PRE-CALCULUS · SELF-STUDY EDITION

# Trigonometry 100

## Angle Basics

01 EASY

### How many degrees are in one full rotation?

**HINT & CONCEPT**

📐 Concept: A full circle = 360°. Half turn = 180°, quarter = 90°.

- (A) 90°
- (B) 180°
- (C) 270°
- (D) 360°

02 EASY

### What is the radian measure of a full circle?

**HINT & CONCEPT**

📐 Concept: Circumference = $2\pi r$, so arc of full circle / r = $2\pi$.

- (A) $\pi$
- (B) $2\pi$
- (C) $\pi/2$
- (D) $3\pi$

03 EASY

## Convert 180° to radians.

**HINT & CONCEPT**

📐 Formula: multiply degrees × $\pi/180$.

- (A) $\pi/2$
- (B) $\pi$
- (C) $2\pi$
- (D) $3\pi/2$

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04 EASY

## Convert $\pi/3$ radians to degrees.

**HINT & CONCEPT**

📐 Formula: multiply radians × $180/\pi$.

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

---

05 EASY

## An angle of 90° in radians is:

**HINT & CONCEPT**

📐 90° is a quarter of 360°, and a quarter of $2\pi$ is $\pi/2$.

- (A) $\pi/4$
- (B) $\pi/3$
- (C) $\pi/2$
- (D) $\pi$

---

06 EASY

## Which angle is coterminal with 30°?

**HINT & CONCEPT**

📐 Coterminal angles differ by a multiple of 360°.

- (A) 150°
- (B) 210°
- (C) 390°
- (D) 330°

---

07 MEDIUM

## Convert 5π/6 radians to degrees.

**HINT & CONCEPT**

📐 $(5\pi/6) \times (180/\pi) = 5\times30 = 150$.

- (A) 120°
- (B) 150°
- (C) 160°
- (D) 130°

---

08 MEDIUM

## Which of the following is a negative angle coterminal with 120°?

**HINT & CONCEPT**

📐 Subtract 360°: 120° − 360° = −240°.

- (A) -120°
- (B) -240°
- (C) -300°
- (D) -60°

---

09 MEDIUM

An arc of length 6 cm is cut from a circle of radius 4 cm. What is the central angle in radians?

**HINT & CONCEPT**

📐 Arc length formula: $s = r\theta$, so $\theta = s/r$.

- (A) 1
- (B) 1.5
- (C) 2
- (D) 2.5

10 MEDIUM

What is the reference angle for 210°?

**HINT & CONCEPT**

📐 210° is in QIII. Reference = 210° − 180° = 30°.

- (A) 30°
- (B) 60°
- (C) 45°
- (D) 150°

## Unit Circle

11 EASY

On the unit circle, what are the coordinates of the point at 0° (or 0 rad)?

**HINT & CONCEPT**

🔵 Unit circle: $(\cos\theta, \sin\theta)$. At $\theta=0$, $\cos 0=1$, $\sin 0=0$.

- (A) (0,1)
- (B) (1,0)
- (C) (-1,0)
- (D) (0,-1)

12 EASY

## What is sin(90°)?

**HINT & CONCEPT**

🔵 At 90° the point is at the top of the circle: (0,1). The y-coordinate is sin.

- (A) 0
- (B) 1
- (C) -1
- (D) $\sqrt{2}/2$

---

13 EASY

## What is cos(180°)?

**HINT & CONCEPT**

🔵 At 180° the point is (−1, 0). Cosine = x-coordinate.

- (A) 0
- (B) 1
- (C) -1
- (D) $\sqrt{3}/2$

---

14 EASY

## What is sin(30°)?

**HINT & CONCEPT**

🔵 30-60-90 triangle: opposite side to 30° = 1, hypotenuse = 2, so sin30 = 1/2.

- (A) $\sqrt{3}/2$
- (B) 1/2
- (C) $\sqrt{2}/2$
- (D) 1

---

15 EASY

## What is cos(60°)?

**HINT & CONCEPT**

🔵 cos 60° = sin 30° due to complementary angle. Or: 30-60-90 adjacent/hyp = 1/2.

- (A) $\sqrt{3}/2$
- (B) 1/2
- (C) $\sqrt{2}/2$
- (D) 0

---

16 EASY

## What is sin(45°)?

**HINT & CONCEPT**

🔵 45-45-90 triangle: both legs = 1, hyp = $\sqrt{2}$, so sin45 = $1/\sqrt{2} = \sqrt{2}/2$.

