

Probability Mastery

100 ESSENTIAL PROBLEMS · COMPLETE COURSE

1. BASIC PROBABILITY · ●

A fair coin is flipped once. What is the probability of getting Heads?

- A) 1
B) 1/4
C) 1/2
D) 3/4

| 💡 $P(\text{event}) = \text{favorable outcomes} \div \text{total outcomes}$.

2. BASIC PROBABILITY · ●

A fair die is rolled. What is the probability of rolling a 3?

- A) 1/2
B) 1/6
C) 1/3
D) 1/4

| 💡 A standard die has 6 equally likely faces.

3. BASIC PROBABILITY · ●

A bag contains 3 red and 2 blue marbles. What is the probability of drawing a red marble?

- A) 2/5
B) 3/2
C) 1/5
D) 3/5

| 💡 Count red marbles over total marbles.

4. BASIC PROBABILITY · ●

If $P(A) = 0.4$, what is $P(A')$? (complement of A)

- A) 0.4
B) 0.6
C) 1.4
D) 0

| 💡 $P(A) + P(A') = 1$.

5. BASIC PROBABILITY · ●

What is the probability of an impossible event?

- A) 1
B) 0.5
C) 0
D) -1

| 💡 Impossible means it can never happen.

6. BASIC PROBABILITY · ●

A fair die is rolled. What is $P(\text{even number})$?

- A) 1/6
B) 1/3
C) 1/2
D) 2/3

| 💡 Even faces: 2, 4, 6 — that's 3 of them.

7. BASIC PROBABILITY · ●

What is the probability of a certain (sure) event?

- A) 0
B) 0.5
C) > 1
D) 1

| 💡 A certain event always occurs.

8. BASIC PROBABILITY · ●

A deck has 52 cards. What is $P(\text{drawing an Ace})$?

- A) 4/52
B) 1/52
C) 13/52
D) 4/13

| 💡 There are 4 Aces in a 52-card deck.

9.

BASIC PROBABILITY · ●

A number is chosen at random from {1, 2, 3, 4, 5}. $P(\text{number} > 3) = ?$

A) $1/5$

B) $3/5$

C) $2/5$

D) $4/5$

| 💡 List numbers greater than 3 in the set.

10.

BASIC PROBABILITY · ●

A bag has 5 identical balls numbered 1–5. What is $P(\text{drawing ball \#2})$?

A) $2/5$

B) $1/5$

C) $1/2$

D) $5/1$

| 💡 Each ball is equally likely.

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PROBLEMS 11-20

11. ADDITION RULE · ●

$P(A) = 0.3$, $P(B) = 0.5$, A and B are mutually exclusive. Find $P(A \cup B)$.

- | | |
|---------|--------|
| A) 0.15 | B) 0.2 |
| C) 0.8 | D) 0.7 |

| 💡 Mutually exclusive: $P(A \cup B) = P(A) + P(B)$.

12. ADDITION RULE · ●

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

- | | |
|--------|--------|
| A) 0.9 | B) 0.7 |
| C) 1.1 | D) 0.3 |

| 💡 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

13. ADDITION RULE · ●

A die is rolled. $A = \{1,2,3\}$, $B = \{3,4,5\}$. $P(A \cup B) = ?$

- | | |
|--------|--------|
| A) 5/6 | B) 1/6 |
| C) 4/6 | D) 6/6 |

| 💡 List all elements in A or B (no repeats).

14. ADDITION RULE · ●

Are A and B mutually exclusive if $P(A \cap B) = 0$?

- | | |
|-------------------|---------------------|
| A) No | B) Yes |
| C) Only sometimes | D) Cannot determine |

| 💡 Definition: mutually exclusive $\leftrightarrow P(A \cap B) = 0$.

15. ADDITION RULE · ●●

$P(A \cup B) = 0.75$, $P(A) = 0.5$, $P(B) = 0.4$. Find $P(A \cap B)$.

