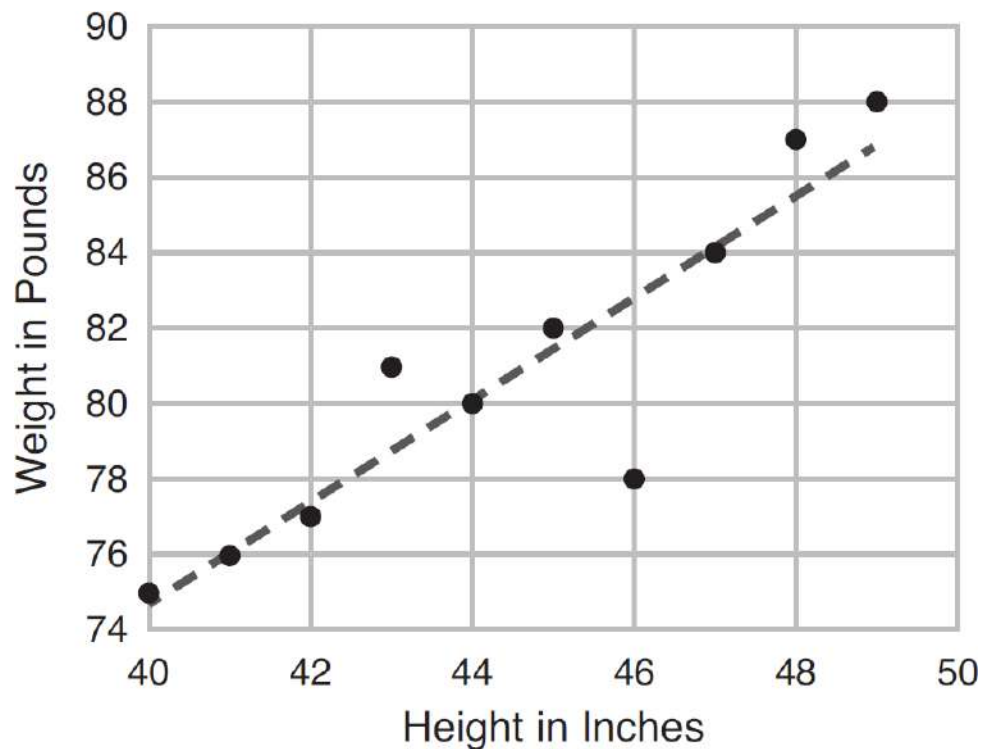
If the city of Detroit has a population of 4,900 persons per square mile, and there are 139 square miles of land within the city, what would most closely approximate the total population of the city?

- (A) 35,000
- (B) 47,000
- (C) 681,000
- (D) 1,240,000

13

If $\frac{x^2 - 4}{3} = 7$ and $x - 2 = 3$, what is the value of $x + 2$?

14



The height in inches and weight in pounds for several female orangutans is graphed in the scatterplot above. Based on the data in the graph, which of these statements best summarizes the overall trend in the data?

- Ⓐ The greater the height of the orangutan, the greater the weight.
- Ⓑ The lower the height of the orangutan, the greater the weight.
- Ⓒ The greater the height of the orangutan, the lower the weight.
- Ⓓ The height and the weight of the orangutan are equivalent throughout the set of data.

15

A cylindrical tank has base with a diameter of 10 feet and a volume of 300π cubic feet. What is the height of this tank in feet?

- Ⓐ 8
- Ⓑ 10
- Ⓒ 12
- Ⓓ 16

16

If the function $f(x) = \frac{x^2 + 3}{2x}$ when $x \neq 0$, what is the value of $f(6)$?

- Ⓐ $-\frac{7}{3}$
- Ⓑ $\frac{2}{9}$
- Ⓒ 1
- Ⓓ $\frac{7}{4}$

17

After a presidential debate, viewers could go to an online poll and vote for which of the two candidates they thought won the debate. Approximately 2,500 viewers chose to fill out responses. Would the results from the online

poll provide an accurate representation of opinions of all likely voters as to whom they thought won the debate?

- Ⓐ Yes, because the sample of viewers was random.
- Ⓑ Yes, because the number of respondents was sufficiently large.
- Ⓒ No, because the respondents were self-selected instead of randomly selected.
- Ⓓ No, because the number of respondents was insufficient.

18

x	y
2	5
6	15
10	25

The relationship between the values in the table above can be expressed as the function $y = kx$, in which k is a constant value. What is the value of k ?

19

In the xy -plane, which of these equations would represent the graph of $y = x^2 - 4$ shifted upward five units?

- Ⓐ $y = x^2 - 5$
- Ⓑ $y = x^2 - 1$
- Ⓒ $y = x^2 + 1$
- Ⓓ $y = 5x^2 - 4$

20

$$y \geq x + n$$

When the ordered pair (2, 5) is a solution for (x, y) in the inequality above, what is the greatest possible value for n ?

- Ⓐ 3
- Ⓑ 5
- Ⓒ 8
- Ⓓ 12

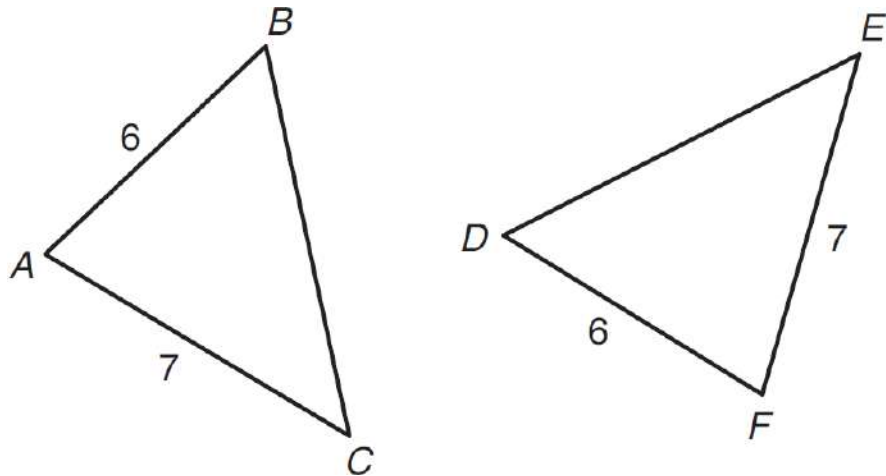
21

	Actually Has Illness	Actually Does Not Have Illness
Positive Test Result	5	20
Negative Test Result	0	800

If a randomly selected patient who participated in the study with the results given above has a positive test result (i.e., the test indicates that someone has an illness), what is the probability that the result is a *false* positive?

- Ⓐ 0.5
- Ⓑ 0.8
- Ⓒ 0.9
- Ⓓ 1.2

22



Triangles ABC and DEF both have sides of length 6 and 7 as shown above. Which of the following would, if true, be sufficient to prove that the triangles are congruent?

- Ⓐ That angle BAC equals angle DEF
- Ⓑ That angle CBA equals angle EDF
- Ⓒ That angle BAC equals angle DFE
- Ⓓ That angle ABC equals angle FED

Section 2, Module 2: Math

35 MINUTES, 22 QUESTIONS

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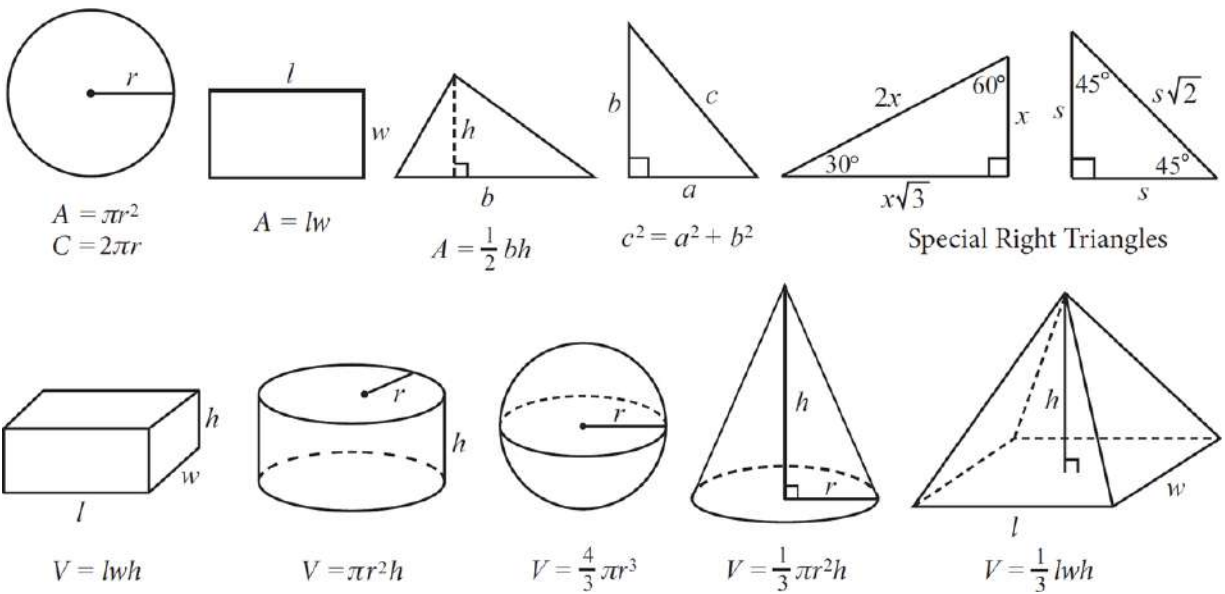
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 - If your answer is a **mixed number** (like $4\frac{1}{2}$), enter it as an improper fraction ($\frac{9}{2}$) or its decimal equivalent (4.5).
 - Do not enter **symbols** like a comma, dollar sign, or percent sign.
-

Examples

Answer	Acceptable Entries	Unacceptable Entries That Will Receive Zero Credit
4.5	4.5 4.50	$4\frac{1}{2}$

	$\frac{9}{2}$	$4\frac{1}{2}$
$\frac{8}{9}$	$\frac{8}{9}$	0.88
	.8888	.88
	.8889	.89
	0.888	0.89
	0.889	
$-\frac{1}{9}$	$-\frac{1}{9}$	-.11
	-.1111	-0.11
	-0.111	

REFERENCE ▼



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

2

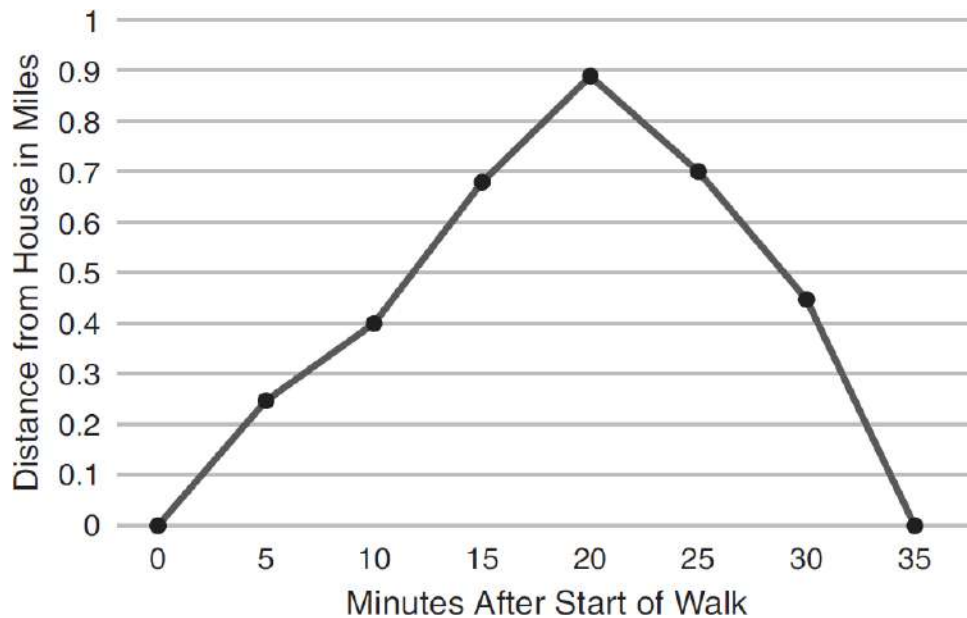
MODULE 2

2

If $5x + 2 = 4(x - 3)$, what is the value of x ?

- (A) -14
- (B) -11
- (C) -7
- (D) 3

2



Tyrese takes a walk around his neighborhood. The distance he is in miles from his house at a particular number of minutes after beginning his walk is represented in the graph above. At what interval of time in his walk did the distance he is from his house decrease most rapidly?

- (A) 0–5 minutes
- (B) 15–20 minutes
- (C) 25–30 minutes
- (D) 30–35 minutes

3

If $\frac{1}{3}(x - 2) = y$ and $x = 5$, what is the value of y ?

4

If a line has a slope of 5 and a y -intercept of k , what is the value of k if the points $(1, 8)$ and $(3, 18)$ are in the line?

- Ⓐ 3
- Ⓑ 5
- Ⓒ 9
- Ⓓ 24

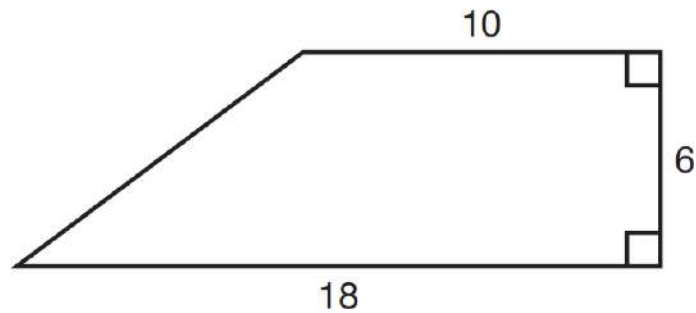
5

$$\frac{x^2 - 9}{x - 3}$$

Which of the following is equivalent to the above expression, in which $x \neq 3$?

- Ⓐ $x^2 - 2$
- Ⓑ $x^3 - 1$
- Ⓒ $x - 9$
- Ⓓ $x + 3$

6



How many units is the perimeter of the above trapezoid?

7

A fruit stand charges a different amount depending on the number of pieces of fruit in a box:

Number of Pieces of Fruit in the Box	Cost of Box
6	\$13.40
12	\$25.40
20	\$41.40
32	\$65.40

What function could be used to determine the total cost, C , of a box that has x pieces of fruit?

- Ⓐ $C(x) = -x + 2.40$
- Ⓑ $C(x) = 2x + 1.40$
- Ⓒ $C(x) = 1.4x + 3$
- Ⓓ $C(x) = 2.8x + 1$

8

Kinetic energy, K , is calculated using the following formula, in which m = mass, and v = velocity.

$$K = \frac{1}{2}mv^2$$

What is the velocity in terms of the other variables?

- Ⓐ $v = 2Km$
- Ⓑ $v = \sqrt{\frac{2K}{m}}$
- Ⓒ $v = \sqrt{\frac{2m}{K}}$
- Ⓓ $v = 2m^2\sqrt{K}$

9

A data set of 15 different numbers has a median of 25 and a mean of 30. If the smallest member of the set has 5 subtracted from it and the largest member has 20 added to it, while all the other elements remain the same, which of these is a correct statement?

- Ⓐ The standard deviation of the set changes more than the range of the set
- Ⓑ The range of the set changes more than the standard deviation of the set.
- Ⓒ The range of the set remains the same.
- Ⓓ The standard deviation of the set remains the same.

10

Consider the sum of the expressions $-x^2 + 4x - 3$ and $5x^2 - 2x + 8$. If the sum is written in the form $gx^2 + hx + k$, where g , h , and k are constants, what is the value of k ?

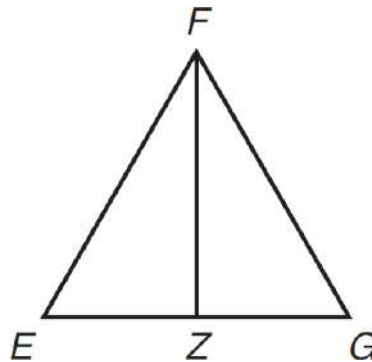
- Ⓐ 1

- Ⓐ 3
- Ⓑ 4
- Ⓒ 5

11

In the xy -plane, a line passes through the points $(5, 0)$ and $(0, 3)$. Another line is perpendicular to this line. What would be this perpendicular line's slope?

12



Triangle EFG is equilateral. If $\overline{EF}$ is 10 units long, and $\angle EFZ$ is 30° , what is the length of $\overline{FZ}$?

- Ⓐ $5\sqrt{3}$
- Ⓑ $5\sqrt{2}$
- Ⓒ 10
- Ⓓ $10\sqrt{2}$

13

A particular multivitamin provides 300 mg of calcium, giving 23% of the recommended daily value of calcium for an adult. If another multivitamin provides 210 mg of calcium, approximately what percentage of a recommended daily value of calcium for an adult would it provide, to the nearest whole percent?