- (A) 1/2
- (B) $\sqrt{3}/2$
- (C) $\sqrt{2}/2$
- (D) 1

---

17 MEDIUM

## What is tan(π/4)?

**HINT & CONCEPT**

🔵 tan θ = sin θ/cos θ. At π/4: both sin and cos = $\sqrt{2}/2$.

- (A) 0
- (B) 1
- (C) $\sqrt{3}$
- (D) undefined

---

18 MEDIUM

## What is cos(3π/2)?

**HINT & CONCEPT**

🔵 $3\pi/2 = 270°$. Point on unit circle: $(0, -1)$. Cosine = x-coordinate.

- (A) 0
- (B) 1
- (C) -1
- (D) $-\sqrt{3}/2$

---

19 MEDIUM

## In which quadrant is sin θ < 0 and cos θ > 0?

**HINT & CONCEPT**

🔵 Mnemonic: All Students Take Calculus (ASTC) — QIV: cos+, sin−.

- (A) QI
- (B) QII
- (C) QIII
- (D) QIV

---

20 MEDIUM

## What is sin(5π/6)?

**HINT & CONCEPT**

🔵 $5\pi/6 = 150°$. Reference angle = 30°. QII: sin is positive. $\sin 30° = 1/2$.

- (A) −1/2
- (B) 1/2
- (C) $\sqrt{3}/2$
- (D) $-\sqrt{3}/2$

---

21 EASY

## In a right triangle, sin θ equals:

**HINT & CONCEPT**

SOH-CAH-TOA: Sin = Opposite/Hypotenuse.

- A adjacent/hypotenuse
- B opposite/hypotenuse
- C opposite/adjacent
- D hypotenuse/opposite

---

22 EASY

## In a right triangle with opposite = 3 and hypotenuse = 5, what is sin θ?

**HINT & CONCEPT**

sin θ = opposite/hypotenuse = 3/5.

- A 3/4
- B 4/5
- C 3/5
- D 5/3

---

23 EASY

## csc θ is defined as:

**HINT & CONCEPT**

Reciprocal functions: csc = 1/sin, sec = 1/cos, cot = 1/tan.

- A 1/sin θ
- B 1/cos θ
- C 1/tan θ
- D sin θ/cos θ

---

24 EASY

What is tan θ expressed in terms of sin and cos?

**HINT & CONCEPT**

tan θ = sin θ / cos θ. This is a fundamental identity.

- (A) sin θ × cos θ
- (B) cos θ/sin θ
- (C) sin θ/cos θ
- (D) 1/(sin θ × cos θ)

---

25 MEDIUM

If sin θ = 5/13 and θ is in QI, what is cos θ?

**HINT & CONCEPT**

Use Pythagorean theorem: adj = $\sqrt{(13^2-5^2)}$ = $\sqrt{144}$ = 12. cos = 12/13.

- (A) 5/12
- (B) 12/13
- (C) 13/12
- (D) 12/5

---

26 MEDIUM

If cos θ = −3/5 and θ is in QIII, what is tan θ?

**HINT & CONCEPT**

opp = −4 (QIII, both neg), adj = −3, hyp = 5. tan = opp/adj = (−4)/(−3) = 4/3.

- (A) −4/3
- (B) 4/3
- (C) 3/4
- (D) −3/4

---

27 MEDIUM

## What is sec(60°)?

**HINT & CONCEPT**

sec = 1/cos. cos 60° = 1/2, so sec 60° = 2.

- A $2$
- B $\sqrt{3}$
- C $1/2$
- D $\sqrt{2}$

---

28 MEDIUM

## What is cot(π/3)?

**HINT & CONCEPT**

cot = cos/sin. cos(π/3)=1/2, sin(π/3)=$\sqrt{3}/2$. cot = (1/2)/($\sqrt{3}/2$)=1/$\sqrt{3}$.

- A $\sqrt{3}$
- B $1/\sqrt{3}$
- C $\sqrt{2}$
- D $1$

---

29 HARD

## If tan θ = 2 and θ is in QI, what is sin θ?

**HINT & CONCEPT**

Draw right triangle: opp=2, adj=1, hyp=$\sqrt{5}$. sin = 2/$\sqrt{5}$.

- A $2/\sqrt{5}$
- B $1/\sqrt{5}$
- C $2/\sqrt{3}$
- D $\sqrt{5}/2$

---

30 HARD

If sec θ = 5/3 and θ is in QIV, what is tan θ?

**HINT & CONCEPT**

📏 sec=5/3 → cos=3/5. sin=−4/5 (QIV, sin neg). tan = sin/cos = −4/3.

(A) 4/3

(B) −4/3

(C) 3/4

(D) −3/4

---

## Graphs of Trig Functions

31 EASY

What is the period of y = sin x?