- | | |
|---------|---------|
| A) 0.15 | B) 0.25 |
| C) 0.35 | D) 0.1 |

| 💡 Rearrange the addition rule: $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

16. ADDITION RULE · ●●

A card is drawn from 52 cards. $P(\text{Heart or King}) = ?$

- | | |
|----------|----------|
| A) 16/52 | B) 17/52 |
| C) 13/52 | D) 4/52 |

| 💡 Hearts = 13, Kings = 4, King of Hearts = 1. Use addition rule.

17. ADDITION RULE · ●●

$P(A) = 1/3$, $P(B) = 1/4$, $P(A \cap B) = 1/12$. Find $P(A' \cap B')$.

- | | |
|---------|---------|
| A) 1/2 | B) 7/12 |
| C) 5/12 | D) 1/3 |

| 💡 $P(A' \cap B') = 1 - P(A \cup B)$. First find $P(A \cup B)$.

18. ADDITION RULE · ●●

If A and B are mutually exclusive, $P(A) = 0.3$, $P(B) = 0.4$. Find $P(A' \cap B')$.

- | | |
|---------|---------|
| A) 0.3 | B) 0.7 |
| C) 0.12 | D) 0.88 |

| 💡 $P(A' \cap B') = 1 - P(A \cup B)$. For mutually exclusive: $P(A \cup B) = P(A) + P(B)$.

19.

ADDITION RULE · ●●

A student can pass Math or English. $P(\text{Math}) = 0.6$, $P(\text{English}) = 0.7$, $P(\text{both}) = 0.5$. $P(\text{at least one}) = ?$

A) 0.8

B) 0.9

C) 1.3

D) 0.7

💡 $P(\text{at least one}) = P(M \cup E) = P(M) + P(E) - P(M \cap E)$.

20.

ADDITION RULE · ●●

$P(A) = 0.4$, $P(B) = 0.35$, $P(A \cup B) = 0.6$. Are A and B mutually exclusive?

A) Yes, because $P(A) + P(B) = P(A \cup B)$

B) No, because $P(A \cap B) = 0.15 \neq 0$

C) Yes, $P(A \cap B) = 0$

D) Cannot determine

💡 Check if $P(A) + P(B)$ equals $P(A \cup B)$.

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PROBLEMS 21-30

21. CONDITIONAL PROBABILITY · ●

$P(A \cap B) = 0.12$, $P(B) = 0.4$. Find $P(A|B)$.

A) 0.3

B) 0.48

C) 0.08

D) 0.52

| 💡 $P(A|B) = P(A \cap B) / P(B)$.

22. CONDITIONAL PROBABILITY · ●

A die is rolled. Given the result is even, what is $P(\text{result} = 4)$?

A) $1/6$

B) $1/2$

C) $1/3$

D) $2/3$

| 💡 Restrict sample space to $\{2,4,6\}$.

23. CONDITIONAL PROBABILITY · ●●

A bag has 4 red, 6 blue. Two drawn without replacement. $P(\text{2nd red} | \text{1st red}) = ?$

A) $4/10$

B) $3/10$

C) $3/9$

D) $4/9$

| 💡 After drawing 1 red, 3 red remain out of 9 total.

24. CONDITIONAL PROBABILITY · ●●

$P(A) = 0.5$, $P(B|A) = 0.4$. Find $P(A \cap B)$.

A) 0.9

B) 0.1

C) 0.2

D) 0.8

| 💡 $P(A \cap B) = P(A) \times P(B|A)$.

25. CONDITIONAL PROBABILITY · ●●

In a class 60% study Math (M), 40% study Physics (P), 20% study both. Given a student studies Physics, $P(\text{studies Math}) = ?$

A) 0.5

B) 0.6

C) 0.2

D) 0.4

| 💡 $P(M|P) = P(M \cap P) / P(P)$.

26. CONDITIONAL PROBABILITY · ●●

$P(A|B) = 0.6$, $P(B) = 0.5$, $P(A) = 0.4$. Find $P(B|A)$.

A) 0.75

B) 0.5

C) 0.3

D) 1.2

| 💡 $P(B|A) = P(A \cap B) / P(A)$. First find $P(A \cap B) = P(A|B) \times P(B)$.