- Ⓐ 8%
- Ⓑ 16%
- Ⓒ 20%
- Ⓓ 44%

14

The measure of an angle is $\frac{2}{3}\pi$ radians. How many angles of this measure would be equivalent to 360 degrees?

15

For the system of inequalities $y \leq x + 2$ and $y \leq -x + 4$, when y is at its greatest possible value, what is the corresponding value of x ?

- Ⓐ -6
- Ⓑ -2
- Ⓒ 1
- Ⓓ 4

16

x	y
1	2
2	$\frac{1}{2}$
3	$\frac{2}{9}$
4	$\frac{1}{8}$
5	$\frac{2}{25}$

Which function expresses the relationship between x and y ?

- Ⓐ $y = \frac{2}{x^2}$
- Ⓑ $y = \frac{4}{x^2}$
- Ⓒ $y = \frac{1}{2}x^2$
- Ⓓ $y = \frac{1}{6}x^3$

17

What is the result when $4x^2 - 3x + 2$ is divided by $x - 3$?

- Ⓐ $4x - 9$
- Ⓑ $12x^2 + 9$
- Ⓒ $2x - 3 + \frac{1}{(x - 3)}$
- Ⓓ $4x + 9 + \frac{29}{(x - 3)}$

18

$$y = x(x - 4)^2(x + 3)^2$$

In the above equation, how many unique x -intercepts does the graph of the function have?

- Ⓐ 0

- Ⓐ 1
- Ⓑ 3
- Ⓒ 5

19

	Bus	Walk	Total
Elementary	a	b	c
Middle	d	e	f
Total	80	70	150

Consider the information in the table above with the variables representing numerical values. There are twice as many elementary school students as there are middle school students, and 60% of elementary school students take the bus. How many middle school students walk to school?

- Ⓐ 20
- Ⓑ 30
- Ⓒ 40
- Ⓓ 60

20

What values of x would make the product of $\frac{2}{x}$ and $\frac{3}{(x-1)}$ undefined?

- Ⓐ 0 only
- Ⓑ -1 only
- Ⓒ 0 and 1 only
- Ⓓ -1 and 0 only

21

For a parabola with the equation $y = 7(x - 5)(x + 2)$, what is the distance between the x intercepts of the parabola?

22

If $(x + 5)(x + 3) = ax^2 + bx + c$, in which a , b , and c are all constants, what is the value of c ?

ANSWER KEY

Practice Test 1

Section 1, Module 1: Reading and Writing

1. D
2. C
3. B
4. C
5. D
6. C
7. B
8. B
9. D
10. B
11. C
12. D
13. B
14. D
15. C
16. A
17. D
18. D
19. A

20. B
21. C
22. C
23. A
24. D
25. A
26. C
27. B

Section 1, Module 2: Reading and Writing

1. D
2. A
3. C
4. B
5. A
6. B
7. A
8. B
9. A
10. B
11. C
12. D
13. C
14. B
15. D
16. C
17. B
18. D
19. C

- 20. A
- 21. C
- 22. A
- 23. C
- 24. B
- 25. B
- 26. B
- 27. A

Section 2, Module 1: Math

- 1. C
- 2. D
- 3. B
- 4. B
- 5. B
- 6. A
- 7. A
- 8. C
- 9. 14
- 10. A
- 11. 6
- 12. C
- 13. 7
- 14. A
- 15. C
- 16. D
- 17. C
- 18. 2.5 or $\frac{5}{2}$
- 19. C

20. A

21. B

22. C

Section 2, Module 2: Math

1. A

2. D

3. 1

4. A

5. D

6. 44

7. B

8. B

9. B

10. D

11. 1.666, 1.667, or $\frac{5}{3}$

12. A

13. B

14. 3

15. C

16. A

17. D

18. C

19. B

20. C

21. 7

22. 15

Digital SAT Scoring Chart

This will give you an approximation of the score you would earn on the Digital SAT.¹ Tally the number of correct answers from the Reading and Writing section (out of 54) and the Math section (out of 44). Take the total for each of these and find the corresponding section score in the tables below.

Number of Correct Reading and Writing Questions (Out of 54)	Reading and Writing Test Score (Out of 800)
0	200
1	210
2	220
3	230
4	240
5	250
6	260
7	270
8	280
9	290
10	300
11	310
12	330
13	340
14	350
15	360
16	380
17	390
18	400
19	410
20	430

Number of Correct Reading and Writing Questions (Out of 54)	Reading and Writing Test Score (Out of 800)
21	440
22	460
23	470
24	490
25	500
26	510
27	520
28	530
29	540
30	550
31	560
32	570
33	580
34	590
35	600
36	610
37	620
38	630
39	640
40	650
41	660
42	670
43	680
44	690
45	700
46	710
47	720
48	730
49	740

Number of Correct Reading and Writing Questions (Out of 54)	Reading and Writing Test Score (Out of 800)
50	750
51	760
52	770
53	790
54	800

¹ Keep in mind that some of the questions on an actual SAT test will be research questions that will not count towards your actual score. For the sake of simplicity, we are including possible research questions in your calculation.

Number of Correct Math Questions (Out of 44)	Math Section Score (Out of 800)
0	200
1	220
2	230
3	240
4	250
5	280
6	300
7	320
8	340
9	350
10	360
11	380
12	390
13	400
14	410
15	430
16	440

Number of Correct Math Questions (Out of 44)	Math Section Score (Out of 800)
17	450
18	460
19	480
20	490
21	500
22	510
23	520
24	530
25	540
26	550
27	560
28	570
29	580
30	590
31	600
32	610
33	620
34	630
35	650
36	660
37	680
38	690
39	700
40	720
41	750
42	770
43	790
44	800

Add the Reading and Writing section score and the Math section score to find your total SAT test score:

_____ Reading and Writing Section Score +

_____ Math Section Score =

_____ **Total SAT Test Score (between 400 and 1600)**

Approximate your testing percentiles (1st to 99th) using this chart:²

Total Score	Section Score	Total Percentile	Reading and Writing Percentile	Math Percentile
1600	800	99+	99+	99
1500	750	98	98	96
1400	700	94	94	91
1300	650	86	86	84
1200	600	74	73	75
1100	550	59	57	61
1000	500	41	40	42
900	450	25	24	27
800	400	11	11	15
700	350	3	3	5
600	300	1	1	1
500	250	1	1	1
400	200	1	1	1

² Scoring Data based on information at collegeboard.org

Answer Explanations

Section 1, Module 1: Reading and Writing

1. **(D)** “To call it a day” means to give up working. The boss in this text acknowledges that work is too difficult under the conditions, so it would be logical for him to suggest that everyone give up working. It is not choice (A) or (B) because continuing the work under these circumstances would not be advisable. It is not (C) because the focus is on work not pleasure.
2. **(C)** In this context, “reservations” means “doubts.” This would be logical given that the two people had not gotten along very well the last time they hung out. It is not choice (A) because to detain someone is to hold them as a prisoner, which is not logical here. Choice (B) does not make sense because someone would not have land or “territories” when thinking about an invitation. It is not (D) because this is the opposite of the needed meaning.
3. **(B)** While all of the choices are in some way related to money or business, only “compensation” makes sense in terms of referring to the likely salaries that engineers could receive. “Expenditures” refers to general spending, “enterprises” refers to business activities, and “reimbursement” refers to receiving a refund for one’s paid expenses.
4. **(C)** Later in the sentence with the underlined portion, the narrator states that the crash test dummy only represents the “smallest 5 percent of women,” indicating that it is a smaller size. To say something is “scaled down” means that it is made to be smaller, making choice (C) the most logical option. It is not choice (A) or (B) because these would show that the model is increased in size. It is not (D) because this word means to shorten a non-physical object, like time or a word.
5. **(D)** The sentence is focused on scientific measurement, so “objectively” would be most appropriate. The other options would not

be used to describe high-quality scientific measurement, but would instead apply to human emotions.

6. **(C)** This sentence provides a statistic and other context intended to show that advertising is pervasive, so choice (C) is correct. The other options don't consider the context prior to the sentence in which the author tries to persuade the reader that advertising is effective despite common opinion that one is immune to influence.
7. **(B)** The final line makes the comparison made throughout the poem most clear in that just as the United States now spans toward the Western Sea, so will the narrator's love. It is not choice (A) because the narrator does not indicate that he intends to lead the country. It is not (C) because the economic features of the states are not compared. It is not (D) because no direct allusion is made to the need for national harmony and a bloody conflict.
8. **(B)** Amory's relationship with his mother can be best seen through their dialogue. Amory's use of his mother's first name indicates that their affiliation is familiar, while her anxiety over him rising too early shows that it is a kindly relationship. So, choice (B) works best here, in which "friendly" refers to "approachable and informal." Using his mother's first name is far from traditional, as in choice (A). Since the mother finds it agreeable, there is no evidence that Amory is disrespectful as in (C). Choice (D) can be tempting because Amory not only uses Beatrice's first name in addressing her, but the narrator also tells us that Amory holds no illusions about his vain mother. However, "cold" and "distant" are too negative.
9. **(D)** This option most directly shows that the narrator wishes to reunify with her departed husband, since she states that if he is coming, he should be sure to let her know ahead of time so that she can meet him. Choice (A) focuses on the narrator's appearance and outdoor actions. Choice (B) would show that the narrator was not especially eager for a reunification. Choice (C) does not express emotion about missing the husband, but just states information about his journey.

10. **(B)** The final sentence of the text is providing an example to elaborate on the claim made in the previous sentence, which is that crops can aid the crop rotation process by either fixing or removing nitrogen. Since the text indicates that a legume is a nitrogen-fixing plant, the example would be completed by presenting a crop that would remove nitrogen from the soil—choice (B) is the best option since corn silage has the highest rate of nitrogen removal of any of the crops in the table. It is not choice (A) because barley has the lowest rate of nitrogen removal. It is not choice (C) or (D) since these incorrectly suggest that something with a low rate of nitrogen removal would be needed.
11. **(C)** The author makes an anecdotal argument in favor of lawsuit reform in this paragraph—the argument would be bolstered by using concrete statistics in support. Choice (C) would provide direct statistical support to make the case that implementing tort reform would result in businesses relocating to more tort-friendly locations. It is not choice (A) because this does not connect to tort reform. It is not choice (B) because this would undermine the author’s argument, showing that having a greater possibility of lawsuit payouts would lead to greater social happiness. It is not choice (D) because this gives a reason unrelated to tort reform for business relocation.
12. **(D)** In these lines, our narrator voices his discontent with the requirements of school and his attention toward other “wild sensations,” supporting the idea that his interest toward his studies is only half-hearted. Choices (A), (B), and (C) are incorrect because they do not provide evidence of the character’s attitude toward following and participating in school requirements.
13. **(B)** The final sentence of the text starts by giving information that would seem to indicate that polar bears may be at risk for heart problems given their high bodily fat content and cholesterol levels. The transition “even so” shows that what follows in the sentence should contrast with what comes before. Choice (B) is the most logical option since it states that polar bears are in fact not at risk for heart attacks. Choice (A) does not provide a contrast, and choices (C)

and (D) would be in agreement with the first part of the sentence instead of showing a contrast with it.

14. **(D)** Since the text states that Mendeleev's predictions cause the popularity of his theory to skyrocket, it makes sense that the periodic table would become extremely popular, making choice (D) the most logical option. It is not choice (A) because it would not make sense to state that Mendeleev received only a limited degree of credit because his theory skyrocketed in popularity. It is not choice (B) because these fields would not directly relate to chemistry. It is not choice (C) because Mendeleev had outlets in chemistry years before the periodic table became quite popular.
15. **(C)** Choice (C) properly uses parentheses to set aside the parenthetical phrase that describes what a baffle is. Choice (A) does not have an independent clause after the semicolon. Choice (B) incorrectly inserts a dash, which provides an unnecessary interruption. Choice (D) is too choppy with an unnecessary comma after "device."
16. **(A)** "Someone else" is a singular subject that agrees with the singular verb "has." All the other verbs are plural.
17. **(D)** Choice (D) puts the words in the most logical order, starting with a verb and then using the idiom "a bit out of style." The other options have illogical word order.
18. **(D)** "Help" would match both the plural subject of "professionals" and the present tense of the sentence as indicated by the verb "allow." Choice (A) is a singular verb, and (B) and (C) are not present tense plural verbs.
19. **(A)** Choice (A) uses the apostrophe to show that the singular serval possesses its diet. "Serval" must be singular since the word "a" precedes it. Choice (B) does not show possession, and (C) and (D) inappropriately use plural apostrophes on servals' and diets' respectively.
20. **(B)** The verb refers to the plural "states" and so requires a plural verb—"become." Also, the tense of the other verbs in the sentence is

present, so “become” makes sense because it is also in the present tense. Choice (A) is singular, and (C) and (D) use incorrect verb tense.

21. **(C)** No punctuation is needed to break up this sentence. Choice (A) is incorrect since there is not a complete sentence after the semicolon. Choice (B) incorrectly breaks up a two-part list with a comma. Choice (D) incorrectly inserts a parenthetical phrase.
22. **(C)** Choice (C) appropriately uses dashes to set aside the parenthetical phrase that clarifies what the two things are. Choice (A) has an unnecessary comma after the parentheses—the parentheses would be sufficient by themselves to set this phrase off to the side. Choice (B) does not have a comma before the parenthetical phrase, and (D) uses inconsistent punctuation around the parenthetical phrase.
23. **(A)** “Whereas” is the only option that provides a contrast within the sentence between the ideas that a resume should be concise and that an online profile can be more thorough.
24. **(D)** “Hence” is the only option to indicate a cause-and-effect relationship between the fact that the housing market is challenging with its cost and unpredictability and the fact that as a result, many working-class families will face further financial instability.
25. **(A)** Choice (A) best emphasizes the physical demands of being a Gyoja monk since it states that they walk an incredible 52 miles a day with very little food and sleep. While choice (B) also refers to the physical demands, 25 miles a day is not as rigorous as 52 miles a day. Choices (C) and (D) do not focus on the physical demands of becoming Gyoja monks, instead focusing on the time and spiritual commitment involved.
26. **(C)** Choice (C) succeeds in making a generalization about the makeup of typical furniture since it states that the springs in the everyday furniture are more likely to be sinuous than coil, which is consistent with the notes. It is not choice (A) because this is the opposite of what the notes indicate. It is not choice (B) because the labor-intensive process only applies to coil springs as described in the notes. It is not

choice (D) because the notes indicate that sinuous springs are more common, not that the springs are equally likely to be found.

27. **(B)** According to the notes, the wind speeds offshore “tend to be higher than on land,” making choice (B) a logical reason why having an offshore wind farm could be better than a land-based one. It is not choice (A) because both offshore and land-based wind farms would provide renewable energy. It is not choice (C) because this is an argument in favor of land-based wind farms. It is not choice (D) because it points out a cost drawback of offshore wind farms.

Section 1, Module 2: Reading and Writing

1. **(D)** The author uses an idiom in an ironic way, stating that despite a lack of common understanding, chemistry is extremely applicable to cooking, implying that this is something to think about. Choice (A) incorrectly understands it as stimulating one’s appetite. Choices (B) and (C) inaccurately assume action rather than contemplation.
2. **(A)** “Drawback” is another way of stating that something is a shortcoming or problem. The narrator is discussing the problems with the ABC-X model, so “drawback” would make sense to introduce a sentence that describes another problem with this theory. It is not “assessment” or “rumination” as the narrator already has a negative opinion on the theory. It is not “trauma” because this is too negative.
3. **(C)** To “assure” is to remove doubt, and it is generally done to another person. I assured my father that I would be home by curfew. To “ensure” means to make sure a thing will or won’t happen. I studied last night to ensure that I wouldn’t fail the test. To “insure” refers to car insurance, life insurance, health insurance, etc. To “reassure” is simply to “assure” someone multiple times. In this sentence, the classrooms are ensuring that something will happen—students will be prepared for the next grade. Choice (C) is the correct answer.
4. **(B)** The narrator is drawing a contrast between something that may appear to be “trivial” while in actuality turning out to be significant—therefore, “substantive” makes the most sense because it means that

whatever is being evaluated has substance. Choices (A) and (C) are overly negative, while “exuberant” does not draw a logical contrast with “trivial.”