**HINT & CONCEPT**

📊 The sine function completes one full cycle in 2π radians.

(A) π

(B) 2π

(C) π/2

(D) 4π

---

32 EASY

What is the amplitude of y = 3 sin x?

**HINT & CONCEPT**

📊 Amplitude = |A| for y = A sin x. It's the peak height.

(A) 1

(B) 2

(C) 3

(D) 6

---

33 EASY

## What is the period of $y = \cos(2x)$?

**HINT & CONCEPT**

📊 Period = $2\pi/|B|$ for $y = \cos(Bx)$. Here $B = 2$, so period = $2\pi/2 = \pi$.

- (A) $4\pi$
- (B) $\pi$
- (C) $2\pi$
- (D) $\pi/2$

---

34 MEDIUM

## What is the range of $y = \sin x$?

**HINT & CONCEPT**

📊 Sine only produces values between $-1$ and 1.

- (A) $[0, 1]$
- (B) $[-1, 1]$
- (C) $(-\infty, \infty)$
- (D) $[0, 2\pi]$

---

35 MEDIUM

## What is the period of $y = \tan x$?

**HINT & CONCEPT**

📊 Unlike sin/cos, tangent has period $\pi$.

- (A) $2\pi$
- (B) $\pi$
- (C) $\pi/2$
- (D) $4\pi$

---

36 MEDIUM

## For $y = 2 \sin(3x - \pi)$, what is the phase shift?

**HINT & CONCEPT**

📊 Phase shift = C/B where $y = A \sin(Bx - C)$. Here $C=\pi$, $B=3$. Shift = $\pi/3$ right.

- (A) $\pi/3$ to the right
- (B) $\pi$ to the right
- (C) $\pi/3$ to the left
- (D) 3 to the right

---

37 MEDIUM

## What is the vertical shift of $y = \sin x + 4$?

**HINT & CONCEPT**

📊 $y = \sin x + D$ shifts the graph D units vertically.

- (A) 4 down
- (B) 1 up
- (C) 4 up
- (D) no shift

---

38 MEDIUM

## Where does $y = \tan x$ have vertical asymptotes?

**HINT & CONCEPT**

📊 $\tan x = \sin x/\cos x$, undefined when $\cos x = 0$, i.e. $x = \pi/2 + n\pi$.

- (A) $x = n\pi$
- (B) $x = \pi/2 + n\pi$
- (C) $x = 2n\pi$
- (D) $x = n\pi/2$

---

39 **HARD** For $y = -3\cos(\pi x/2) + 1$, what is the amplitude?

**HINT & CONCEPT**

📊 Amplitude = $|A|$. The negative sign flips the graph but doesn't change amplitude.

(A) 1

(B) 3

(C) $\pi/2$

(D) $-3$

---

40 **HARD** What is the period of $y = \sin(\pi x)$?

**HINT & CONCEPT**

📊 Period = $2\pi/B = 2\pi/\pi = 2$.

(A) 2

(B) $2\pi$

(C) 1

(D) $\pi$

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## Pythagorean Identities

41 **EASY** Which is the fundamental Pythagorean identity?

**HINT & CONCEPT**

🔑 Derived from the equation of the unit circle: $x^2 + y^2 = 1$.

(A) $\sin^2\theta + \cos^2\theta = 2$

(B) $\sin^2\theta - \cos^2\theta = 1$

(C) $\sin^2\theta + \cos^2\theta = 1$

(D) $\tan^2\theta + 1 = \sin^2\theta$

42 EASY

### Which identity follows from dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$?

**HINT & CONCEPT**

🔑 Divide every term by $\cos^2\theta$: $\sin^2/\cos^2 + 1 = 1/\cos^2$. That's $\tan^2 + 1 = \sec^2$.

- (A) $1 + \cot^2\theta = \csc^2\theta$
- (B) $\tan^2\theta + 1 = \sec^2\theta$
- (C) $\sin^2\theta + 1 = \sec^2\theta$
- (D) $\tan^2\theta - 1 = \sec^2\theta$

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43 EASY

### If $\sin\theta = 3/5$, what is $\cos^2\theta$?

**HINT & CONCEPT**

🔑 $\cos^2\theta = 1 - \sin^2\theta = 1 - 9/25 = 16/25$.