27. CONDITIONAL PROBABILITY · ●●

Cards numbered 1–10. Given a card > 5 is drawn, $P(\text{card is odd}) = ?$

A) $2/5$

B) $1/2$

C) $3/5$

D) $2/10$

| 💡 Restrict to $\{6,7,8,9,10\}$. Odd ones?

28. CONDITIONAL PROBABILITY · ●●

A family has 2 children. Given at least one is a boy, $P(\text{both boys}) = ?$

A) $1/4$

B) $1/3$

C) $1/2$

D) $2/3$

| 💡 Sample space with at least one boy: $\{BB, BG, GB\}$.

29. CONDITIONAL PROBABILITY · ●●●

$P(A) = 0.3$, $P(B) = 0.5$, $P(A \cup B) = 0.65$. Find $P(A|B)$.

A) 0.3

B) 0.6

C) 0.15

D) 0.5

💡 Find $P(A \cap B)$ first using the addition rule.

30. CONDITIONAL PROBABILITY · ●●●

A box has 3 white, 2 black balls. 2 drawn without replacement. $P(\text{both white}) = ?$

A) $9/25$

B) $3/10$

C) $1/5$

D) $6/20$

💡 $P(W_1 \cap W_2) = P(W_1) \times P(W_2|W_1)$.

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PROBLEMS 31-40

31. INDEPENDENCE · ●

Events A and B are independent. $P(A) = 0.4$, $P(B) = 0.5$. Find $P(A \cap B)$.

- A) 0.9
B) 0.1
C) 0.2
D) 0.8

💡 Independent: $P(A \cap B) = P(A) \times P(B)$.

32. INDEPENDENCE · ●

A coin is flipped and a die is rolled. $P(\text{Head AND } 6) = ?$

- A) $1/12$
B) $1/6$
C) $1/2$
D) $7/12$

💡 These are independent. Multiply the probabilities.

33. INDEPENDENCE · ●●

$P(A) = 0.6$, $P(B) = 0.5$, $P(A \cap B) = 0.3$. Are A and B independent?

- A) Yes, because $P(A) \times P(B) = P(A \cap B)$
B) No, $P(A) \times P(B) \neq P(A \cap B)$
C) Yes, because $P(A \cap B) > 0$
D) Cannot determine

💡 Check: $P(A) \times P(B) = 0.6 \times 0.5 = 0.3$. Compare to $P(A \cap B)$.

34. INDEPENDENCE · ●●

A and B are independent. $P(A) = 0.7$. Find $P(A' \cap B')$ if $P(B) = 0.4$.

- A) 0.28
B) 0.18
C) 0.12
D) 0.42

💡 A' and B' are also independent. $P(A') = 0.3$, $P(B') = 0.6$.

35. INDEPENDENCE · ●●

Two dice rolled. $P(\text{first die} = 2 \text{ AND second die} = 5) = ?$

- A) $7/36$
B) $2/36$
C) $1/36$
D) $1/6$

💡 Dice rolls are independent.

36. INDEPENDENCE · ●●

$P(A|B) = P(A)$. What does this tell us?

- A) A and B are mutually exclusive
B) A and B are independent
C) $P(B) = 0$
D) $P(A) = 0.5$

💡 This is the definition of independence.

37. INDEPENDENCE · ●●

A and B are independent with $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A \cup B)$.

- A) 0.8
B) 0.65
C) 0.15
D) 0.85

💡 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For independent: $P(A \cap B) = P(A)P(B)$.

38. INDEPENDENCE · ●●●

$P(A \cap B) = 0.12$, $P(A') = 0.6$. If A and B are independent, find $P(B)$.

- A) 0.2
B) 0.3
C) 0.48
D) 0.5

💡 $P(A) = 1 - 0.6 = 0.4$. Then $P(B) = P(A \cap B) / P(A)$.

39.

INDEPENDENCE · ●●●

Three independent events A, B, C with $P(A)=P(B)=P(C)=0.5$. $P(\text{all three occur}) = ?$

A) 1.5

B) 0.125

C) 0.25

D) 0.5

💡 Multiply all three probabilities.