5. **(A)** The text asserts that certain things help cause the spread of bedbugs, making “agent” an appropriate word since in one of the less common definitions of “agent,” it can mean something that helps bring about a specific effect. It is not “excursions” or “meanderings” since these buildings do not travel anywhere. It is not “hygienics” because the buildings are not sanitary.
6. **(B)** The text begins by stating that since young children are better suited to learning multiple languages, they can benefit from a wide range of positive effects. The text goes on to highlight some of these effects, such as increased cultural awareness and overall improvement in academic skill. Thus, choice (B) makes the most sense because the text provides evidence in favor of early learning of multiple languages. It is not choice (A) because the narrator does not argue that a specific language should be learned; rather, the narrator argues that multiple language learning in general is beneficial. It is not (C) because no specific curriculum is suggested. It is not (D) because the narrator argues that biological obstacles to early language learning are minimized if the language is learned earlier in life.
7. **(A)** The second text suggests that some emotional responses that Spielberg tries to evoke in his films may not always be “earned” because they are prompted by watching the character reactions instead of considering the content of what happened. The example in Text 1 has quite a bit of intellectual activity behind it, implying that an emotional response to this speech is based on something substantive instead of something manipulative. It is not choice (B) because the author of Text 2 does not express a concern with political activism. It is not choice (C) because the author of Text 2 clearly believes that emotion can be prompted by the expressions that actors have on the screen. It is not (D) because the author of Text 2 implies that some emotional responses can be earned.

8. **(B)** The text tells us that Jim was struggling to make it in New York. Then, he had a “brilliant marriage.” The connection to “the daughter of a distinguished man” helped Jim advance in life. The implication, then, is that Jim married his wife for her connections. Additionally, the marriage between them prompted “sharp comment” at the time, suggesting that the motives for the marriage were considered suspicious. This makes choice (B) the best answer—the narrator believes Jim married his wife to advance his career. Choices (A) and (C) are incorrect as there is no evidence that Jim and his wife share any love. (D) is incorrect as we have no evidence that his wife sympathized with his poverty.
9. **(A)** In the second paragraph, the author describes the tremendous impact that printing has had on human history, saying that it “changed the destiny of Europe,” and that it has “governed opinion,” reigning over the mind of man. So, this influence can best be described as “extremely significant.” It is not choice (B) because the influence of printing is relevant, not irrelevant. It is not (C) because the author suggests that printing has had a widespread influence, not that it has had a serious negative impact on society. It is not (D) because the impact of printing is more cultural, not militaristic.
10. **(B)** This option shows that the speaker believes that the “fiend of gambling” has made his father’s “good resolutions transient, his promises vain,” showing that he has an inability to follow through on his commitments. This is a great obstacle to the narrator’s father since it has “fully possessed him.” Choice (A) describes the father’s early adulthood but does not indicate that it is a great obstacle. Choices (C) and (D) show a single obstacle his father faced (the declining favor of the king) but do not indicate that this was a large overall issue in his life.
11. **(C)** We can see in [Figure 1](#) that the evening hours have the highest normalized occurrence rates of any time of day. We also see that in [Figure 2](#), the Northern Hemispheric spring months of April, May, and June have the highest rate of tornado formation. Combined, this makes the evening hours in the spring the most likely time for a

tornado to form. Choice (C) is the best option. The other options all have a lower likelihood of spawning a tornado.

12. **(D)** To give oneself entirely to the state means that someone would give up their personal ambitions and desires so that the society as a whole can benefit. Choice (D) most directly illustrates this since the individual would “alienate” himself to the whole community, giving up his rights. Choices (A) and (B) do not address the individual sacrifice of rights. It is not (C) because this suggests an opportunity for individuals to recover their rights should the social contract be violated.
13. **(C)** The economists hypothesize that if we look at the months after 2020, interest rates should decrease. Choice (C) correctly points out that mortgage rates generally decreased during this time period, providing an example in support of the claim. Choice (A) would not show a decrease. Choices (B) and (D) would show the opposite of what would support the hypothesis.
14. **(B)** Group 2 argues that the reason why Species X has been growing in population despite a loss of habitat is that the landscape has become more suitable for the bird. Choice (B) would directly support this idea because if there is more construction taking place, it stands to reason that there might be more fragmentation of land creating more edges along tree lines that the birds might prefer. It is not choice (A) because this does not show why south Ohio would be a good choice for the birds. It is not (B) because an increase in predators would likely kill some of the birds, making the habitat less desirable. Also, choice (D) is incorrect because having more pollution in southern Ohio would be likely be detrimental to the birds.
15. **(D)** The text states that the bees cannot see into the capped cell, so they do not know if mites are growing inside the cells. Thus, choice (D) makes the most sense because it underscores how there are mites breeding that the treatments may not reach because they are inside the capped cells. It is not choice (A) or (C) because these options would instead show that the mites could be gotten rid of. It is not (B)

because this would not be directly relevant to the claim made in the final sentence of the text.

16. **(C)** “Became” is consistent with the past tense seen in surrounding verbs, like “wasn’t,” “used,” and “called.” The other options are not in the proper conjugation for past tense.
17. **(B)** “Which” is used to set aside a nonessential description, while “that” is used at the beginning of an essential description. In this case, the clarification about the constancy of interest rates could be removed from the sentence and the sentence would still be logical. Therefore, surrounding the description of the fixed interest rates with commas make sense. It is not choice (A) because this is a nonessential description, and commas typically do not precede the word “that”. It is not (C) because there is no pause before “which.” It is not (D) because there is not a complete sentence after the semicolon.
18. **(D)** The surrounding sentences use verbs that are in the present tense, like “can” and “have.” So, “allows” is both consistent with the present tense and the singular subject of “method.” The other options are not in the present tense.
19. **(C)** This option uses the correct form of “they’re,” which means “they are.” Also, it puts a comma after the dependent clause that introduces the sentence. Choices (B) and (D) use “their” and “there,” which are incorrect in this context. Choice (A) incorrectly uses a semicolon to provide a break after a dependent clause—there needs to be an independent clause on either side of the semicolon when it is used to break up sentences.
20. **(A)** This needs a possessive to match the idea: the contribution of either scientist. As possession and an apostrophe are required, eliminate choice (C). Eliminate choices (B) and (D) for using the plural word “scientists” since the initial idea was the contribution of either scientist, not the contribution of both scientists.
21. **(C)** Choice (C) is the only option that correctly uses the comma and semicolon to indicate that this is part of a list—the items within the list have commas within them (separating each city name and country

name/description), so semicolons are logically used to separate items in this list.

22. **(A)** The text begins by stating that Sweden had to spend quite a bit of money during the Great Depression, which would make it understandable if Sweden had not been able to repay its debts. The text goes on to state that Sweden quickly paid back its creditors, making a contrast to introduce the second sentence necessary. “Despite” is the only option that expresses a contrast.
23. **(C)** “Further” correctly indicates that what follows in this sentence will build upon the argument already presented. Choice (A) does not make a strong tie to the argument, (B) shows contrast, and (D) shows cause and effect.
24. **(B)** “Even” in this context is used to emphasize that severe market downturns should not incite panic. (A) causal link is already established with the “if” at the beginning of the sentence, making “thus” and “so” unnecessary. It is not “while” because there is not a contrast.
25. **(B)** If an audience is already familiar with the concept of IQ, they will know that IQ is associated with intelligence. Choice (B) builds off this understanding by drawing a contrast between AQ and IQ. It is not choice (A) because the notes indicate that AQ is an important component to success but does not indicate that it is the most important component. It is not choice (C) because the text does not suggest that IQ is totally unimportant to success. It is not (D) because the last bullet point implies that AQ is not necessarily key to academic prowess.
26. **(B)** While there are many ways that sustainable agriculture could be implemented, the student wishes to suggest a way of doing so that does not involve much technology. The last bullet point suggests that simply planting crops close to each other will increase their success, making (B) the most logical answer. The other options all involve more high-tech ways of implementing sustainable agricultural practices.

27. **(A)** According to the notes, a utilitarian looks at the outcomes of an action to determine if it is ethical, while a deontologist looks at whether the action was carried out in a rule-based way. Choice (A) is a good example of something that would be approved of by a utilitarian but not a deontologist: the politician is violating a rule against lying, but while in office the politician carries out policies that serve the greater good. It is not choice (B) because a utilitarian would not like the resulting lack of collaboration among the students. It is not choice (C) because a utilitarian would not appreciate the outcome of a collapsed bridge. It is not choice (D) because it is not clear whether any ethical rules would be violated in this action, making it uncertain as to whether a deontologist would disapprove.

Section 2, Module 1: Math

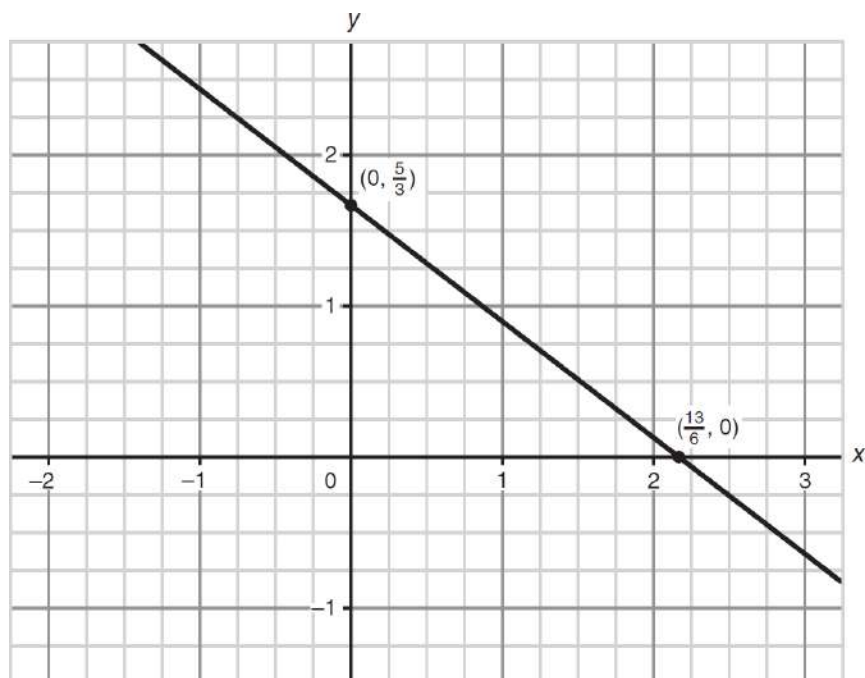
1. **(C)** Plug 5 in for x into the expression to find its value:

$$\begin{aligned} 3(x + 2) - 2x &\rightarrow \\ 3(5 + 2) - 2(5) &\rightarrow \\ 3(7) - 10 &= 21 - 10 = 11 \end{aligned}$$

2. **(D)**

$$\begin{aligned} -3(x^2 - 2x + 4) &\rightarrow \\ \text{Distribute the } -3 &\rightarrow \\ -3x^2 + 6x - 12 & \end{aligned}$$

3. **(B)** The total number of students in the class is $26 + 33 + 21 + 13 + 7 = 100$. So, the median grade will be the average of the 50th and 51st term. The 50th and 51st terms, when the grades are put in order from least to greatest, are both (B). So, the median grade for the class is (B).
4. **(B)**



Take the change in y divided by the change in x to find the slope of the line. Use the points $(0, \frac{5}{3})$ and $(\frac{13}{6}, 0)$, and the formula $slope = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{5}{3} - 0}{0 - \frac{13}{6}} = \frac{\frac{5}{3}}{-\frac{13}{6}} = \frac{5}{3} \times -\frac{6}{13} = -\frac{5 \times 2}{13} = -\frac{10}{13}$$

5. **(B)** The equation is already in a factored form that allows you to identify the solutions. If x is $-\frac{1}{4}$ or $\frac{2}{3}$, the value of the right-hand side of the equation would be zero. Therefore, these two numbers are solutions. So, multiply them together to find the product:

$$-\frac{1}{4} \times \frac{2}{3} = -\frac{2}{12} = -\frac{1}{6}$$

6. **(A)** Find 27% of 4,000 to see how many of the students would have a science or math major. Move the decimal point of 27 to the left two spots, then multiply this by 4,000 to find the number of students:

$$0.27 \times 4,000 = 1,080$$

7. **(A)** There are 7 days in one week, so multiply the 8 hours Mark sleeps per day by 7 to find out the total number of hours he sleeps in a week: $7 \times 8 = 56$. Then, multiply 56 by the number of weeks, W , to find the number of hours that Mark would sleep in W weeks: $56 \times W$.
8. **(C)** Take the cube root of both sides of the equation to solve for x :

$$\begin{aligned}x^3 &= \frac{27}{8} \rightarrow \\ \sqrt[3]{x^3} &= \sqrt[3]{\frac{27}{8}} \rightarrow \\ x &= \frac{3}{2}\end{aligned}$$

If $x^3 = \frac{27}{8}$, what is the value of x ?

9. **(14)** Plug 40 in for B and 4 in for E . Then solve for P :

$$\begin{aligned}B &= 2P + 3E \rightarrow \\ 40 &= 2P + 3(4) \rightarrow \\ 40 &= 2P + 12 \rightarrow \\ 40 - 12 &= 2P \rightarrow \\ 28 &= 2P \rightarrow \\ P &= 14\end{aligned}$$

10. **(A)** Multiply 16π by 1.5625 to find what the area of a circle that is 56.25% greater in area would be. (This would represent 156.25% of the original circle—that is why we multiply by 1.5625 instead of just 0.5625.)

$$16\pi \times 1.5625 = 25\pi$$

Note that we only multiplied the 16 by the 1.5625 so that we can identify the radius more easily. So, the new circle will have a radius of 5, since the area of a circle is πr^2 and the square root of 25 is 5.

11. **(6)** You can use substitution to solve for x . Take the second equation and find the value of y in terms of x :

$$\begin{aligned}x + y &= 11 \rightarrow \\y &= 11 - x\end{aligned}$$

Then, substitute this in to the first equation to solve for x :

$$\begin{aligned}3x - 4y &= -2 \rightarrow \\3x - 4(11 - x) &= -2 \rightarrow \\3x - 44 + 4x &= -2 \rightarrow \\7x &= 42 \rightarrow \\x &= 6\end{aligned}$$

12. **(C)** Multiply the number of persons per square mile by the total number of square miles to find an approximation of the total population of the city:

$$4,900 \times 139 \approx 681,000$$

13. **(7)** Be sure to solve for “ $x + 2$ ” and not just for x . The expression $x^2 - 4$ can be rewritten as $(x + 2)(x - 2)$, enabling you to plug in 3 for $x - 2$ and solve for $x + 2$:

$$\begin{aligned}\frac{x^2 - 4}{3} &= 7 \rightarrow \\\frac{(x - 2)(x + 2)}{3} &= 7 \rightarrow \\\frac{3(x + 2)}{3} &= 7 \rightarrow \\x + 2 &= 7\end{aligned}$$

14. **(A)** As the height of the orangutans increases, their weight also increases. Choice (A), “The greater the height of the orangutan, the greater the weight,” correctly summarizes this trend.
15. **(C)** Use the volume formula for a cylinder, $V = \pi r^2 h$, in which r is the radius and h is the height. Note that this formula is provided in the formulas at the beginning of the section, so refer to it if you do not remember the formula. Since the tank has a base with a diameter of 10

feet, the radius will be 5. Plug in the volume and the radius into the formula to solve for height:

$$\begin{aligned}V &= \pi r^2 h \rightarrow \\300\pi &= \pi(5^2)h \rightarrow \\300\pi &= 25\pi h \rightarrow \\\frac{300\pi}{25\pi} &= h \rightarrow \\12 &= h\end{aligned}$$

16. **(D)** Plug 6 in for x to solve for the value of $f(6)$:

$$\begin{aligned}f(x) &= \frac{x^2 + 3}{2x} \rightarrow \\f(6) &= \frac{6^2 + 3}{2(6)} = \frac{36 + 3}{12} = \frac{39}{12} = \frac{13}{4}\end{aligned}$$