- (A) 9/25
- (B) 16/25
- (C) 7/25
- (D) 4/5

---

44 MEDIUM

### Simplify: $\sin^2\theta/\cos^2\theta + 1/\cos^2\theta$

**HINT & CONCEPT**

🔑 Factor out $1/\cos^2\theta$: $(\sin^2\theta + 1)/\cos^2\theta$ = ... wait, it's $(\sin^2\theta+1)/\cos^2\theta$? No — it's $(\sin^2+1)/\cos^2$? Re-read: $\sin^2/\cos^2 + 1/\cos^2 = (\sin^2+1)/\cos^2$? Actually $(\sin^2\theta + 1)/\cos^2\theta$... use $\sin^2+\cos^2=1 \rightarrow$ check the expression equals $\sec^2\theta$.

- (A) 1
- (B) $\sec^2\theta$
- (C) $\csc^2\theta$
- (D) $\tan^2\theta$

---

45 MEDIUM

## Which identity is correct?

**HINT & CONCEPT**

🔑 Divide $\sin^2+\cos^2=1$ by $\sin^2\theta$: $1 + \cot^2\theta = \csc^2\theta$.

- (A) $1 + \tan^2\theta = \csc^2\theta$
- (B) $1 + \cot^2\theta = \csc^2\theta$
- (C) $\cot^2\theta - 1 = \csc^2\theta$
- (D) $1 + \csc^2\theta = \cot^2\theta$

---

46 MEDIUM

## Simplify: $(1 - \sin^2\theta)/\cos\theta$

**HINT & CONCEPT**

🔑 $1 - \sin^2\theta = \cos^2\theta$. So expression becomes $\cos^2\theta/\cos\theta = \cos\theta$.

- (A) $\cos\theta$
- (B) $\sin\theta$
- (C) $\tan\theta$
- (D) $\sec\theta$

---

47 MEDIUM

## If $\tan\theta = 1$, what is $\sec^2\theta$?

**HINT & CONCEPT**

🔑 $\sec^2\theta = \tan^2\theta + 1 = 1 + 1 = 2$.

- (A) 1
- (B) $\sqrt{2}$
- (C) 2
- (D) 4

---

48 HARD

## Simplify: $\tan^2\theta - \sec^2\theta$

**HINT & CONCEPT**

🔑 From $\tan^2\theta + 1 = \sec^2\theta$, rearrange: $\tan^2\theta - \sec^2\theta = -1$.

(A) 1

(B) $-1$

(C) 0

(D) 2

---

49 HARD

## Simplify: $\sin^4\theta - \cos^4\theta$

**HINT & CONCEPT**

🔑 Factor as difference of squares: $(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)$. Since $\sin^2+\cos^2=1$...

(A) $\sin^2\theta + \cos^2\theta$

(B) $\sin^2\theta - \cos^2\theta$

(C) $2\sin^2\theta$

(D) 1

---

50 HARD

## If $\sin\theta + \cos\theta = 1$, what is $\sin\theta \cdot \cos\theta$?

**HINT & CONCEPT**

🔑 Square both sides: $(\sin\theta+\cos\theta)^2=1 \rightarrow \sin^2+2\sin\cos+\cos^2=1 \rightarrow 1+2\sin\cos=1$.

(A) 0

(B) 1/2

(C) $-1/2$

(D) 1

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# Sum & Difference Formulas

51 MEDIUM

## Which formula correctly expresses sin(A + B)?

**HINT & CONCEPT**

+ Key formula: sin(A+B) = sinA cosB + cosA sinB.

- (A) sin A cos B − cos A sin B
- (B) sin A cos B + cos A sin B
- (C) cos A cos B − sin A sin B
- (D) cos A cos B + sin A sin B

---

52 MEDIUM

## Evaluate sin(75°) using sin(45° + 30°).

**HINT & CONCEPT**

+ sin(45+30) = sin45·cos30 + cos45·sin30 = (√2/2)(√3/2) + (√2/2)(1/2).

- (A) (√6+√2)/4
- (B) (√6−√2)/4
- (C) √3/2
- (D) (√2+√3)/2

---

53 MEDIUM

## cos(A − B) =

**HINT & CONCEPT**

+ For cosine: difference formula reverses the operation compared to sum.

- (A) cos A cos B − sin A sin B
- (B) cos A cos B + sin A sin B
- (C) sin A cos B − cos A sin B
- (D) sin A cos B + cos A sin B

---

54 MEDIUM

## Evaluate cos(15°) = cos(45° − 30°).

**HINT & CONCEPT**

+ cos(45−30)=cos45·cos30+sin45·sin30 = (√2/2)(√3/2)+(√2/2)(1/2) = (√6+√2)/4. Wait: cos45=√2/2, cos30=√3/2, sin45=√2/2, sin30=1/2. cos(A-B)=cosAcosB+sinAsinB = √6/4+√2/4.