40.

INDEPENDENCE · ●●●

A machine works if parts A AND B both work. $P(A \text{ works})=0.9$, $P(B \text{ works})=0.8$. They work independently. $P(\text{machine works}) = ?$

A) 0.72

B) 0.98

C) 0.17

D) 0.8

💡 Multiply for independent AND condition.

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PROBLEMS 41-50

41. BAYES' THEOREM · ●●

$P(A) = 0.4$, $P(B|A) = 0.3$, $P(B|A') = 0.2$. Find $P(B)$.

- A) 0.24
B) 0.5
C) 0.14
D) 0.12

| 💡 Total probability: $P(B) = P(B|A)P(A) + P(B|A')P(A')$.

42. BAYES' THEOREM · ●●

$P(A) = 0.5$, $P(B|A) = 0.4$, $P(B|A') = 0.2$. Find $P(A|B)$.

- A) 0.5
B) 0.667
C) 0.4
D) 0.3

| 💡 Bayes: $P(A|B) = P(B|A)P(A)/P(B)$. First find $P(B)$.

43. BAYES' THEOREM · ●●

A test for a disease is 90% accurate. Disease rate is 1%. $P(\text{positive test}) = ?$

- A) 0.9
B) 0.1071
C) 0.01
D) 0.099

| 💡 $P(+) = P(+|D)P(D) + P(+|D')P(D')$. $P(+|D') = 0.1$ (10% false positive).

44. BAYES' THEOREM · ●●

Box 1 has 3R,2B. Box 2 has 1R,4B. A box is chosen at random, then a ball. $P(R) = ?$

- A) 0.4
B) 0.3
C) 0.5
D) 0.45

| 💡 $P(R) = P(R|B1) \times 1/2 + P(R|B2) \times 1/2$.

45. BAYES' THEOREM · ●●●

$P(A) = 0.6$, $P(B|A) = 0.5$, $P(B|A') = 0.3$. Find $P(A|B)$.

- A) 0.5
B) 0.625
C) 0.714
D) 0.6

| 💡 Find $P(B)$ first, then apply Bayes.

46. BAYES' THEOREM · ●●●

3 factories make bulbs: 30%, 50%, 20% of total. Defect rates: 2%, 1%, 3%. A random bulb is defective. $P(\text{from factory 1}) = ?$

- A) 0.35
B) 0.30
C) 0.375
D) 0.25

| 💡 $P(D) = \text{sum of } P(D|Fi)P(Fi)$. Then use Bayes.

47. BAYES' THEOREM · ●●●

Two urns: Urn A has 4W,2B; Urn B has 2W,4B. Urn chosen with $P(A) = 1/3$. A white ball drawn. $P(\text{urn A} | \text{white}) = ?$

- A) 1/3
B) 2/3
C) 4/9
D) 1/2

| 💡 $P(W|A) = 4/6 = 2/3$. $P(W|B) = 2/6 = 1/3$. Apply Bayes.

48.

BAYES' THEOREM · ●●●

$P(H) = 0.4$ (hypothesis). $P(E|H) = 0.8$, $P(E|H') = 0.3$. Evidence E observed. $P(H|E) = ?$

A) 0.64

B) 0.64/0.50

C) 0.64

D) 0.64

💡 $P(E) = P(E|H)P(H) + P(E|H')P(H')$. Then Bayes.

49.

BAYES' THEOREM · ●●●

$P(B|A) = 0.7$, $P(A) = 0.3$, $P(B) = 0.4$. Find $P(A|B)$.

A) 0.21

B) 0.5

C) 0.525

D) 0.3

💡 $P(A|B) = P(B|A) \times P(A) / P(B)$.

50.

BAYES' THEOREM · ●●●

A disease affects 2% of population. A test has 95% sensitivity, 90% specificity. $P(\text{disease} | \text{positive test}) = ?$

A) 0.95

B) 0.162

C) 0.02

D) 0.90

💡 $P(+) = P(+|D)P(D) + P(+|D')P(D')$. $P(+|D') = 1 - \text{specificity} = 0.10$.