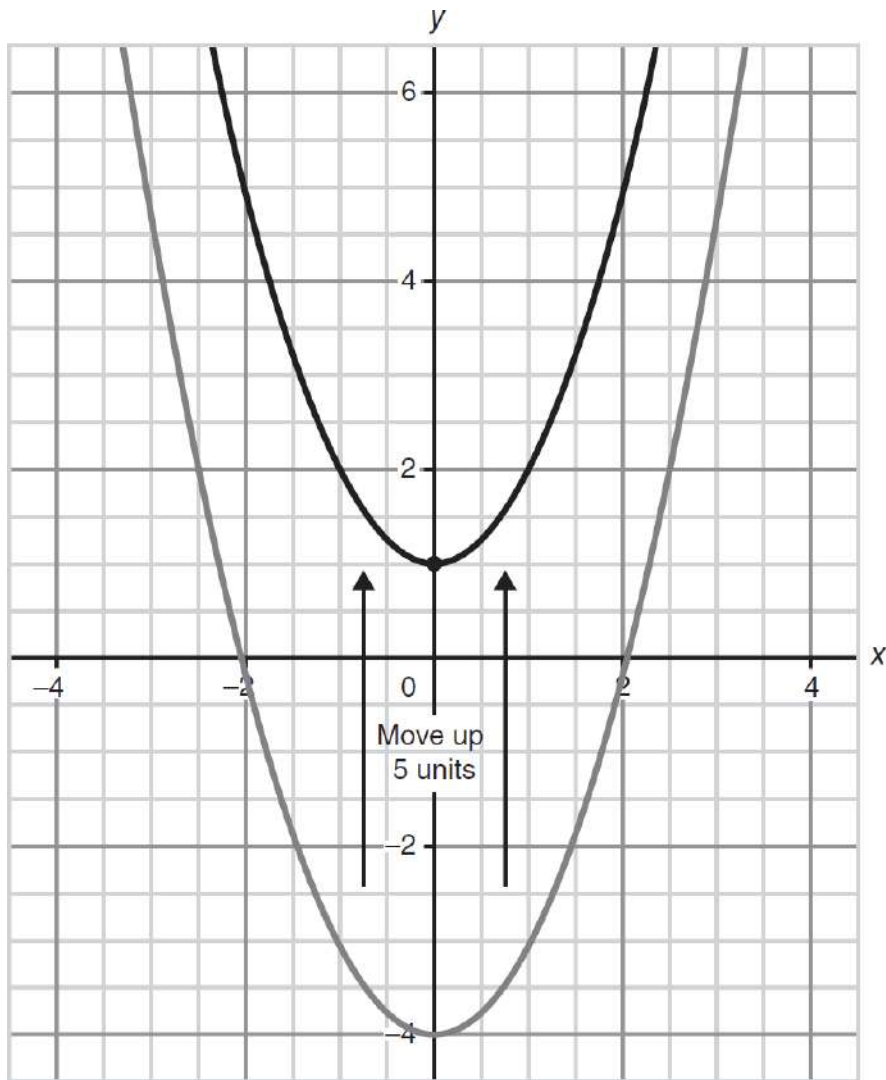
17. **(C)** The number of respondents is sufficient to generalize on the voter preferences. However, the respondents were not randomly selected—they all chose to go to an online poll to vote for a particular candidate. People who chose to respond to the online poll are more likely enthusiastic about their political preferences; this self-selected group would therefore not accurately represent a cross-section of voters. The survey would have been more accurate if the respondents had been randomly selected instead of self-selected.
18. **(2.5 or $\frac{5}{2}$)** Pick an ordered pair, (2, 5) would work, and plug it in to the equation $y = kx$ to solve for the constant k :

$$\begin{aligned}y &= kx \rightarrow \\5 &= k(2) \rightarrow \\k &= 2.5\end{aligned}$$

19. **(C)** Add 5 to the equation to shift the equation upwards by 5 units:

$$y = x^2 - 4 \rightarrow y = x^2 - 4 + 5 \rightarrow y = x^2 + 1$$

You can see the effect of this translation in the following graph:



20. **(A)** Plug in 2 for x and 5 for y , since this ordered pair represents a solution to the inequality:

$$y \geq x + n \rightarrow$$

$$5 \geq 2 + n \rightarrow$$

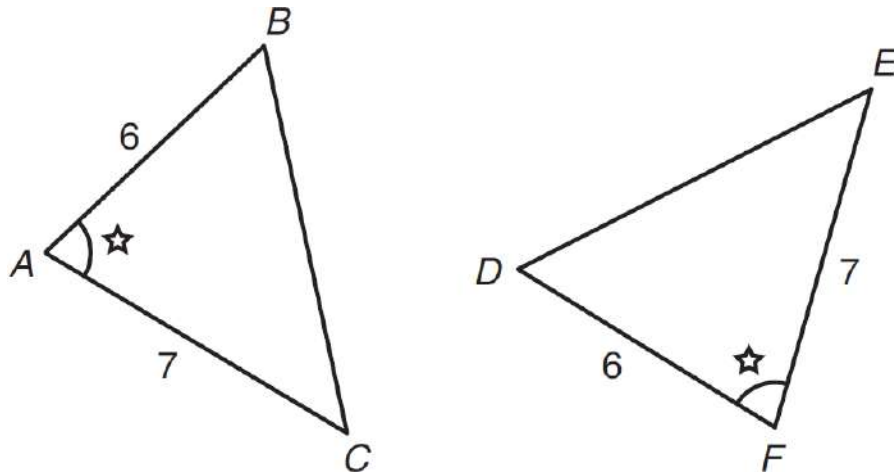
$$3 \geq n$$

Since n must be less than or equal to 3, the greatest possible value for n is 3.

21. **(B)** There are 25 patients who had a positive test result. Out of those 25 patients, 20 actually did not have the illness, meaning they had a false positive. So, divide 20 by 25 to find the probability that one of the positive test results was actually a false positive:

$$\frac{20}{25} = 0.8$$

22. **(C)**



If angle BAC equals angle DFE , as labelled above, the triangles would be congruent because of the Side-Angle-Side theorem. This theorem states that if two sides and the angle formed by these two sides are equal to two sides and the included angle in another triangle, the two triangles are congruent.

Section 2, Module 2: Math

1. **(A)**

$$5x + 2 = 4(x - 3) \rightarrow$$

$$5x + 2 = 4x - 12 \rightarrow$$

$$x + 2 = -12 \rightarrow$$

$$x = -14$$

2. **(D)** The interval from 30–35 minutes has a decrease in distance of 0.45 miles. This is the greatest decrease in distance over any time period portrayed in the graph, so choice (D) is the correct answer.
3. **(1)** Plug in 5 for x to the equation to solve for the value of y :

$$\frac{1}{3}(x - 2) = y \rightarrow$$

$$\frac{1}{3}(5 - 2) = y \rightarrow$$

$$\frac{1}{3}(3) = 1 = y$$

4. **(A)** The equation can be written in slope-intercept form since it is a line. Use k as the y -intercept:

$$y = mx + b$$

$$y = 5x + k$$

Now, plug in a point—we can use $(1, 8)$ —for x and y to solve for k :

$$y = 5x + k \rightarrow$$

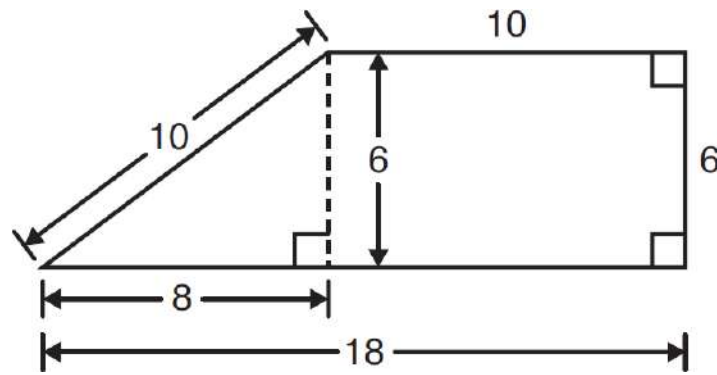
$$8 = 5(1) + k \rightarrow$$

$$3 = k$$

5. **(D)**

$$\frac{x^2 - 9}{x - 3} \rightarrow \frac{(x + 3)(x - 3)}{x - 3} = \frac{(x + 3)\cancel{(x - 3)}}{\cancel{x - 3}} = x + 3$$

6. **(44)** Since this is a trapezoid, the top and bottom sides are parallel to one another. So, you can inscribe a right triangle within the trapezoid to find the length of the unknown side, as seen below.



Then, add up the lengths of the sides to find the perimeter of the figure:

$$10 + 6 + 18 + 10 = 44.$$

7. **(B)** The functions are all in slope-intercept form, and they all have different values for the slope. So, find the slope of the function by using the slope formula and plugging in two points. The cost corresponds to the y values and the pieces of fruit will be the x value. Use the points $(6, 13.40)$ and $(12, 25.40)$ in the slope formula:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25.40 - 13.40}{12 - 6} = \frac{12}{6} = 2$$

The only option that has 2 as its slope is choice (B). Alternatively, you could plug in values to the equations to see which one correctly models the numbers in the table.

8. **(B)** Manipulate the equation to isolate v :

$$\begin{aligned} K &= \frac{1}{2}mv^2 \rightarrow \\ \frac{2K}{m} &= v^2 \rightarrow \\ \sqrt{\frac{2K}{m}} &= v \end{aligned}$$

9. **(B)** The range of the set (difference between the least and greatest terms) will increase by 25, since the smallest member of the set has 5

subtracted from it and the largest member has 20 added to it. The standard deviation (the deviation from the mean) would increase slightly with these changes, but not nearly as much as the range would since the values of the other 13 members of the set will remain constant. So, choice (B) is the correct answer.

10. **(D)** Add the two functions together, combining like terms to put it in a simplified quadratic form:

$$\begin{array}{r} -x^2 + 4x - 3 \\ + 5x^2 - 2x + 8 \\ \hline 4x^2 + 2x + 5 \end{array}$$

In this combined equation, the number 5 corresponds to the constant k , making choice (D) correct.

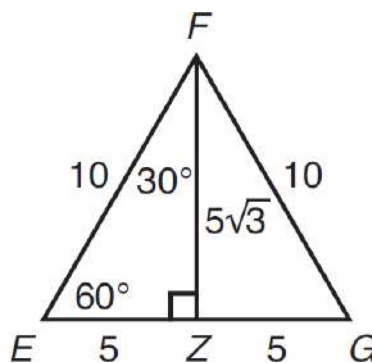
11. **($\frac{5}{3}$, 1.666, 1.667)** First, find the slope of the line comprised of the two points (5, 0) and (0, 3).

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{0 - 5} = -\frac{3}{5}$$

Now, take the negative reciprocal of $-\frac{3}{5}$ to find the slope of a line perpendicular to this one:

$$-\frac{3}{5} \rightarrow \text{Flip the fraction and multiply by } -1 \rightarrow \frac{5}{3}$$

12. **(A)**



Since triangle EFG is equilateral, you can label $\angle FEZ$ as 60 degrees. This turns triangle FEZ into a special right triangle—a 30-60-90 triangle. The ratios of the sides in a 30-60-90 triangle are x , $\sqrt{3}x$, $2x$. So, the length of $\overline{FZ}$ will be $5\sqrt{3}$ since the x in this case is 5.

13. **(B)** Since 23% of the recommended daily value of calcium for an adult is 300 mg, calculate the total recommended daily value of calcium for an adult, x , using this equation:

$$\begin{aligned} 0.23x &= 300 \rightarrow \\ x &= \frac{300}{0.23} \approx 1,304 \end{aligned}$$

Now, find what percent of 1,304 that 210 is:

$$\frac{210}{1,304} \times 100 \approx 16\%$$

14. **(3)** 2π radians and 360 degrees both represent an entire circle. To see how many angles of the measure of $\frac{2}{3}\pi$ radians are in 360 degrees, divide 2π by $\frac{2}{3}\pi$:

$$\frac{2\pi}{\left(\frac{2}{3}\pi\right)} = \frac{2}{\left(\frac{2}{3}\right)} = 2 \times \frac{3}{2} = 3$$

So, there are 3 angles of this measure within 360 degrees.

15. **(C)** The greatest possible value for y occurs when $y = x + 2$, since that would maximize its value within the inequality. Set $x + 2$ equal to $-x + 4$, since the value of y would be at its maximum in both inequalities, and then solve for x :

$$\begin{aligned} x + 2 &= -x + 4 \rightarrow \\ 2x &= 2 \rightarrow \\ x &= 1 \end{aligned}$$

The correct answer is therefore choice (C).

16. **(A)** Try plugging in values from the table to determine which function correctly represents the relationship between the numbers. Before just plugging in numbers to every equation, think about the overall relationship of the numbers. As x increases, y decreases. So, choices (C) and (D) would not work because they would result in increasing functions. Then, you just need to try options (A) and (B). Use the ordered pair (1, 2) to see which of these would work:

$$y = \frac{2}{x^2} \rightarrow 2 = \frac{2}{1^2} \text{ which is true.}$$
$$y = \frac{4}{x^2} \rightarrow 2 \neq \frac{4}{1^2}, \text{ so choice (B) is incorrect.}$$

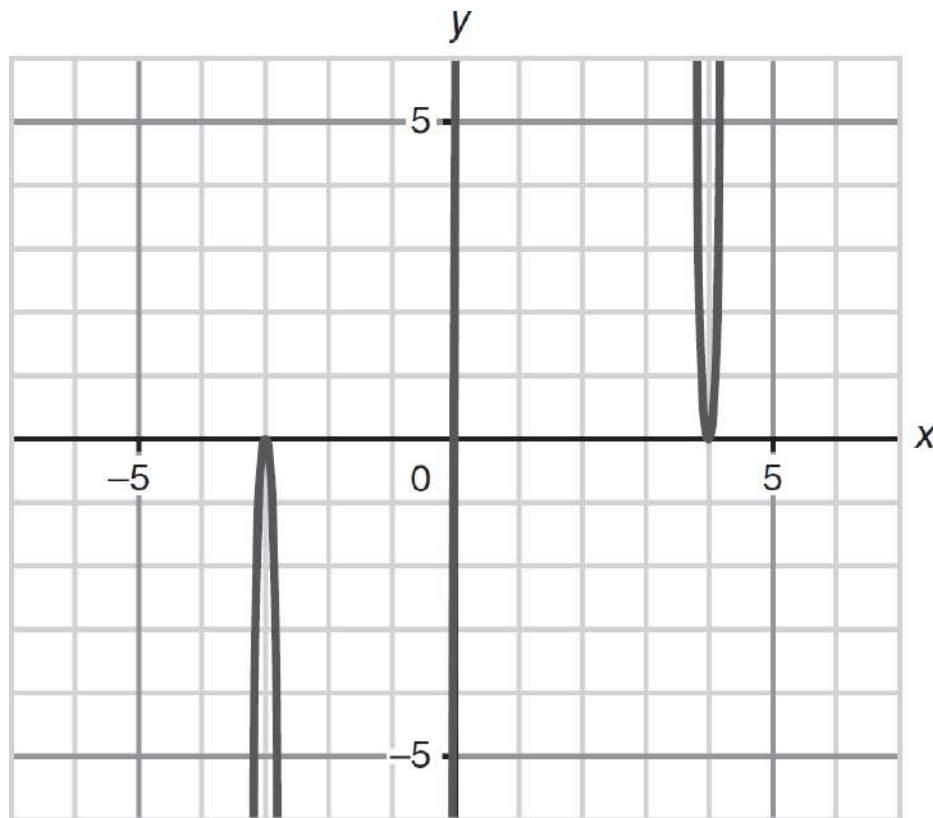
Therefore, the correct function is in choice (A).

17. **(D)** Use synthetic division to divide. Use 4, -3 , and 2 as the values in the top of the synthetic division format and use $+3$ to go through the synthetic division steps.

$$\begin{array}{r|rrr} 3 & 4 & -3 & 2 \\ & & 12 & 27 \\ \hline & 4 & 9 & 29 \end{array}$$

This results in $4x + 9 + \frac{29}{(x-3)}$, in which $\frac{29}{(x-3)}$ is the remainder. You could also use long division to solve.

18. **(C)** A selection of the function is graphed below:



While the function has a high degree, it has only three unique x intercepts: 0, 4, and -3 . You can see this by examining the equation to see what values of x would cause y to equal zero. If $x = 0$, $x = 4$, or $x = -3$, the value of y will be equal to zero.

19. **(B)**

Since there are twice as many elementary school students as middle school students, $c = 2f$ and $f = \frac{c}{2}$. The number of elementary school students and middle school students is 150, so find the value of c :

$$c + f = 150 \rightarrow$$

$$c + \frac{c}{2} = 150 \rightarrow$$

$$1.5c = 150 \rightarrow c = 100$$

So, we know that c is 100.

Since 60% of elementary school students take the bus, find 60% of 100 to find the value for a : $0.6 \times 100 = 60$. Since the number of elementary school students is 100 total, subtract 60 from 100 to find the number of elementary school students who walk: $100 - 60 = 40$.

The total number of students who walk to school is 70 based on the table. So, find the number of middle school students who walk by subtracting 40 from 70:

$$70 - 40 = 30$$

To more easily visualize this, all the solved values for the variables are plugged into the spots in the table below:

	Bus	Walk	Total
Elementary	60	40	100
Middle	20	30	50
Total	80	70	150

20. (C) The product of these two expressions will be undefined if the denominator on either expression is equal to zero. For $\frac{2}{x}$, if $x = 0$, the expression is undefined. For $\frac{3}{(x-1)}$, if $x = 1$, the expression is undefined since the denominator $x - 1$ would equal zero if $x = 1$. So, the correct answer is choice C.
21. (7) The equation will equal zero when $x = 5$ and $x = -2$, as you can see below:

$$y = 7(x - 5)(x + 2) \rightarrow$$

$$y = 7(5 - 5)(5 + 2) \rightarrow$$

$$y = 7(0)(7) = 0$$

and

$$\begin{aligned}
 y &= 7(-2 - 5)(-2 + 2) \rightarrow \\
 y &= 7(-2 - 5)(-2 + 2) \rightarrow \\
 y &= 7(-7)(0) = 0
 \end{aligned}$$

So, the points (5, 0) and (-2, 0) are the x intercepts of the parabola. Since the y values do not change, simply find the difference between the x values to find the distance between the x intercepts:

$$5 - (-2) = 7$$

22. (15)

$$\begin{aligned}
 (x + 5)(x + 3) &= ax^2 + bx + c \rightarrow \\
 x^2 + 3x + 5x + 15 &= ax^2 + bx + c \rightarrow \\
 x^2 + 8x + 15 &= ax^2 + bx + c
 \end{aligned}$$

Since 15 is the only numerical constant, it will equal c because the c term on the right-hand side does not have an x in it. So, the value of c is 15.