- (A) (√6−√2)/4
- (B) (√6+√2)/4
- (C) √3/2
- (D) √2/2

55 MEDIUM

## tan(A + B) =

**HINT & CONCEPT**

+ tan(A+B) = (tanA + tanB)/(1 − tanA tanB). The denominator has a minus.

- (A) (tan A − tan B)/(1 + tan A tan B)
- (B) (tan A + tan B)/(1 − tan A tan B)
- (C) (tan A + tan B)/(1 + tan A tan B)
- (D) tan A + tan B

56 MEDIUM

## Using the sum formula, sin(π/2 + x) simplifies to:

**HINT & CONCEPT**

+ sin(π/2+x) = sin(π/2)cosx + cos(π/2)sinx = 1·cosx + 0·sinx.

- (A) −cos x
- (B) cos x
- (C) sin x
- (D) −sin x

57 HARD

## If $\sin A = 1/2$ (QI) and $\sin B = \sqrt{3}/2$ (QI), find $\sin(A + B)$.

**HINT & CONCEPT**

+ A=30°, B=60°. sin(30+60)=sin90=1.

(A) 0

(B) 1

(C) $\sqrt{3}/2$

(D) $\sqrt{2}/2$

---

58 HARD

## Simplify: $\cos(\pi - \theta)$

**HINT & CONCEPT**

+ $\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta = (-1)\cos\theta + 0 = -\cos\theta$.

(A) $\cos\theta$

(B) $-\cos\theta$

(C) $\sin\theta$

(D) $-\sin\theta$

---

59 HARD

## Simplify: $\sin(\pi + \theta)$

**HINT & CONCEPT**

+ $\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta = 0 - \sin\theta = -\sin\theta$.

(A) $\sin\theta$

(B) $-\sin\theta$

(C) $\cos\theta$

(D) $-\cos\theta$

---

60 HARD Evaluate $\tan(\pi/4 + \pi/4)$ using the tan sum formula.

**HINT & CONCEPT**

➕ $\tan(\pi/2)$ = undefined. tan sum: $(1+1)/(1-1\times1) = 2/0$ = undefined.

(A) 1

(B) 0

(C) undefined

(D) 2

## Double & Half-Angle Formulas

61 MEDIUM $\sin(2\theta) =$

**HINT & CONCEPT**

🔄 $\sin(2\theta) = \sin(\theta+\theta)$. Apply sum formula: $\sin\theta\cos\theta + \cos\theta\sin\theta = 2\sin\theta\cos\theta$.

(A) $2 \sin \theta$

(B) $2 \sin \theta \cos \theta$

(C) $\sin^2\theta - \cos^2\theta$

(D) $2 \cos^2\theta - 1$

62 MEDIUM $\cos(2\theta)$ expressed as 1 minus something is:

**HINT & CONCEPT**

🔄 $\cos(2\theta) = \cos^2\theta - \sin^2\theta = (1-\sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta$.

(A) $1 - 2\cos^2\theta$

(B) $1 - 2\sin^2\theta$

(C) $2\cos^2\theta - 1$

(D) $\cos^2\theta - \sin^2\theta$

63 MEDIUM

If $\sin\theta = 1/2$ (QI), find $\sin(2\theta)$.

**HINT & CONCEPT**

🔄 $\cos\theta = \sqrt{3}/2$ (since sin=1/2 in QI → θ=30°). $\sin(2\times30°)=\sin60°=\sqrt{3}/2$.

Ⓐ 1/2

Ⓑ $\sqrt{3}/4$

Ⓒ $\sqrt{3}/2$

Ⓓ 1

---

64 MEDIUM

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$. If $\cos\theta = 3/5$, $\sin\theta = 4/5$, find $\cos(2\theta)$.

**HINT & CONCEPT**

🔄 $(3/5)^2 - (4/5)^2 = 9/25 - 16/25 = -7/25$.

Ⓐ 7/25

Ⓑ −7/25

Ⓒ 24/25

Ⓓ −24/25

---

65 MEDIUM

What is $\tan(2\theta)$ in terms of $\tan\theta$?

**HINT & CONCEPT**

🔄 $\tan(2\theta) = \tan(\theta+\theta) = (\tan\theta+\tan\theta)/(1-\tan\theta\cdot\tan\theta) = 2\tan\theta/(1-\tan^2\theta)$.