Probability Mastery

PROBLEMS 51-60

51. PERMUTATIONS · ●

How many ways can 3 students be arranged in a line from 5 students?

- A) 10 B) 20
C) 60 D) 120

| 💡 $P(5,3) = 5 \times 4 \times 3$.

52. PERMUTATIONS · ●

In how many ways can 4 people sit in 4 chairs?

- A) 4 B) 8
C) 16 D) 24

| 💡 $4! = 4 \times 3 \times 2 \times 1$.

53. PERMUTATIONS · ●●

How many 3-letter codes can be formed from {A,B,C,D,E} if no letter repeats?

- A) 15 B) 60
C) 125 D) 10

| 💡 $P(5,3) = 5 \times 4 \times 3$.

54. PERMUTATIONS · ●●

How many ways can the word 'MATH' be arranged?

- A) 12 B) 16
C) 24 D) 4

| 💡 4 distinct letters: $4!$

55. PERMUTATIONS · ●●

How many arrangements of 'LEVEL' are there?

- A) 120 B) 30
C) 60 D) 20

| 💡 LEVEL has 5 letters: L×2, E×2, V×1. Formula: $5!/(2!2!)$.

56. PERMUTATIONS · ●●

How many 4-digit numbers can be formed from digits 1–9 with no repetition?

- A) 3024 B) 6561
C) $9 \times 9 \times 9 \times 9$ D) $9!/5!$

| 💡 $P(9,4) = 9 \times 8 \times 7 \times 6$.

57. PERMUTATIONS · ●●

6 people sit around a circular table. How many arrangements?

- A) 720 B) 360
C) 120 D) 6

| 💡 Circular permutations: $(n-1)!$

58. PERMUTATIONS · ●●●

How many ways can 5 people be arranged in a row if 2 specific people must be adjacent?

- A) 24 B) 48
C) 120 D) 60

| 💡 Treat the 2 as a block: $4! \times 2!$.

59.

PERMUTATIONS · ●●●

How many arrangements of 'PROBABILITY' contain all the letters? (11 letters: P, R, O, B, A, B, I, L, I, T, Y — B×2, I×2, rest unique)

A) 39916800

B) 9979200

C) 19958400

D) 4989600

| 💡 $11!/(2!2!)$ for the repeated B and I.

60.

PERMUTATIONS · ●●●

8 books on a shelf: 3 Math, 3 Science, 2 English. Books of same subject must be together. How many arrangements?

A) 432

B) 1296

C) 864

D) 216

| 💡 Arrange 3 groups: 3! ways. Within: $3! \times 3! \times 2!$.

Probability Mastery

PROBLEMS 61-70

61. PERMUTATIONS · ●●●

8 books on a shelf: 3 Math, 3 Science, 2 English. Books of same subject together. Total arrangements?

- A) 432
B) 864
C) 1296
D) 72

| 💡 $3! \text{ ways to arrange subject groups} \times \text{internal arrangements.}$

62. COMBINATIONS · ●

$C(6,2) = ?$

- A) 12
B) 15
C) 30
D) 6

| 💡 $C(n,r) = n!/(r!(n-r)!).$

63. COMBINATIONS · ●

A committee of 3 is chosen from 7 people. How many ways?

- A) 210
B) 35
C) 21
D) 7

| 💡 $C(7,3) = 7!/(3!4!).$

64. COMBINATIONS · ●●

From 10 students, 4 are selected. How many ways?

- A) 5040
B) 210
C) 3024
D) 240

| 💡 Order doesn't matter, so use $C(10,4).$

65. COMBINATIONS · ●●

A hand of 5 cards is dealt from 52. How many possible hands?

- A) 2598960
B) 311875200
C) 54145
D) 10!

| 💡 $C(52,5).$

66. COMBINATIONS · ●●

From 5 men and 4 women, a committee of 3 men and 2 women is formed. How many ways?

- A) 60
B) 120
C) 45
D) 126

| 💡 $C(5,3) \times C(4,2).$ Multiply.