Practice Test 2

Section 1, Module 1: Reading and Writing

32 MINUTES, 27 QUESTIONS

DIRECTIONS ▼

You will be tested on a variety of important reading and writing skills. Each question has one or more passages, possibly including a graph or table. Carefully read each passage and question and choose the best answer to the question based on the passage(s).

Every question in this section is multiple-choice, with four possible answers. Each question has only one best answer.

1

MODULE 1

1

In science class, the two friends frantically mixed their chemical ingredients together. They had failed to meet over the weekend and their assignment was nowhere close to being finished. When they received a failing grade, the teacher remarked that the _____ to their problem lay in adequate preparation.

1

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ solution
- Ⓑ mixture
- Ⓒ choice
- Ⓓ origin

Thad walked through the tunnel to a loud applause. He strolled up to the pitching mound and breathed deeply to calm his nerves. He threw hard and fast but was _____. The catcher had to run and retrieve the wild pitch.

2

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ off his mark
- Ⓑ highly motivated
- Ⓒ not on the mound
- Ⓓ serene

The judge decided to _____ on some mistakes as it was his first case. She could remember fumbling through her opening argument the first time she was in court.

3

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ provide the prosecutor with consequences
- Ⓑ evoke carte blanche
- Ⓒ give the prosecutor leeway
- Ⓓ grant the prosecutor amnesty

Some bystanders overly concerned about potential litigation have chosen not to help for fear they would make the situation worse, thereby opening themselves up to lawsuits. Some states have passed varying degrees of protection known as “good Samaritan laws” so that people can feel comfortable helping. Some countries have even gone so far as to pass “duty to help” laws that generally require people to assist. While these laws are

often criticized as legislating morality, they do help alleviate the negative _____ of the bystander effect.

4

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ impacts
- Ⓑ collisions
- Ⓒ clashes
- Ⓓ accomplishments

Poker players are fond of saying that if you look around the table and can't spot the fool, run away as fast as you can; you're it. The same goes for the world of finance, which has quickly become as adversarial as a duel at high noon. Such is the nature of equities, futures, bonds, and derivatives (to name a few): any transaction has both a winner and a loser. Either the buyer has purchased an instrument that is undervalued and will be worth more tomorrow, or the seller has unloaded a bloated instrument that will fall back closer to its "true" market valuation in the near future. Either way, dollars (both unrealized and actual) will flow from one pocket to another.

5

As used in the text, what does the word "bloated" most nearly mean?

- Ⓐ overvalued
- Ⓑ distended
- Ⓒ full
- Ⓓ waterlogged

The following text is from Christina Rossetti's 19th-century poem "Who Has Seen the Wind?"

Who has seen the wind?
Neither I nor you.
But when the leaves hang trembling,
The wind is passing through.
Who has seen the wind?
Neither you nor I.
But when the trees bow down their heads,
The wind is passing by.

6

Which choice best describes the overall structure of the text?

- Ⓐ It analyzes the patterns of weather formation.
- Ⓑ It contemplates the meaning of new growth in nature.
- Ⓒ It presents different ways that the invisible wind may be seen.
- Ⓓ It considers the emotional impact that the wind can have on observers.

The following text is adapted from Daniel Defoe's 1719 novel *The Life and Adventures of Robinson Crusoe*. In it, the title character has left home despite his parents' wishes and sets out to find adventures on the sea.

We were not much more than a quarter of an hour out of our ship till we saw her sink, and then I understood for the first time what was meant by a ship foundering in the sea. I could hardly watch, for from the moment that they rather put me into the boat than that I might be said to go in, my heart was, as it were, dead within me, partly with fright, partly with horror of mind, and the thoughts of what was yet before me.

We got all safe on shore, where, as unfortunate men, we were used with great humanity, as well by the magistrates of the town, who assigned us good quarters, as by particular merchants and owners of ships, and had

money given us sufficient to carry us either to London or back to Hull as we thought fit.

7

Which choice best describes the function of the underlined sentence in the text as a whole?

- Ⓐ To show the scorn with which people treat shipwrecked sailors
- Ⓑ To describe how those who experienced shipwrecks are treated with hospitality
- Ⓒ To explain the envy that others have of those sailors who have been shipwrecked
- Ⓓ To demonstrate how those who experienced shipwrecks are treated with humor

The following text is adapted from W. E. B. Du Bois's 1903 work "Of Sorrow Songs."

What are these songs, and what do they mean? I know little of music and can say nothing in technical phrase, but I know something of men, and knowing them, I know that these songs are the articulate message of the slave to the world. They tell us in these eager days that life was joyous to the black slave, careless and happy. I can easily believe this of some, of many. But not all the past South, though it rose from the dead, can gainsay the heart touching witness of these songs. They are the music of an unhappy people, of the children of disappointment; they tell of death and suffering and unvoiced longing toward a truer world, of misty wanderings and hidden ways.

8

Based on the text, Du Bois's attitude toward those who would suggest that the songs of slaves expressed their happiness and joy is most likely one of

- Ⓐ complete agreement.

- Ⓐ qualified skepticism.
- Ⓑ outright dismissal.
- Ⓒ hopeful optimism.

The following text is from Harriet Beecher Stowe’s 1852 novel *Uncle Tom’s Cabin*. She presents a scene from a time when slavery was widely practiced.

Late in the afternoon of a chilly day in February two gentlemen were sitting over their wine, in a well-furnished parlour in the town of P_____ in Kentucky in the midst of an earnest conversation.

“That is the way I should arrange the matter,” said Mr. Shelby, the owner of the place. “The fact is, Tom is an uncommon fellow; he is certainly worth that sum anywhere; steady, honest, capable, manages my farm like a clock. You ought to let him cover the whole of the debt; and you would, Haley, if you’d got any conscience.”

“Well, I’ve got just as much conscience as any man in business can afford to keep,” said Haley, “and I’m willing to do anything to ‘blige friends; but this yer, ye see, is too hard on a feller, it really is. Haven’t you a boy or gal you could throw in with Tom?”

9

Based on the text, Haley is best characterized as a/an

- Ⓐ humane empathizer.
- Ⓑ financial amateur.
- Ⓒ aggressive negotiator.
- Ⓓ passive mediator.

Career Office Service	Percent of Students Who Used Service	Percent of Students Who Found Service Helpful

Practice Interviewing	31.2%	69.9%
Resume Assistance	74.5%	68.4%
Workshops	37.8%	63.0%
Job Listings	55.4%	62.1%
Researching Employers	32.2%	60.5%
Career Skills Testing	29.4%	52.4%

Information adapted from the *Class of 2017 Student Survey*, National Association of Colleges and Employers

The career services office at a college is analyzing the effectiveness of its offerings by evaluating student opinions about the services offered. Some students suggest that the career services office should offer services that students are more likely to find helpful. The administrator of the career services office argues, however, that career services can only be helpful to the extent that students use the services offered.

10

Which choice most effectively uses data from the graph to support the office administrator's position?

- Ⓐ While 52.4% of surveyed students thought that career services testing would be helpful, only 29.4% of students took advantage of this offering.
- Ⓑ Interview practice and resume assistance are among the most sought-out services by college students today.
- Ⓒ Although 74.5% of surveyed students used career services resume assistance, only 68.4% of students found this service useful.
- Ⓓ Nearly identical percentages of students found that the workshops and job listings provided through career services were helpful.

Physicians claim that broken-heart syndrome often presents symptoms like those of a heart attack but without the defining arterial blockage. Instead of plaque blocking the arteries, scientists believe surrounding muscles

constrict the arteries, reducing the flow of blood to the heart and mimicking a heart attack. Unlike a heart attack, broken-heart syndrome is rarely fatal, and can even resolve itself within hours to weeks. Although patients often recover from this condition quickly, the stress that is thought to cause this condition may persist long after their so-called recovery. Unlike broken-heart syndrome, prolonged stress can have consequences on long-term heart health, resulting in heart disease, hypertension, and even stroke. Such consequences exemplify the importance of psychological health, as it can significantly affect one's physical health.

11

Which finding, if true, would most directly support the physicians' claim?

- Ⓐ Researchers find that they can successfully differentiate between broken-heart syndrome and a heart attack by determining whether plaque is blocking the arteries.
- Ⓑ Sociologists report that increasing numbers of people are experiencing a decrease in their mental health due to prolonged stress.
- Ⓒ Psychiatrists find that there is a roughly equivalent likelihood of dying of broken-heart syndrome as there is from dying of a heart attack.
- Ⓓ A randomly controlled survey of adults between 50 and 70 years of age finds that heart attacks are a greater source of worry than broken-heart syndrome.

“On the Firing Line of Education” is a 1919 paper by A.J. Ladd, a professor of education at the State University of North Dakota. In the paper, Professor Ladd discusses the future of vocational education in American high schools. He asserts that schools should help guide students into their most appropriate vocational paths.

12

Which quotation from “On the Firing Line of Education” most effectively illustrates the assertion?

- Ⓐ “Children in these early years are not yet ready to choose their work of life; that they do not yet sufficiently know themselves—their own tastes and capacities for such serious choice.”
- Ⓑ “To place before children such attractive objective features would result in swerving many from the normal pathway of their development and check it midway. The result has been what might be called a compromise.”
- Ⓒ “Thru the child-study movement the teacher comes to know child nature so well that direct application can be made to the individual child and an intimate knowledge gained of his tastes, capacities, ambitions, and dominant interests.”
- Ⓓ “The high school is an institution established by a community for community purposes—to take its young people—all of them—and guide them thru the difficult and transitional period of adolescence, directing, inspiring, shaping, checking, developing for the largest manhood and womanhood possible and providing the community with efficient workmen in various lines.”

A student is studying Edmund Burke’s 18th-century work *Selections from the Speeches and Writings of Edmund Burke*. The student is specifically analyzing the selection under the title “Colonies and British Constitution.” This selection is believed to have been written shortly before the American Revolution and discusses the relationship between the United States and Great Britain. The student argues that Burke asserted that at the time he wrote the text, the recognition of civil rights for the citizens of countries around the world was relatively limited.

Which quotation from “Colonies and British Constitution” most effectively illustrates the argument?

- Ⓐ “These are ties, which, though light as air, are as strong as links of iron.”
- Ⓑ “Slavery they can have anywhere. It is a weed that grows in every soil.”
- Ⓒ “Do not entertain so weak an imagination, as that your registers and your bonds . . . are what form the great securities of your commerce.”
- Ⓓ “Dead instruments, passive tools as they are, it is the spirit of the English communion that gives all their life and efficacy to them.”

Microplastics are microscopic particles of plastic that tend to accumulate toxins and are in some cases toxic themselves. Unfortunately, microplastics tend to end up being eaten by various organisms. The primary reason for this problem is that in recent years, approximately 380 million tons of plastic are produced annually. Plastic is not biodegradable, which means that once it enters the environment it can take several hundred years to degrade. Some have proposed plastic-eating microorganisms as a revolutionary solution to this critical issue. Others are skeptical of the claims made about _____

14

Which choice most logically completes the text?

- Ⓐ whether a bacteria could have any role in harming the environment.
- Ⓑ the capacity of microbiologists to analyze bacteria in a laboratory setting.
- Ⓒ this bacteria’s utility in consuming massive plastic deposits.
- Ⓓ the possibility of plastic extending the length it needs to biodegrade.

According to Florentine records between the end of the fourteenth century and the middle of the fifteenth, the number of women participating in

tribunals dropped by 84 percent. The implications of this backslide are significant. By 1450, the power of women to defend themselves in court had suffered a significant blow. And while weakness in the courtroom may not have significantly impacted the elite women of the upper classes, its effects were undoubtedly felt by working women of the lower classes, who were, by the late 1460s, _____

15

Which choice most logically completes the text?

- (A) essentially at the mercy of their accusers.
- (B) able to successfully cite legal precedents in court proceedings.
- (C) empowered with a spirit of judicial advocacy.
- (D) strong advocates of violent means to obtain justice.

Career websites like LinkedIn, Dice, and Monster make it simple to get started with your job search. _____ work experience, intern or research positions, and volunteer efforts.

16

Which choice completes the text so that it conforms to the conventions of Standard English?

- (A) Within your online, profiles it is critical that you include information, about your educational background, previous
- (B) Within your online profiles it is critical that you include information about your educational background, previous
- (C) Within your online profiles, it is critical that you include information about your educational background, previous
- (D) Within your online profiles, it is critical that you include information about: your educational background, previous

Capsaicin, the compound in peppers that makes them spicy, only affects mammals, so _____ effective in driving away squirrels and rodents. Like humans, these animals are often sensitive to spicy foods.

17

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ it's
- Ⓑ its
- Ⓒ their
- Ⓓ there

According to Rognlie, the solution put forth in *Capital in the Twenty-First Century*, since it will do little to limit _____ returns on assets, is no solution at all.

18

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ homeowners'
- Ⓑ homeowner's
- Ⓒ the home owner's
- Ⓓ homeowners

The machine shown in Picabia's drawing demonstrated his appreciation for the technology of his time. However, the machine he depicted did not serve any functional purpose, which he used to mock the excessive pride inventors _____ after successfully creating an invention.

19

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ feeling
- Ⓑ have feeling
- Ⓒ will felt
- Ⓓ felt

Among the poems that he wrote during his lifetime, Bashō's 1686 haiku "The Old Pond" is his most well known and is, arguably, the best haiku of all time. _____ was inspired by nature: it describes the splashes a frog makes after it jumps into a pond.

20

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ Like many other haikus: Bashō's poem
- Ⓑ Like many other haikus, Bashō's' poem
- Ⓒ Like many other haikus—Bashō's poem
- Ⓓ Like many other haikus, Bashō's poem

The Kafala System is a migrant worker system in the Middle East. In 2016, the United Arab Emirates (UAE) implemented reforms that _____ in employment and do not tie the worker to the employer.

21

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ allowing flexibility
- Ⓑ allow flexibility
- Ⓒ has allow flexing

- Ⓓ allow flexing

Around 10,000 years ago, humans started farming in the Fertile Crescent. Farming brought advantages like a more reliable food supply and a less nomadic _____

22

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ lifestyle but it also brought a new problem; rodents.
- Ⓑ lifestyle; but it also, brought a new problem—rodents.
- Ⓒ lifestyle, but it also brought a new problem rodents.
- Ⓓ lifestyle, but it also brought a new problem: rodents.

Once a birder has identified the kinds of food they want to put out, the next step is to make sure the environment around the feeder is welcoming and safe. Feeders should be placed either within three feet of or farther than 30 feet away from windows to avoid collisions as birds land and take off. Birds _____ like to have cover as they eat; nearby shrubs and trees can provide safety for birds when they feel threatened. Some birders even like to provide water to go with the food.

23

Which choice completes the text with the most logical transition?

- Ⓐ also
- Ⓑ on the other hand
- Ⓒ due to this
- Ⓓ as a result

The process of carbon fixation involves combining an enzyme called RuBP oxygenase-carboxylase (rubisco) with carbon dioxide _____ form an

organic molecule; however, the rubisco molecule also has the potential to use O_2 as a substrate instead of CO_2 , which contributes to a side reaction called photorespiration.

24

Which choice completes the text with the most logical transition?

- Ⓐ because
- Ⓑ since
- Ⓒ while
- Ⓓ in order to

While researching a topic, a student has taken the following notes:

- Ibn Battuta was a famous explorer from the 1300s.
- He explored most of the Muslim world in his lifetime.
- He is believed to have travelled as far east as China and the Philippines, and as far west as Morocco.
- Because of his extensive studies in Mecca, Ibn Battuta was prized as an Islamic scholar.
- Ibn Battuta successfully used his intellect to obtain influence with rulers and merchants around the world.

25

The student wants to underscore the time period during which Ibn Battuta conducted his exploration. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Ibn Battuta traveled as far east as China and as far west as Morocco.
- Ⓑ Many rulers and merchants valued Ibn Battuta's impressive academic background.

- Ⓒ During his travels, Ibn Battuta used his personal charisma to make connections.
- Ⓓ Ibn Battuta travelled through much of the world during the 1300s.