Ⓐ $2\tan\theta/(1-\tan^2\theta)$

Ⓑ $2\tan\theta/(1+\tan^2\theta)$

Ⓒ $\tan^2\theta/(1-\tan^2\theta)$

Ⓓ $2/(1-\tan^2\theta)$

---

66 HARD

## The half-angle formula for $\cos(\theta/2)$ is:

**HINT & CONCEPT**

🔄 From $\cos(2\alpha)=2\cos^2\alpha-1$, set $\alpha=\theta/2$: $\cos\theta = 2\cos^2(\theta/2)-1 \rightarrow \cos^2(\theta/2)=(1+\cos\theta)/2$.

- (A) $\pm\sqrt{((1-\cos\theta)/2)}$
- (B) $\pm\sqrt{((1+\cos\theta)/2)}$
- (C) $\pm\sqrt{(\cos\theta/2)}$
- (D) $\sin\theta/2$

---

67 HARD

## Using the half-angle formula, find $\sin(\pi/8) = \sin(22.5°)$.

**HINT & CONCEPT**

🔄 $\sin(\pi/8) = \sqrt{((1-\cos(\pi/4))/2)} = \sqrt{((1-\sqrt{2}/2)/2)} = \sqrt{((2-\sqrt{2})/4)} = \sqrt{(2-\sqrt{2})}/2$.

- (A) $\sqrt{(2+\sqrt{2})}/2$
- (B) $\sqrt{(2-\sqrt{2})}/2$
- (C) $\sqrt{2}/2$
- (D) 1/2

---

68 HARD

## Simplify: $2\sin(\theta/2)\cos(\theta/2)$

**HINT & CONCEPT**

🔄 This matches $\sin(2\alpha)=2\sin\alpha\cos\alpha$ with $\alpha = \theta/2$. So result $= \sin(2\cdot\theta/2) = \sin\theta$.

- (A) $\cos\theta$
- (B) $\sin\theta$
- (C) $2\sin\theta$
- (D) $\sin(2\theta)$

---

69 HARD

If $\cos\theta = 7/25$ and $\theta$ is in QI, find $\sin(2\theta)$.

**HINT & CONCEPT**

🔄 $\sin\theta = 24/25$ (since $7^2+24^2=25^2$). $\sin(2\theta)=2\cdot(24/25)\cdot(7/25)=336/625$.

(A) 336/625

(B) 168/625

(C) 24/25

(D) 336/25

70 HARD

Rewrite $2\cos^2\theta - 1$ using a double-angle identity.

**HINT & CONCEPT**

🔄 $\cos(2\theta) = 2\cos^2\theta - 1$. So $2\cos^2\theta - 1 = \cos(2\theta)$.

(A) $\sin(2\theta)$

(B) $\cos(2\theta)$

(C) $\tan(2\theta)$

(D) $2\cos(2\theta)$

## Inverse Trig Functions

71 MEDIUM

What is arcsin(1/2)?

**HINT & CONCEPT**

🔃 arcsin asks: which angle in $[-\pi/2, \pi/2]$ has sin = 1/2? Answer: $\pi/6 = 30°$.

(A) $\pi/6$

(B) $\pi/4$

(C) $\pi/3$

(D) $\pi/2$

72 MEDIUM

## What is the range of y = arcsin x?

**HINT & CONCEPT**

🔃 The arcsin function outputs angles restricted to [−π/2, π/2] to be a true function.

(A) [0, π]

(B) [−π, π]

(C) [−π/2, π/2]

(D) (−π/2, π/2)

---

73 MEDIUM

## What is arctan(1)?

**HINT & CONCEPT**

🔃 tan(π/4) = 1. arctan(1) = π/4.

(A) π/6

(B) π/4

(C) π/3

(D) π/2

---

74 MEDIUM

## What is arccos(0)?

**HINT & CONCEPT**

🔃 cos(π/2) = 0. So arccos(0) = π/2. Range of arccos is [0, π].

(A) 0

(B) π/4

(C) π/2

(D) π

75 MEDIUM

## What is arcsin(−1)?

**HINT & CONCEPT**

🔃 $\sin(-\pi/2) = -1$. arcsin is restricted to $[-\pi/2, \pi/2]$.

- A $\pi$
- B $3\pi/2$
- C $-\pi/2$
- D $-\pi$

---

76 HARD

## Simplify: sin(arccos(x))

**HINT & CONCEPT**

🔃 Let $\theta = \arccos(x)$, so $\cos\theta = x$. Then $\sin^2\theta+\cos^2\theta=1 \rightarrow \sin\theta=\sqrt{(1-x^2)}$.

- A $x$
- B $1-x^2$
- C $\sqrt{(1-x^2)}$
- D $1/x$

---

77 HARD

## What is the domain of arcsin x?

**HINT & CONCEPT**

🔃 arcsin is only defined when the input is a valid sine value: between −1 and 1.