67. COMBINATIONS · ●●

$C(n,2) = 10.$ Find $n.$

- A) 4
B) 5
C) 6
D) 3

| 💡 $n(n-1)/2 = 10 \rightarrow n(n-1) = 20.$

68. COMBINATIONS • ●●

A pizza can be topped with any selection of 4 toppings from 8 available. How many different pizzas?

- A) 32
C) 1680
- B) 70
D) 56

💡 $C(8,4)$.

69. COMBINATIONS · ●●●

From 12 students, 4 are picked. At least 2 must be girls. There are 5 girls and 7 boys. How many ways?

- A) 310
B) 280
C) 35
D) 175

💡 At least 2 girls: (exactly 2G + exactly 3G + exactly 4G). Add them.

70. COMBINATIONS · ●●●

From 5 girls and 7 boys, select 4 students with AT LEAST 2 girls. How many ways?

- A) 285
B) 210
C) 495
D) 310

💡 Sum: 2G2B + 3G1B + 4G0B.

Probability Mastery

PROBLEMS 71-80

71. COMBINATIONS · ●●●

In how many ways can 6 different colored balls be divided into two groups of 3?

- A) 10 B) 20
C) 15 D) 40

💡 $C(6,3)=20$. But are the groups labeled or not? If unlabeled, divide by 2.

72. COMBINATIONS · ●●●

6 different colored balls divided into two DISTINCT groups of 3. How many ways?

- A) 10 B) 20
C) 60 D) 15

💡 $C(6,3)$ — once first group is chosen, second is determined.

73. DISCRETE DISTRIBUTIONS · ●●

$X \sim B(10, 0.3)$. $E[X] = ?$

- A) 0.3 B) 10
C) 3 D) 7

💡 For Binomial: $E[X] = np$.

74. DISCRETE DISTRIBUTIONS · ●●

$X \sim B(n, p)$. $\text{Var}(X) = ?$

- A) np B) np^2
C) npq where $q=1-p$ D) n^2p

💡 Variance of Binomial = $np(1-p)$.

75. DISCRETE DISTRIBUTIONS · ●●

A fair coin flipped 8 times. $P(\text{exactly 3 Heads}) = ?$

- A) $C(8,3)(0.5)^8$ B) $C(8,3)(0.5)^3$
C) $(0.5)^3$ D) $3/8$

💡 Binomial: $P(X=k) = C(n,k)p^k(1-p)^{(n-k)}$.

76. DISCRETE DISTRIBUTIONS · ●●

$X \sim \text{Geometric}(p=0.2)$. $E[X] = ?$

- A) 5 B) 0.2
C) 4 D) 0.8

💡 For Geometric (# of trials until first success): $E[X]=1/p$.

77. DISCRETE DISTRIBUTIONS · ●●

$X \sim \text{Poisson}(\lambda=3)$. $E[X] = ?$

- A) $1/3$ B) 9
C) 3 D) $\sqrt{3}$

💡 For Poisson: $E[X] = \lambda$.

78. DISCRETE DISTRIBUTIONS · ●●

$X \sim \text{Poisson}(\lambda=2)$. $\text{Var}(X) = ?$

- A) 4 B) $\sqrt{2}$
C) 2 D) $1/2$

💡 Poisson: $\text{Var}(X) = \lambda$.

79.

DISCRETE DISTRIBUTIONS · ●●●

 $X \sim B(5, 0.4)$. $P(X \geq 1) = ?$

A) 0.4

B) 0.92224

C) 0.07776

D) 0.5

💡 $P(X \geq 1) = 1 - P(X = 0)$. $P(X = 0) = (0.6)^5$.

80.

DISCRETE DISTRIBUTIONS · ●●●

 $X \sim B(n=100, p=0.02)$. Using Poisson approximation, $P(X=0) \approx ?$ A) e^{-2} B) $2e^{-2}$ C) $e^{-0.02}$ D) $1/e$

💡 $\lambda = np = 2$. Poisson: $P(X=0) = e^{-(\lambda)}$.