While researching a topic, a student has taken the following notes:

- Pumpkins come in a variety of colors beyond orange, including red, blue, pink, and white.
- Deep reddish orange pumpkins, commonly referred to as Cinderella pumpkins, are a French variety called Rouge Vif d'Etampes that was introduced to the United States in the late 19th century.
- Jarrahdale pumpkins are bluish grey with a golden orange interior and were developed in Australia.
- Porcelain Doll pumpkins were developed in the United States and are pale pink. They also have a wonderful taste, making them a good choice for pumpkin pie.
- Casperita pumpkins are small and white. While they are edible, their size makes them more common for decorations than for food.

26

The student wants to highlight two pumpkin varieties that can be consumed as food. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Blue, white, and orange pumpkins will often be found on the dinner table.
- Ⓑ Jarrahdale and Cinderella pumpkins are considered delicacies.
- Ⓒ Both Porcelain Doll and Casperita pumpkins can be eaten.
- Ⓓ Porcelain Doll pumpkins are considered the most delicious pumpkin varietal.

While researching a topic, a student has taken the following notes:

- The Rice's whale is a unique species of whale discovered in 2021.

- It is named after whale researcher Dale Rice.
- It can grow to a little over 40 feet long and has baleen instead of teeth.
- This whale is found in the Gulf of Mexico.
- The Rice's whale is on the brink of extinction, with fewer than 100 of the whales believed to still be alive.
- Major threats to the Rice's whale include pollution, especially oil spills.

27

The student wants to suggest a reason why the Rice's whale is quite difficult to observe. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ With so few of the Rice's whales left in the wild, whale spotting opportunities are quite rare.
- Ⓑ The Rice's whale is unsurprisingly named for the man who discovered it—Dale Rice.
- Ⓒ The geographic vicinity in which the Rice's whale is found is yet to be established.
- Ⓓ A potential oil spill poses a major threat to the long-term viability of the Rice's whale.

Section 1, Module 2, Reading and Writing

32 MINUTES, 27 QUESTIONS

DIRECTIONS ▼

You will be tested on a variety of important reading and writing skills. Each question has one or more passages, possibly including a graph or table. Carefully read each passage and question and choose the best answer to the question based on the passage(s).

Every question in this section is multiple-choice, with four possible answers. Each question has only one best answer.

The vestibular system is _____ in the inner ear and is comprised of three tubes on each side of the head. Each tube on the right has a corresponding tube on the left, making up three pairs.

1

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ residing
- Ⓑ located
- Ⓒ lodged
- Ⓓ dwelling

Staging can be a win-win situation for both the aspiring chef and the mentor: the mentor receives help in the kitchen, and the aspiring chef acquires new skills that he can apply in a future position. Stages at prestigious restaurants can _____ a chef's resume for future opportunities.

2

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ cushion
- Ⓑ maintain
- Ⓒ bolster
- Ⓓ rebuild

Soap is made up of small molecules that have hydrophilic heads and hydrophobic tails; the hydrophilic heads are attracted to water, while the hydrophobic tails are _____ by water. The soap molecules, when dissolved in water, float around and can either act independently or can join with other soap molecules to form bubbles.

3

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ repelled
- Ⓑ levitated
- Ⓒ captivated
- Ⓓ narrowed

Organisms that must consume oxygen to live are known as aerobic organisms, while those that do not need oxygen are considered anaerobic. Aerobic organisms _____ most of their energy from a process known as cellular respiration. This process is oxygen-requiring and cannot be completed by anaerobic organisms. Cellular respiration produces 18 times more energy units per glucose molecule consumed in glycolysis than the anaerobic alternative, fermentation.

4

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ discover
- Ⓑ inhale
- Ⓒ derive
- Ⓓ discern

Since its development in 1932 by Soviet engineer Konstantin Khrenov, marine welding has become a highly specialized and _____ career. As the name suggests, marine welders construct or repair infrastructures like oil rigs and bridges underwater. Welding occurs when structures like metals or thermoplastics are heated to high temperatures and melted together, causing the structures to bind.

5

Which choice completes the text with the most logical and precise word or phrase?

- Ⓐ monetary
- Ⓑ exorbitant
- Ⓒ budgetary
- Ⓓ lucrative

The following text is adapted from the 1918 American Federation of Labor Records. In it, the American Federation of Labor discusses the possible ramifications of labor uprisings and revolts in Europe as the end of World War I approaches.

The peace then offered would be even worse than that of Haase, namely, . . . adaptation of the status quo ante and “no annexations, no indemnities” formula, with neighboring nations bound by coerced economic treaties and “readjustments of the frontiers.”

To aid the German Socialists (positively) and the Bolsheviki (negatively) is not only playing with fire, it is almost certain to end the war before German defeat or American victory—with all the consequences that must inevitably follow such an indecisive outcome.

6

Which choice best describes the function of the underlined selection in the text as a whole?

- Ⓐ To underscore potentially negative consequences
- Ⓑ To predict pending physical harm
- Ⓒ To demonstrate possible sources of reconciliation
- Ⓓ To forecast a coming perpetual peace

Text 1

Sitting at the Art Institute of Chicago is a famous work of art—*A Sunday on La Grande Jatte*. The piece took artist Georges Seurat more than two years to complete. The painting depicts 48 Parisian citizens spending their afternoon along the Seine River. He sought to capture the timelessness seen in historical art using pointillist technique. By positioning dots of contrasting color closely together, the viewers’ eyes would naturally blend the pigments when they looked upon it.

Text 2

Claude Monet’s painting, *Impression: Sunrise*, marked the beginning of the impressionist movement. It was characterized by large, rough brushstrokes that stood out on the canvas. Nature was the preferred muse of its painters, focusing on the shifting light patterns across the environment. Because of the focus on landscapes, most impressionist painters worked outside on small canvases. They wanted to capture the scenery in its fleeting moments, sometimes coming back and repainting the scene in different seasons.

Based on the texts, the author of Text 2 is more concerned with what objective than is the author of Text 1?

- Ⓐ Describing the specific painting technique used in a work
- Ⓑ Placing the analysis of a single painting in a larger artistic context
- Ⓒ Analyzing the setting in which an artistic scene takes place
- Ⓓ Noting the biographical details of a famous artist

“Antigenic drift” and “antigenic shift” are the main ways influenza circumvents our adaptive immunity; they are likely the causes of epidemic and pandemic influenza, respectively. To date, 16 HA and nine NA subtypes have been identified, only a fraction of which are currently infectious to humans. However, because the influenza genome is split into segments, when an animal—a bird, for instance—is co-infected with a strain specific to its species, as well as one capable of infecting humans, the segments may become intermixed during replication in a process called “viral reassortment.” When these reassorted genes include either HA or NA, antigenic shift occurs, and the resulting viral particles will express novel proteins to which the entire human race is vulnerable. The aggregation of many point mutations over time is known as antigenic drift, and eventually results in renewed vulnerability to viral strains against which an individual was previously immune.

Based on the text, would antigenic drift or antigenic shift result in greater fundamental changes to genetic structure?

- Ⓐ Antigenic drift because it results in increasing vulnerability to viruses.
- Ⓑ Antigenic drift because it can easily spread throughout the body.
- Ⓒ Antigenic shift because it entails genetic replication.
- Ⓓ Antigenic shift because it involves interspecies genome exchange.

The line between over-sterilization and under-sterilization seems to be a tight rope with falling on either side leading to disaster. However, with common sense and moderation, most healthy people can remain healthy with little effort: Wash your hands when you suspect you may have been exposed to something that could make you ill (raw meat, bathroom germs) and before you eat. The rest of the time, let less harmful germs interact with your immune system to keep you healthy.

9

Which of the following, if true, would best support the narrator's position?

- Ⓐ A research study finds that those who wash their hands approximately eight times per day have substantially lower incidence of illness than those who wash their hands more than 20 times per day.
- Ⓑ Those who only wash their hands before a meal have less illness on average than those who wash their hands only after using the restroom.
- Ⓒ The germs that are most affected by hand washing are more prominent in urban areas than in rural areas.
- Ⓓ A chemist finds that for the vast majority of germs, hand sanitizer has the same impact on germ reduction as does hand washing.

The Jungle is a 1906 novel by Upton Sinclair. In this story, the characters Jurgis, Ona, and Teta find it extremely difficult to plan their budget because of the high likelihood of unpredictable expenses: _____

10

Which quotation from *The Jungle* most effectively illustrates the claim?

- Ⓐ “She had had to give up all idea of marrying then; the family could not get along without her—though for that matter she was likely soon to become a burden even upon them”

- Ⓑ “So Jurgis and Ona and Teta Elzbieta would hold anxious conferences until late at night, trying to figure how they could manage this too without starving.
- Ⓒ “This was in truth not living; it was scarcely even existing, and they felt that it was too little for the price they paid.”
- Ⓓ “There seemed never to be an end to the things they had to buy and to the unforeseen contingencies.”

	Demographic	Percentage
Gender	Men	72%
	Women	76%
Ages	Ages 18-29	89%
	Ages 30-49	82%
	Ages 50-64	65%
	Ages 65+	49%
Education Level	High school graduate or less	72%
	Some college	78%
	College graduate and beyond	73%
Yearly Salary	Less than \$30,000 per year	79%
	Between \$30,000 and \$49,000	73%
	Between \$50,000 and \$74,000	70%
		78%

	Greater than \$75,000	
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Source: Pew Research Center

A social scientist is researching the general characteristics of people who use social networking websites. The scientist is putting together a presentation and wants to give a concise example of a typical social networking website user. Based on the latest research from the Pew Research Center, the social scientist could best cite this type of person as an example for her presentation: _____.

11

Which choice most effectively uses data from the graph to complete the example?

- Ⓐ A 45-year-old man who has an advanced graduate degree
- Ⓑ A 35-year-old man who has a master's degree in engineering
- Ⓒ A 25-year-old woman who left college without completing her degree
- Ⓓ A 15-year-old girl who is a sophomore in high school

The primary electrolytes lost during sweating are Na^+ and K^+ . Water follows Na^+ in the body, so when one sweats profusely, a decent number of electrolytes are lost as well. Researchers are looking at electrolyte loss in warm versus hot environments, wondering if there is a difference in performance from the amount of sweat lost. They hypothesize that losing body water would not be as detrimental during exercise in the cold.

12

Which observation, if true, would most directly support the researchers' hypothesis?

- Ⓐ During the winter months in the Northern Hemisphere, the temperatures in the Southern Hemisphere are generally warmer.
- Ⓑ The electrolytes Na^+ and K^+ can be easily supplemented in the diet through ingestible tablets.
- Ⓒ Those who have a relatively high body mass index tend to sweat more than those with a low body mass index.
- Ⓓ The average body does not sweat as much in a cold environment compared to a warm environment.

Portfolio variance is an estimation of the risk of a portfolio, with a higher number indicating a greater risk and a lower number indicating a lower risk.

Number of Stocks	Expected Portfolio Variance
1	46.6
2	26.8
4	16.9
6	13.7
8	12.0
10	11.0
20	9.0
50	7.8
100	7.5
1,000	7.1

Data gathered from Edwin Elton and Martin Gruber in their article “Risk Reduction and Portfolio Size: An Analytical Solution,” published in The Journal of Business in 1977.

A stockbroker is trying to recommend the best portfolio construction for a hypothetical client. The stockbroker wants to create a recommended course of action for the hypothetical client to reduce risk but does not know the precise makeup of the client’s stock portfolio. The stockbroker anticipates that a significant reduction in portfolio risk would most likely come from the following adjustment: _____



13

Which choice most effectively uses data from the table to complete the statement?

- Ⓐ having two stocks instead of just one.
- Ⓑ having six stocks instead of four.
- Ⓒ having 20 stocks instead of 10.
- Ⓓ having 1,000 stocks instead of 100.

Many countries—including the U.S.—have used Keynesian initiatives to varying degrees of success as a jumpstart for their failing economies. It is difficult even for economists to say with any certainty why the policy has worked so well for Sweden. One possible contributor is the remarkable capacity for compromise demonstrated by Swedish politicians, businessmen, labor unions and farmers, even in times of financial turmoil. Although the Social Democrats have remained in power in Sweden for most of the last century, 90% of Swedish industry remains privately owned; labor disputes, in contrast, _____

14

Which choice most logically completes the text?

- Ⓐ are settled by a governmental bureau trusted by workers and management alike.
- Ⓑ are administered by entrepreneurs who seek to maximize profits for shareholders.
- Ⓒ look to stimulate the economy through increased governmental spending.
- Ⓓ are negotiated by opposing leaders who seek to maximize concessions from the other side.

Guest workers can be subject to abhorrent treatment: they often must pay pre-arrival recruiters exorbitant fees for very minor work, and recruiters

may even hold _____ hostage until additional fees are paid.

15

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ workers passports' and visas'
- Ⓑ workers passports and visas
- Ⓒ workers' passports and visas
- Ⓓ worker's passports and visas

The scientific method is the process that takes a hypothesis, tests it, and with careful observation, may lead to the discovery of new knowledge.

16

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ One of the key components of the scientific method is the ability to replicate findings.
- Ⓑ To replicate findings is, of the scientific method, one of the key components.
- Ⓒ Of the scientific method to replicate findings is one of the key components.
- Ⓓ Replicating findings of one of the components is, of the scientific method, key.

Cats may have been selectively bred by humans for traits like friendliness and tameness, but they may also have developed those traits naturally through generations of coexisting with humans. This is where the phrase “cats domesticated themselves” comes from: no one is certain exactly how

we came to have _____ predators in our homes, but we certainly appreciate them.

17

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ loveable, fluffy
- Ⓑ loveable fluffy
- Ⓒ loveable—fluffy—
- Ⓓ loveable; fluffy

During the first roller derby event called the Transcontinental Roller Derby, each team consisted of _____. Additionally, female and male pairs would compete against players of their respective gender, and all of the players abided by the same roller derby rules.

18

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ two players: one woman; and one man.
- Ⓑ two players, one woman, and one man.
- Ⓒ two players: one woman and one man.
- Ⓓ two players; one woman—and one man.

Comité des Citoyens (the Committee of Citizens) brought *Plessy v. Ferguson* to the Supreme Court, where the Court cemented the separate but equal doctrine into law until it was _____ down in *Brown v. Board of Education*.

19

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ striking
- Ⓑ strucked
- Ⓒ struck
- Ⓓ striked

The group of modern artists came up with their movement's name after a knife that was randomly inserted into a dictionary seemed to point to the word "dada." The _____ to go against conformity and the violence surrounding World War I. Dada was inspired by various art styles like Expressionism, Cubism, Constructivism, and Futurism, although the movement itself was not classified as an art style.

20

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ word "dada" means "hobby-horse" in French which suited the premise of the movement,
- Ⓑ word "dada" means "hobby-horse" in French which suited the premise of the movement
- Ⓒ word "dada" means "hobby-horse" in French; which suited the premise of the movement,
- Ⓓ word "dada" means "hobby-horse" in French, which suited the premise of the movement

Many of the greatest battles in history can be simplified to microcosmic struggles between two strong characters. Take Grant and Lee, for instance. Or consider David and Goliath if you prefer an older story. But not often _____ such a struggle occur when both great leaders are on the same side.

21

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ do
- Ⓑ doing
- Ⓒ does
- Ⓓ have done

For a casual birder—another name for “birdwatcher”—common backyard birds like house sparrows, European starlings, and American robins can be enough. _____ want to see all kinds of birds in their backyards.

22

Which choice completes the text so that it conforms to the conventions of Standard English?

- Ⓐ Some birders however have more ambition they
- Ⓑ Some birders, however have more ambition; they
- Ⓒ Some birders however have more ambition—they
- Ⓓ Some birders, however, have more ambition: they

Some interpret the pond in the poem as symbolizing subconsciousness—specifically, the subconsciousness of an older person. Just as frogs are motivated to jump into the water by instinct, humans relax and meditate because their biology prompts them to do so. _____ the frog jumping into the water symbolizes the meditative state of the older person.

23

Which choice completes the text with the most logical transition?

- Ⓐ Also,
- Ⓑ Despite this,

- Ⓒ Additionally,
- Ⓓ Thus,

Creating the closest precursor of the periodic table, Chancourtois—utilizing the invaluable molar mass work of Amedeo Avogadro—graphed 63 known elements in order of increasing atomic weight on his “telluric helix,” and observed an element’s chemical reactivity. _____ because Chancourtois inexplicably included several polyatomic ions on the helix and published his report using geological rather than chemical terminology, his discovery was largely ignored.

24

Which choice completes the text with the most logical transition?

- Ⓐ Simultaneously,
- Ⓑ Moreover,
- Ⓒ Naturally,
- Ⓓ Unfortunately,

While researching a topic, a student has taken the following notes:

- The submarine *CCS Hunley* was the first to sink another ship, the *USS Housatonic*.
- The *Hunley* was used by the Confederates during the Civil War and it was human-powered.
- The *Hunley* disappeared after its first and only attack that sank another ship.
- The *Hunley* remained missing until 2000 when it was raised from the sea floor.
- The *Hunley* was in surprisingly good condition, which allowed researchers to learn more about its final battle.