- A $(-\infty, \infty)$
- B $[-\pi/2, \pi/2]$
- C $[-1, 1]$
- D $[0, 1]$

---

78 HARD

Evaluate: tan(arcsin(3/5))

**HINT & CONCEPT**

🔃 Let $\theta$=arcsin(3/5). Then sin$\theta$=3/5, cos$\theta$=4/5. tan$\theta$=sin/cos=3/4.

(A) 3/4

(B) 4/3

(C) 3/5

(D) 4/5

---

## Solving Trig Equations

79 MEDIUM

Solve: sin $\theta$ = 1/2 for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ sin is positive in QI and QII. Reference angle = 30°. Solutions: 30° and 180°−30°=150°.

(A) 30° only

(B) 150° only

(C) 30° and 150°

(D) 60° and 120°

---

80 MEDIUM

Solve: cos $\theta$ = 0 for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ cos = 0 at 90° and 270° on the unit circle.

(A) 0° and 180°

(B) 90° and 270°

(C) 45° and 315°

(D) 90° only

---

81 MEDIUM

## Solve: $2\sin\theta - 1 = 0$ for $\theta \in [0, 2\pi)$.

**HINT & CONCEPT**

⚖️ $2\sin\theta=1 \rightarrow \sin\theta=1/2$. Solutions in $[0,2\pi)$: $\pi/6$ and $\pi-\pi/6=5\pi/6$.

- (A) $\pi/3$ and $2\pi/3$
- (B) $\pi/6$ and $5\pi/6$
- (C) $\pi/4$ and $3\pi/4$
- (D) $\pi/6$ only

---

82 MEDIUM

## Solve: $\tan\theta = -1$ for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ tan = −1 in QII (135°) and QIV (315°). Reference angle = 45°.

- (A) 135° and 315°
- (B) 45° and 225°
- (C) 135° only
- (D) 315° only

---

83 MEDIUM

## Solve: $2\cos^2\theta - 1 = 0$ for $\theta \in [0, 2\pi)$.

**HINT & CONCEPT**

⚖️ $\cos^2\theta=1/2 \rightarrow \cos\theta=\pm1/\sqrt{2}$. Solutions at 45°,135°,225°,315°.

- (A) $\pi/4$ and $3\pi/4$
- (B) $\pi/3$ and $2\pi/3$
- (C) $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$
- (D) $\pi/3$, $2\pi/3$, $4\pi/3$, $5\pi/3$

---

84 HARD

Solve: $2\sin^2\theta + \sin\theta - 1 = 0$ for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ Factor: $(2\sin\theta-1)(\sin\theta+1)=0$. $\sin\theta=1/2$ (→30°,150°) or $\sin\theta=-1$ (→270°).

- (A) 30° and 150°
- (B) 30°, 150°, and 270°
- (C) 90° and 210°
- (D) 60° and 300°

---

85 HARD

For how many values of $\theta \in [0, 2\pi)$ does $\sin(2\theta) = 1$?

**HINT & CONCEPT**

⚖️ $\sin(2\theta)=1 \rightarrow 2\theta=\pi/2+2n\pi \rightarrow \theta=\pi/4+n\pi$. In $[0,2\pi)$: $\theta=\pi/4$ and $\theta=5\pi/4$.

- (A) 1
- (B) 2
- (C) 3
- (D) 4

---

86 HARD

Solve: $\cos(2\theta) = \cos\theta$ for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ $2\cos^2\theta-1=\cos\theta \rightarrow 2\cos^2\theta-\cos\theta-1=0 \rightarrow (2\cos\theta+1)(\cos\theta-1)=0$. $\cos\theta=-1/2$ or 1.

- (A) 0°, 120°, 240°
- (B) 0°, 90°, 270°
- (C) 60°, 180°, 300°
- (D) 0°, 120°, 180°, 240°

87 HARD

Solve: $\sin\theta = \cos\theta$ for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ sin=cos → tanθ=1 → θ=45°+n·180°. In [0°,360°): 45° and 225°.

- (A) 45° only
- (B) 45° and 225°
- (C) 135° and 315°
- (D) 45° and 135°

---

88 HARD

Solve: $\sqrt{3}\tan\theta + 1 = 0$ for $\theta \in [0°, 360°)$.

**HINT & CONCEPT**

⚖️ $\tan\theta = -1/\sqrt{3}$. Reference angle = 30°. tan is negative in QII and QIV: 150°, 330°.