Probability Mastery

PROBLEMS 81-90

81. DISCRETE DISTRIBUTIONS · ●●●

$X \sim B(4, 0.5)$. $P(X = 2) = ?$

A) $3/8$

B) $1/4$

C) $1/2$

D) $1/8$

| 💡 $C(4,2)(0.5)^2(0.5)^2$.

82. DISCRETE DISTRIBUTIONS · ●●●

A fair die rolled 3 times. $P(\text{exactly two 6s}) = ?$

A) $5/72$

B) $15/216$

C) $1/36$

D) $5/216$

| 💡 Binomial: $n=3$, $p=1/6$, $k=2$. $C(3,2)(1/6)^2(5/6)$.

83. EXPECTED VALUE · ●●

X takes values 1,2,3 with $P(1)=0.2$, $P(2)=0.5$, $P(3)=0.3$. $E[X] = ?$

A) 2.1

B) 1.8

C) 2.5

D) 2.0

| 💡 $E[X] = \sum x_i P(x_i)$.

84. EXPECTED VALUE · ●●

$E[X] = 4$, $\text{Var}(X) = 9$. Find $E[X^2]$.

A) 25

B) 16

C) 13

D) 81

| 💡 $\text{Var}(X) = E[X^2] - (E[X])^2$. Solve for $E[X^2]$.

85. EXPECTED VALUE · ●●

A game: win \$10 with $P=0.3$, lose \$5 with $P=0.7$. Expected gain per game?

A) \$5

B) $-\$0.50$

C) \$3

D) \$1.50

| 💡 $E = 10 \times 0.3 + (-5) \times 0.7$.

86. EXPECTED VALUE · ●●

$E[3X + 5]$ where $E[X] = 4$.

A) 17

B) 12

C) 20

D) 7

| 💡 $E[aX+b] = aE[X]+b$.

87. EXPECTED VALUE · ●●

$\text{Var}(2X)$ where $\text{Var}(X) = 9$.

A) 18

B) 36

C) 6

D) 9

| 💡 $\text{Var}(aX) = a^2\text{Var}(X)$.

88. EXPECTED VALUE · ●●

$E[X]=3$, $E[Y]=5$, X,Y independent. $E[XY] = ?$

A) 8

B) 15

C) 2

D) 30

| 💡 For independent X,Y : $E[XY]=E[X] \times E[Y]$.

89.

EXPECTED VALUE · ●●●

 $P(X=0)=0.5$, $P(X=2)=0.3$, $P(X=4)=0.2$. $\text{Var}(X) = ?$

A) 2.0

B) 2.36

C) 1.6

D) 3.2

💡 $\text{Var}(X) = E[X^2] - (E[X])^2$. Find $E[X]$ and $E[X^2]$ first.

90.

EXPECTED VALUE · ●●●

 $E[X]=2$, $\text{Var}(X)=4$. Find $\text{Var}(3-2X)$.

A) 16

B) 4

C) 12

D) 8

💡 $\text{Var}(a+bX) = b^2\text{Var}(X)$. Constant doesn't affect variance.

Probability Mastery

PROBLEMS 91-100

91. EXPECTED VALUE · ●●●

X,Y independent. $\text{Var}(X)=3$, $\text{Var}(Y)=4$. $\text{Var}(X+Y) = ?$

- A) 7 B) 12
C) 5 D) 1

💡 For independent: $\text{Var}(X+Y)=\text{Var}(X)+\text{Var}(Y)$.

92. EXPECTED VALUE · ●●●

$E[X]=5$, $E[Y]=2$, X,Y independent. $E[(X-Y)^2] = ?$

- A) 9 B) 16
C) 7 D) 34

💡 $E[(X-Y)^2]=\text{Var}(X-Y)+(E[X-Y])^2$. $\text{Var}(X-Y)=\text{Var}(X)+\text{Var}(Y)$ for independent.

93. EXPECTED VALUE · ●●●

Given $E[X]=5$, $\text{Var}(X)=4$, $E[Y]=2$, $\text{Var}(Y)=3$, X and Y independent. $E[(X-Y)^2] = ?$

- A) 9 B) 16
C) 7 D) 16

💡 $E[(X-Y)^2]=\text{Var}(X-Y)+(E[X-Y])^2$. $\text{Var}(X-Y)=\text{Var}(X)+\text{Var}(Y)$.