25

The student wants to underscore why there was little information to be found about the *Hunley* from the 20th century. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Given the near total destruction of the *Hunley* during a Civil War battle, it makes sense that there was little of historical significance available to analyze.
- Ⓑ Because the *Hunley* was preserved in excellent condition, researchers have frequently been able to gather information about it since the Civil War.
- Ⓒ Since the *Hunley* was only recently raised from the sea floor, it is understandable that there is little 20th-century research on this vessel.
- Ⓓ Lacking a fossil-fuel power source, the *Hunley* did not experience the same type of mechanical decay often observed in Civil War artifacts.

While researching a topic, a student has taken the following notes:

- The increasing digitization of historical records has increased access to those documents and helped increase the popularity of genealogy as a hobby.
- Genealogy is tracing one's family history.
- Knowing a few key kinship terms will help the hobby genealogist get started.
- "Consanguinity" refers to family members you're related to by blood.
- "Affinal" refers to family members you're related to by marriage.

The student wants to use specialized terminology to emphasize what can be accomplished with digitized genealogical records. Which choice most

effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ Digitized genealogical records are made using advanced programming languages like Python and Java.
- Ⓑ With the aid of digitized genealogical records, hobby genealogists can learn about their family history.
- Ⓒ By learning about “family,” one can successfully navigate the complex world of digitized genealogy.
- Ⓓ Utilizing digitized genealogical records can enable you to determine consanguineous and affinal relations.

While conducting research, a student took these notes:

- Antibiotic resistance has been a growing concern in the medical field for the past decade.
- One pathway taken to resolve this is an adjuvant that targets bacterial translation.
- The gene, *efp*, codes for the elongation factor protein, EF-P, which works to increase the efficiency of polyproline synthesis.
- One study showed that the deletion of *efp* in bacteria increases its susceptibility to beta-lactam antibiotics.
- Another study tried this deletion but tested bacteria’s resistance to carbapenem antibiotics instead.

27

The student wants to suggest a course of action that would provide comparative analysis of the effectiveness of certain antibiotic medications. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- Ⓐ The elongation factor protein and the polyproline synthesis should be examined to see which is more effective in minimizing illness.

- Ⓑ The results of the carbapenem study and the beta-lactam study should be compared to evaluate which antibiotic is more helpful.
- Ⓒ Studies comparing which bacteria are more susceptible and less susceptible to a particular antibiotic would go quite far in helping doctors skillfully treat patients.
- Ⓓ A historical survey of the treatments used prior to the advent of antibiotics would educate practitioners about the evolution of medical interventions.

Section 2, Module 1: Math

35 MINUTES, 22 QUESTIONS

DIRECTIONS ▼

- All expressions and variables use real numbers.
- All figures are drawn to scale.
- Every figure lies in a plane.
- The domain of given functions is the set of all real numbers for which the corresponding value of the function is real.

For **multiple-choice questions**, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer.

For **student-produced response questions**, solve each problem and record your answer following these guidelines:

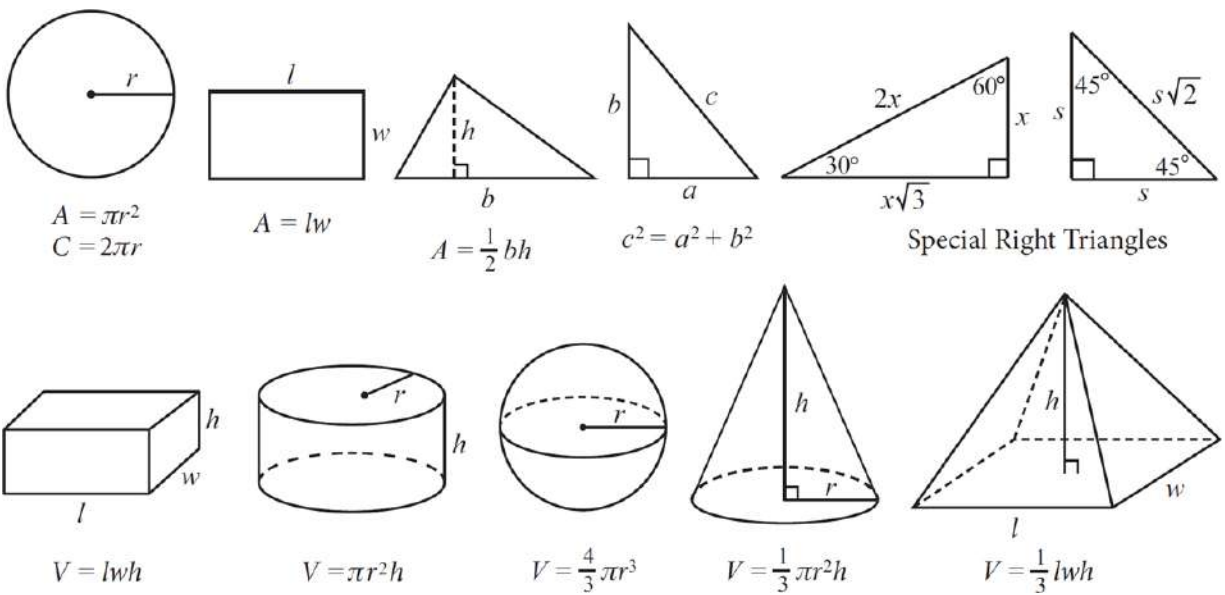
- If you find **more than one correct answer**, enter just one answer.
 - You can enter up to five characters for a **positive** answer and up to six characters (this includes the negative sign) for a **negative** answer.
 - If your answer is a **fraction** that does not fit in the given space, enter the decimal equivalent instead.
 - If your answer is a **decimal** that does not fit in the given space, enter it by stopping at or rounding up at the fourth digit.
 - If your answer is a **mixed number** (like $4\frac{1}{2}$), enter it as an improper fraction ($\frac{9}{2}$) or its decimal equivalent (4.5).
 - Do not enter **symbols** like a comma, dollar sign, or percent sign.
-

Examples

Answer	Acceptable Entries	Unacceptable Entries That Will Receive Zero Credit
4.5	4.5 4.50	$4\frac{1}{2}$ 41/2

	9/2	
$\frac{8}{9}$	8/9	0.88
	.8888	.88
	.8889	0.89
	0.888	.89
	0.889	
$-\frac{1}{9}$	-1/9	-.11
	-.1111	-0.11
	-0.111	

REFERENCE ∨



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

2

MODULE 1

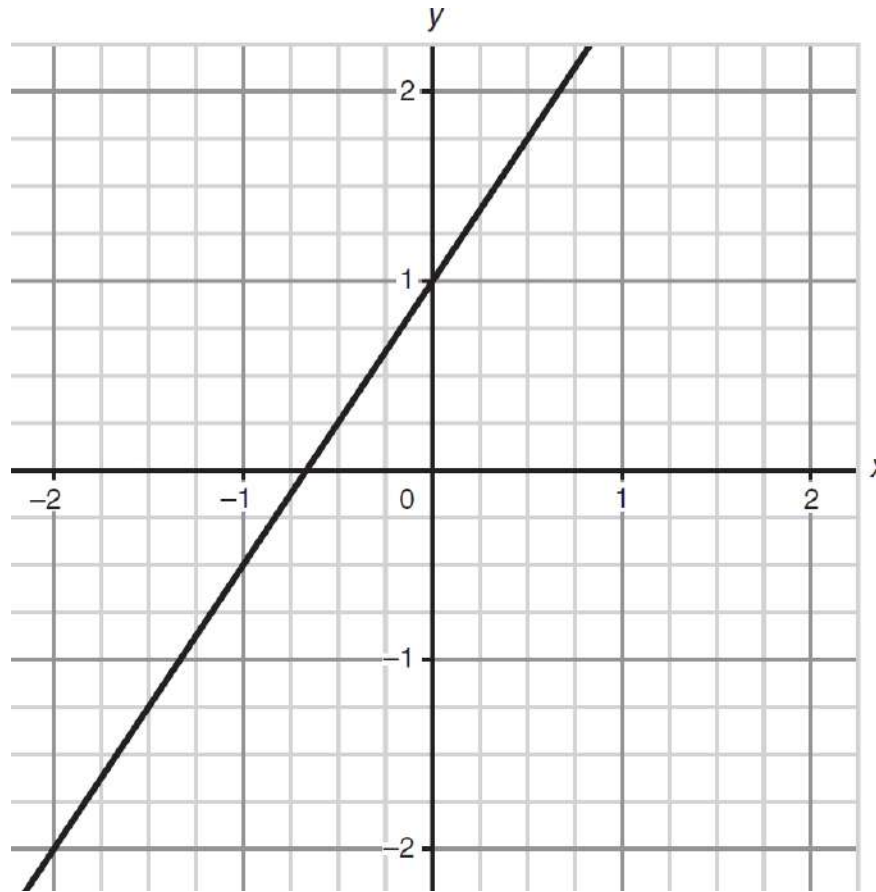
2

1

What is an equivalent form of $(0.8a + 0.7b) - (1.4a - 0.3b)$?

- Ⓐ $2.2a + b$
- Ⓑ $-0.6a + b$
- Ⓒ $1.12a - .21b$
- Ⓓ $-0.6a - 0.4b$

2



Based on the graph of the line above, what is its slope?

- Ⓐ $\frac{2}{3}$
- Ⓑ 1
- Ⓒ $\frac{3}{2}$
- Ⓓ $\frac{4}{3}$

3

If $12x - 10 = 4$, what is $-5 + 6x$?

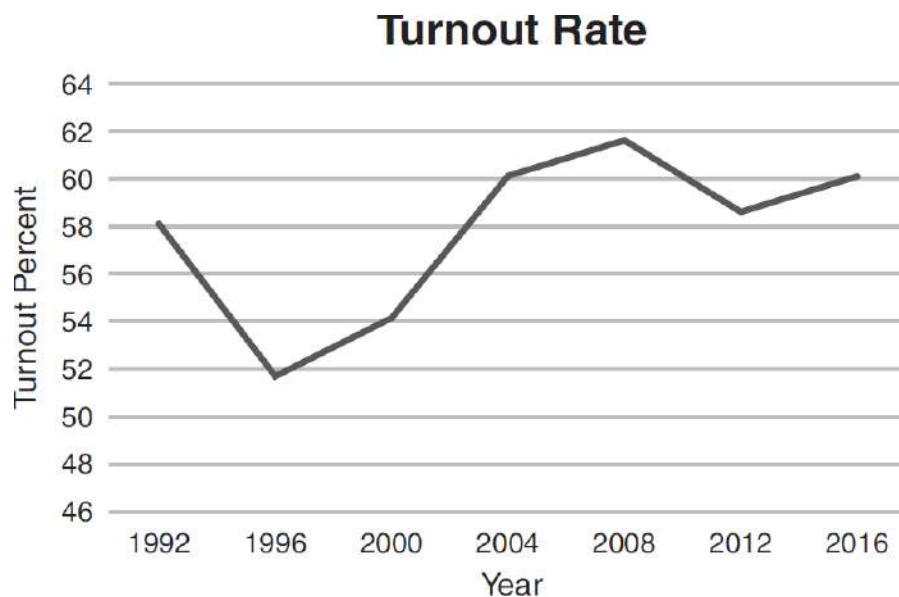
4

The volume of a cylinder, v , is given by the equation $v = \pi r^2 h$, in which r is the radius of the cylinder and h is the height of the cylinder. What is the height in terms of the other variables?

- (A) $h = \frac{v}{\pi r^2}$
(B) $h = \frac{\pi \sqrt{r}}{v}$
(C) $h = \frac{2\pi v}{r}$
(D) $h = \pi r^2 v$

5

Voter turnout rates for different U.S. presidential elections are given in the following graph:



During what year was voter turnout the highest?

- Ⓐ 1996
- Ⓑ 2004
- Ⓒ 2008
- Ⓓ 2016

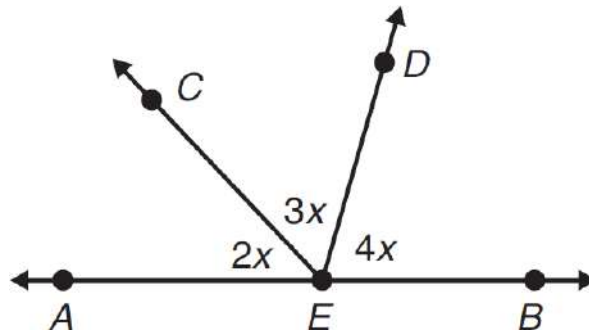
6

Which of these options is equivalent to the following expression?

$$(2y^2 - y) - (2y^2 + 2y)$$

- Ⓐ $4y^4 - 2y^2$
- Ⓑ $4y^2 - 2y$
- Ⓒ $-3y$
- Ⓓ $6y + 2$

7



In the figure above, lines $\overline{AEB}$, $\overline{CE}$, and $\overline{DE}$ all intersect at point E and have measures as indicated. What is the measure of angle $\angle CED$?

- Ⓐ 20°
- Ⓑ 30°

- Ⓒ 45°
- Ⓓ 60°

8

$$(x + 2)^3 = 27$$

What positive value of x would make the above equation true?

9

	Appetizer	No Appetizer	Total
Dessert	15	35	50
No Dessert	25	25	50
Total	40	60	100

A restaurant tabulated meals from 100 of its diners to record if the diner ordered an appetizer and/or a dessert with a meal. Based on the table above, what is the probability that if a diner ordered dessert, they would also order an appetizer?

- Ⓐ $\frac{1}{5}$
- Ⓑ $\frac{3}{10}$
- Ⓒ $\frac{3}{8}$
- Ⓓ $\frac{3}{7}$

10

$$y = 2x - 3$$

What is the x -coordinate of the x -intercept of the line with the above equation?

11

The graph of $y = \frac{1}{2}x - 2$ in the xy -plane is a line. What is the y -intercept of this line?

- (A) -2.5
- (B) -2
- (C) $\frac{1}{2}$
- (D) 2

12

If $3 = \frac{3}{2}(4x - 2y)$ and $y = \frac{1}{2}x$, what is the value of y ?

13

Which of the following would be points at which the function $y = (x + 4)(x - 5)$ intersects the x axis?

- I. $(-4, 0)$
- II. $(5, 0)$
- III. $(0, 0)$

- (A) I only

- Ⓐ II only
- Ⓑ I and III only
- Ⓒ I, II, and III

14

$$B(h) = 1000 \times 2^h$$

The growth of a population of bacteria is modeled by the above function, in which $B(h)$ represents the number of bacteria after h hours have passed. After how many hours will the population of bacteria be 8 times its initial number?

- Ⓐ 1 hour
- Ⓑ 2 hours
- Ⓒ 3 hours
- Ⓓ 4 hours

15

Total of Combined Spins	Number of Spins with That Total Value
2	1
3	2
4	2
5	4
6	5
7	6
8	7
9	4
10	3
11	4

Pete was playing a children's game that had a spinner with values 1-6; with each turn, he spins the spinner twice and finds the total. He tabulated the combined values of two combined spins from a total of 40 different turns as shown above. What fraction of all his combined spins were 6 or less?

- (A) $\frac{1}{40}$
- (B) $\frac{9}{40}$
- (C) $\frac{13}{40}$
- (D) $\frac{7}{20}$

16

The population of the greater metropolitan area of Mexico City was estimated to have been 22,000,000 at the end of the year 2020. If the Mexico City area represented 17% of all of Mexico's population at the end of 2020, what would be the closest estimate of Mexico's total population?

- (A) 4,000,000
- (B) 26,000,000
- (C) 104,000,000
- (D) 129,000,000

17

For the function f as defined below, what is the value of $f(5)$?

$$f(x) = (x - 3)(x + 1)$$

18

The solutions to this set of inequalities are found in which quadrant(s) of the xy -coordinate plane?

$$y > x + 1 \quad \text{and} \quad y < -2x - 3$$

- Ⓐ Only quadrant 1
- Ⓑ Only quadrant 3
- Ⓒ Quadrants 1 and 4 only
- Ⓓ Quadrants 2 and 3 only

19

If a computer costs \$2,000 and depreciates (i.e., the value decreases) by the same amount each year, how much is the annual amount of depreciation if after 6 years the computer is worth \$800?

- Ⓐ \$80
- Ⓑ \$100
- Ⓒ \$200
- Ⓓ \$400

20

If a circle graphed in the xy -coordinate plane has a center at $(3, 4)$ and intersects the origin, what is the equation of the circle?

- Ⓐ $(x - 3)^2 + (y - 4)^2 = 25$
- Ⓑ $(x + 3)^2 - (y + 4)^2 = 25$
- Ⓒ $x^2 + y^2 = 49$
- Ⓓ $x^2 + y^2 = 25$

21

Susan blinks an average of 20 times per minute while awake. In the 16 hours she is awake in a day, how many times will she blink?

- Ⓐ 320
- Ⓑ 7,680
- Ⓒ 12,400
- Ⓓ 19,200

22

Hugo's \$3,000 monthly budget is broken down in this table:

Rent	\$800
Car expenses	\$400
Student loans	\$300
Savings	\$250
Food	\$450
Utilities	\$250
Clothing	\$150
Miscellaneous	\$400

Hugo is trying to allocate more money into savings. If he cut his allocation towards miscellaneous expenses by half and allocated this amount to savings, by what percentage would his monthly savings increase?

- Ⓐ 20%
- Ⓑ 50%
- Ⓒ 65%
- Ⓓ 80%

Section 2, Module 2: Math

32 MINUTES, 27 QUESTIONS

DIRECTIONS ▼

- All expressions and variables use real numbers.
- All figures are drawn to scale.
- Every figure lies in a plane.
- The domain of given functions is the set of all real numbers for which the corresponding value of the function is real.

For **multiple-choice questions**, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer.

For **student-produced response questions**, solve each problem and record your answer following these guidelines:

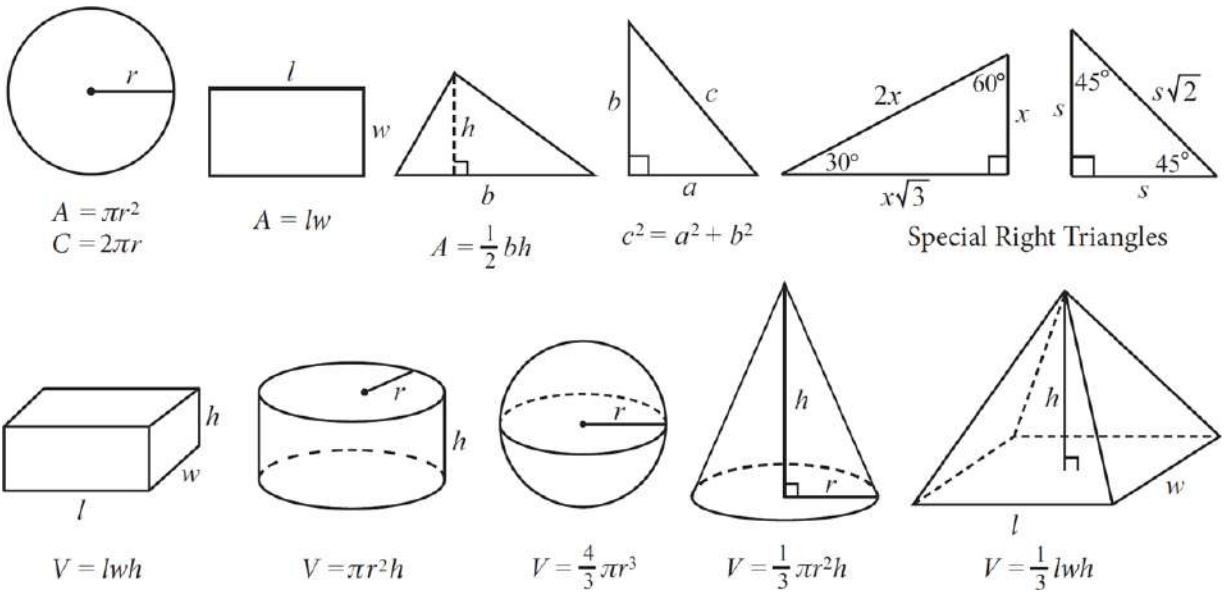
- If you find **more than one correct answer**, enter just one answer.
 - You can enter up to five characters for a **positive** answer and up to six characters (this includes the negative sign) for a **negative** answer.
 - If your answer is a **fraction** that does not fit in the given space, enter the decimal equivalent instead.
 - If your answer is a **decimal** that does not fit in the given space, enter it by stopping at or rounding up at the fourth digit.
 - If your answer is a **mixed number** (like $4\frac{1}{2}$), enter it as an improper fraction ($\frac{9}{2}$) or its decimal equivalent (4.5).
 - Do not enter **symbols** like a comma, dollar sign, or percent sign.
-

Examples

Answer	Acceptable Entries	Unacceptable Entries That Will Receive Zero Credit
4.5	4.5 4.50	$4\frac{1}{2}$ 41/2

	$\frac{9}{2}$	
$\frac{8}{9}$	$\frac{8}{9}$	0.88
	.8888	.88
	.8889	.89
	0.888	0.89
	0.889	
$-\frac{1}{9}$	$-\frac{1}{9}$	-.11
	-.1111	-0.11
	-0.111	

REFERENCE ∨



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

2

MODULE 2

2

1

The function g is defined by $g(x) = \frac{x^3}{2}$. What is the value of $g(-2)$?

- Ⓐ -4
- Ⓑ -2
- Ⓒ -1
- Ⓓ 4

2

x	$f(x)$
3	-1
6	0
9	1

For the linear function f , the table above shows some values of x and the corresponding values of $f(x)$. Which of the following equations correctly defines the function?

- Ⓐ $f(x) = \frac{1}{3}x - 2$
- Ⓑ $f(x) = \frac{2}{3}x - 1$
- Ⓒ $f(x) = x - 6$
- Ⓓ $f(x) = 3x + 2$

3

Which of these is equivalent to $\sqrt[3]{a} \times a^{\frac{5}{3}}$?

- Ⓐ $a^{\frac{2}{3}}$
- Ⓑ $a^{\frac{4}{3}}$
- Ⓒ a^2
- Ⓓ a^5

4

If $y = ax$ in which a is a constant and $y = 16$ when $x = 8$, what is the value of x when y is -3 ?

- (A) -6
- (B) -1.5
- (C) 1
- (D) 8.5

5

$$ax^2 + bx + c = 2x^2 + 16x + 32$$

In the equation above, a , b , and c are all constants. What is the sum of a , b , and c ?

- (A) 48
- (B) 50
- (C) 52
- (D) 64

6

$$S(w) = 500 + 10w$$

Ana receives an allowance of \$10 per week. The function above shows how much money she will have in her savings account after w weeks, assuming she saves all her allowance in the account. What does the number 500 represent in the function?

- (A) The money in her account at the beginning of the time period
- (B) The money in the account at the end of the time period
- (C) The weekly increase in money in the account
- (D) The net profit she receives after expenses

7

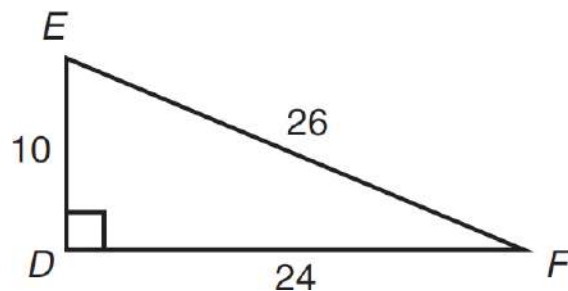
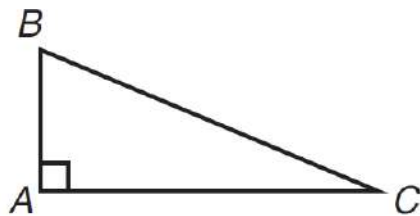
On a map, 1 inch represents 3 miles in actual distance. If there is a rectangular plot of land on the map with a width of 2 inches and a length of 3 inches, what would be the actual area of the rectangular plot of land in square miles?

8

Samara drove on a 400-mile trip. If the first half of the trip took her 4 hours of driving to complete and the second half took her 6 hours of driving to complete, what was her average driving speed for the whole 400-mile trip?

- Ⓐ 20 miles per hour
- Ⓑ 35 miles per hour
- Ⓒ 40 miles per hour
- Ⓓ 80 miles per hour

9

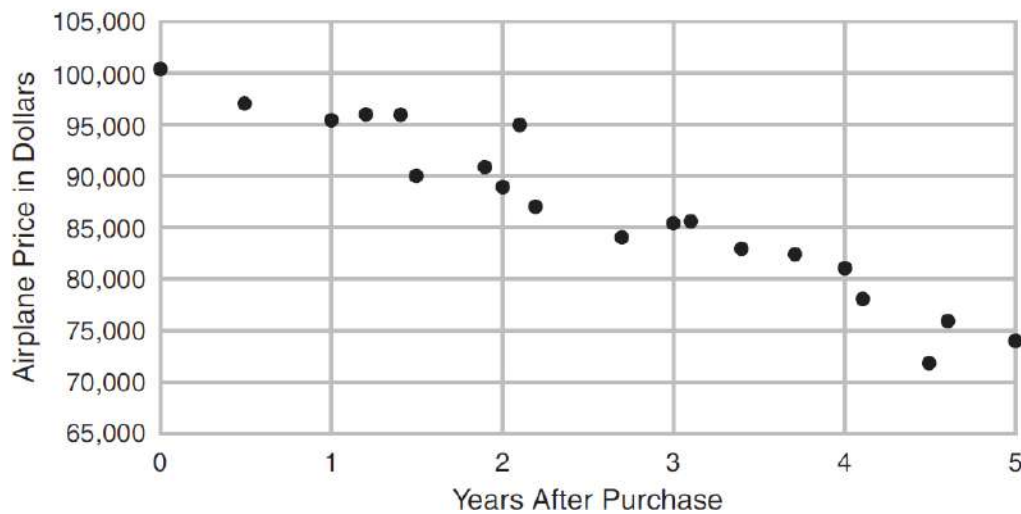


The above triangles ABC and DEF are similar to one another, with angle B corresponding to angle E . What is the tangent of angle C ?

- (A) $\frac{5}{13}$
- (B) $\frac{5}{12}$
- (C) $\frac{10}{13}$
- (D) $\frac{12}{13}$

10

Price of an Airplane Over a Period of Years



If the airplane continues to decrease in value at a linear rate, at what year after purchase will the plane first closely approximate a value of zero dollars?

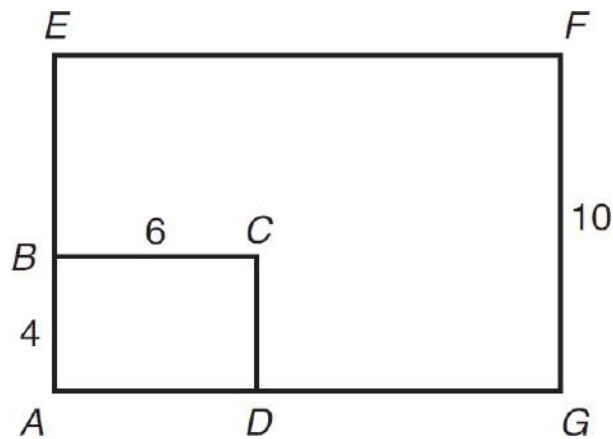
- (A) 10 years after purchase
- (B) 15 years after purchase
- (C) 20 years after purchase
- (D) 25 years after purchase

11

If x is increased by 400%, what is the result in terms of x ?

- Ⓐ z
- Ⓑ $2x$
- Ⓒ $4x$
- Ⓓ $5x$

12



In the figure above, $ABCD$ and $AEFG$ are similar rectangles. If $\overline{AB}$ is 4 units long, $\overline{BC}$ is 6 units long, and $\overline{FG}$ is 10 units long, what is the perimeter of rectangle $AEFG$?

- Ⓐ 24 units
- Ⓑ 48 units
- Ⓒ 50 units
- Ⓓ 72 units

13

How many zeros will a line with the equation $y = k$ have in which k is a constant greater than zero?

- Ⓐ Zero
- Ⓑ One

- Ⓒ Two
- Ⓓ Cannot be determined from the given information

14

Quiz Score (Out of 5 Points)	0 Correct	1 Correct	2 Correct	3 Correct	4 Correct	5 Correct
Class 1 Students' Scores	1	2	4	6	4	3
Class 2 Students' Scores	0	4	7	3	4	2

Based on the information in the table, what is the median quiz score of the combined set of both classes?

15

What would be the value(s) of x that would be a solution or solutions to this equation?

$$\frac{x^2 - 4}{x - 2} = 4$$

- Ⓐ -2
- Ⓑ 2
- Ⓒ 4
- Ⓓ There are no solutions.

16

What is the vertex of a parabola with the equation $y = (x - 6)^2$?

- Ⓐ $(-6, 0)$
- Ⓑ $(0, 0)$

- Ⓒ (6, 0)
Ⓓ (0, 6)

17

Out of all the 44 people to serve as president of the United States before the year 2020, 14 of them had previously been U.S. vice president. Which function expresses the relationship between the number of presidents, P , and vice presidents who became presidents, V ?

- Ⓐ $V = 14P$
Ⓑ $V = \frac{7}{22}P$
Ⓒ $V = \frac{1}{14}P$
Ⓓ $V = \frac{22}{7}P$

18

$$-\frac{1}{4}x + \frac{1}{6}y = 12$$
$$2y = 3x + a$$

If the above set of equations has infinitely many solutions, what is the value of the constant a ?

19

Julian and Maria are budgeting for their upcoming wedding reception. They are inviting people from each side of their families— J people from Julian's side of the family and M people from Maria's side of the family. They are planning on spending \$40 per person invited to the wedding reception. If they want to keep the cost of the wedding reception to less than \$18,000,

and have more than 200 people in attendance, which system of inequalities would represent these conditions?

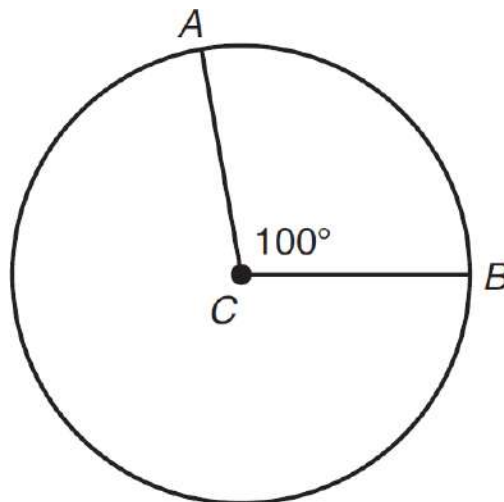
- (A) $M + J \leq 200$
 $40M + J \geq 18,000$
- (B) $M + J > 40$
 $18,000 + M + J > 200$
- (C) $M + J > 18,000$
 $\frac{M + J}{40} < 200$
- (D) $M + J > 200$
 $40(M + J) < 18,000$

20

Given that there are 5,280 feet in a mile, which of the following would most closely approximate 30 miles per hour in feet per second?

- (A) 28 feet/second
- (B) 44 feet/second
- (C) 176 feet/second
- (D) 158,400 feet/second

21



The circle graphed above has a center of C , a radius of 7 units, and a central angle $\angle ACB$ with a measure of 100° . What is the area of the sector of the circle designated by $\angle ACB$ to the nearest whole square unit?

22

At a café, each cup of coffee has 95 milligrams of caffeine, and each cup of tea has 26 milligrams of caffeine. Shantiel goes to the store and purchases only cups of coffee and tea. If the number of cups of coffee that Shantiel purchased was half the number of cups of tea that she purchased, and the total milligrams of caffeine in all the drinks she purchased was 735, how many cups of coffee did she purchase?

- Ⓐ 2
- Ⓑ 3
- Ⓒ 5
- Ⓓ 6

ANSWER KEY

Practice Test 2

Section 1, Module 1: Reading and Writing

1. A
2. A
3. C
4. A
5. A
6. C
7. B
8. B
9. C
10. A
11. A
12. D
13. B
14. C
15. A
16. C
17. A
18. A
19. D
20. D

21. B
22. D
23. A
24. D
25. D
26. C
27. A

Section 1, Module 2: Reading and Writing

1. B
2. C
3. A
4. C
5. D
6. A
7. B
8. D
9. A
10. D
11. C
12. D
13. A
14. A
15. C
16. A
17. A
18. C
19. C
20. D

- 21. C
- 22. D
- 23. D
- 24. D
- 25. C
- 26. D
- 27. B

Section 2, Module 1: Math

- 1. B
- 2. C
- 3. 2
- 4. A
- 5. C
- 6. C
- 7. D
- 8. 1
- 9. B
- 10. 1.5 or $\frac{3}{2}$
- 11. B
- 12. .3333 or $\frac{1}{3}$
- 13. D
- 14. C
- 15. D
- 16. D
- 17. 12
- 18. D
- 19. C
- 20. A

21. D

22. D

Section 2, Module 2: Math

1. A

2. A

3. C

4. B

5. B

6. A

7. 54

8. C

9. B

10. C

11. D

12. C

13. A

14. 3

15. D

16. C

17. B

18. 144

19. D

20. B

21. 43

22. C