94. COUNTING & SAMPLE SPACE · ●

A coin is tossed twice. How many outcomes in the sample space?

- A) 2 B) 3
C) 4 D) 8

💡 Each toss has 2 outcomes. Use multiplication principle.

95. COUNTING & SAMPLE SPACE · ●

A restaurant offers 4 appetizers, 5 mains, 3 desserts. How many different meals (one of each)?

- A) 12 B) 60
C) 15 D) 20

💡 Multiply the number of choices for each course.

96. COUNTING & SAMPLE SPACE · ●●

A password is 3 letters (A–Z) followed by 2 digits (0–9). How many passwords?

- A) 1757600 B) 260000
C) 175760 D) 456976

💡 $26 \times 26 \times 26 \times 10 \times 10$. Repetition allowed.

97. COUNTING & SAMPLE SPACE · ●●

How many subsets does a set with 4 elements have?

- A) 4 B) 8
C) 16 D) 12

💡 Number of subsets = 2^n .

98. COUNTING & SAMPLE SPACE · ●●

Two dice rolled. How many outcomes sum to 7?

- A) 4 B) 5
C) 6 D) 7

💡 List pairs: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1).

99.

COUNTING & SAMPLE SPACE · ●●

A binary string of length 5 is formed (each digit 0 or 1). How many strings have exactly three 1s?

- A) 10
- C) 5

- B) 32
- D) 20

| 💡 $C(5,3)$ = choose 3 positions for the 1s.

100.

COUNTING & SAMPLE SPACE · ●●

How many ways can you choose a president and vice-president from 8 candidates?

- A) 28
- C) 64

- B) 56
- D) 16

| 💡 Order matters (different roles). $P(8,2)=8 \times 7$.

Probability Mastery

PROBLEMS 101-100

101.

COUNTING & SAMPLE SPACE · ●●●

How many integers from 1 to 100 are divisible by 3 or 5?

A) 47

B) 40

C) 33

D) 53

| 💡 Inclusion-exclusion: $|3 \cup 5| = |3| + |5| - |15|$.

102.

COUNTING & SAMPLE SPACE · ●●●

In a group of 30 students: 18 study French, 15 study Spanish, 10 study both. How many study neither?

A) 5

B) 7

C) 8

D) 10

| 💡 Inclusion-exclusion: $|F \cup S| = 18 + 15 - 10 = 23$. Neither = $30 - 23$.

103.

COUNTING & SAMPLE SPACE · ●●●

How many 5-digit numbers (no leading zero, digits 0–9, no repetition) are there?

A) 27216

B) 90000

C) 30240

D) 50400

| 💡 First digit: 9 choices (1–9). Remaining 4 from 9: $P(9,4) = 9 \times 8 \times 7 \times 6$.

Answer Key

All 100 answers – verify after completing the test.

1. C	2. B	3. D	4. B	5. C
6. C	7. D	8. A	9. C	10. B
11. C	12. B	13. A	14. B	15. A
16. A	17. A	18. A	19. A	20. B
21. A	22. C	23. C	24. C	25. A
26. A	27. A	28. B	29. A	30. B
31. C	32. A	33. A	34. B	35. C
36. B	37. B	38. B	39. B	40. A
41. A	42. B	43. B	44. A	45. C
46. C	47. D	48. A	49. C	50. B
51. C	52. D	53. B	54. C	55. B
56. A	57. C	58. B	59. B	60. C
61. A	62. B	63. B	64. B	65. A
66. A	67. B	68. B	69. A	70. A
71. B	72. B	73. C	74. C	75. A
76. A	77. C	78. C	79. B	80. A
81. A	82. B	83. A	84. A	85. B
86. A	87. B	88. B	89. B	90. A
91. A	92. D	93. B	94. C	95. B
96. A	97. C	98. C	99. A	100. B
101. A	102. B	103. A		