- (A) 150° and 330°
- (B) 30° and 210°
- (C) 120° and 300°
- (D) 60° and 240°

---

## Law of Sines & Cosines

89 MEDIUM

The Law of Sines states:

**HINT & CONCEPT**

📐 Law of Sines: ratios of each side to the sine of its opposite angle are equal.

- (A) $a^2=b^2+c^2-2bc\cdot\cos A$
- (B) a/sinA = b/sinB = c/sinC
- (C) sinA/a = cosB/b
- (D) a·sinA = b·sinB

90 MEDIUM

## The Law of Cosines for side a is:

**HINT & CONCEPT**

📐 It's a generalization of Pythagorean theorem. When A=90°, cos A=0 and it reduces to $a^2=b^2+c^2$.

- (A) $a^2 = b^2 + c^2$
- (B) $a^2 = b^2 + c^2 - 2bc \cos A$
- (C) $a = b + c - 2bc \cos A$
- (D) $a^2 = b^2 - c^2 + 2bc \cos A$

---

91 MEDIUM

## In triangle ABC, A = 30°, a = 5, b = 8. How many possible triangles exist (ambiguous case)?

**HINT & CONCEPT**

📐 Ambiguous case (SSA): b > a and a > b·sinA. Check: 8 > 5 and 5 > 8·sin30°=4. Yes → 2 triangles.

- (A) 0
- (B) 1
- (C) 2
- (D) 3

---

92 MEDIUM

## Use the Law of Sines: In triangle ABC, A = 45°, B = 60°, a = 10. Find b.

**HINT & CONCEPT**

📐 b/sinB = a/sinA. b = a·sinB/sinA = 10·sin60°/sin45°. Note B and C give the same expression.

- (A) 10sin45°/sin60°
- (B) 10sin60°/sin45°
- (C) $5\sqrt{6}$
- (D) both B and C are equivalent

---

93 MEDIUM

In a triangle with sides a=5, b=7, c=8, find cos C using the Law of Cosines.

**HINT & CONCEPT**

📐 $c^2=a^2+b^2-2ab\cos C \rightarrow \cos C = (a^2+b^2-c^2)/(2ab) = (25+49-64)/70 = 10/70 = 1/7$.

- (A) 1/7
- (B) 2/7
- (C) 1/14
- (D) 1/5

---

94 MEDIUM

Area of a triangle with sides b, c and included angle A is:

**HINT & CONCEPT**

📐 Area = (1/2)·base·height. Height = c·sinA. So Area = (1/2)·b·c·sinA.

- (A) (1/2)bc
- (B) (1/2)bc sin A
- (C) bc cos A
- (D) bc sin A

---

95 HARD

Find the area of a triangle with sides 6 and 8 and included angle 30°.

**HINT & CONCEPT**

📐 Area = (1/2)·6·8·sin30° = (1/2)·48·(1/2) = 12.

- (A) 24
- (B) 12
- (C) 48
- (D) $6\sqrt{3}$

96 HARD

In a triangle with a = 7, b = 8, C = 120°, find c using Law of Cosines.

**HINT & CONCEPT**

📐 $c^2=7^2+8^2-2(7)(8)\cos 120° = 49+64-112(-1/2) = 113+56 = 169$. Wait, re-check.

- (A) $\sqrt{177}$
- (B) $\sqrt{113}$
- (C) $\sqrt{153}$
- (D) $\sqrt{169}$

---

97 HARD

Heron's formula for triangle area with sides a, b, c (s = semi-perimeter) is:

**HINT & CONCEPT**

📐 $s = (a+b+c)/2$. Area $= \sqrt{(s(s-a)(s-b)(s-c))}$.

- (A) $\sqrt{(s(s-a)(s-b)(s-c))}$
- (B) $s(s-a)(s-b)(s-c)$
- (C) $\sqrt{((s-a)(s-b)(s-c))}$
- (D) $2\sqrt{(s(s-a)(s-b)(s-c))}$

---

98 HARD

The Law of Sines is best used when you are given:

**HINT & CONCEPT**

📐 Law of Sines = angle-side pairs. AAS and ASA give angle-side pairs directly.

- (A) Three sides (SSS)
- (B) Two sides and the included angle (SAS)
- (C) Two angles and one side (AAS or ASA)
- (D) Two sides and an excluded angle with no ambiguity

99 HARD

From a point 100 m from the base of a building, the angle of elevation to the top is 60°. How tall is the building?

**HINT & CONCEPT**

📐 $\tan(60°)$ = height/100 → height = $100 \cdot \tan 60°$ = $100\sqrt{3}$.

- (A) $100/\sqrt{3}$
- (B) $100\sqrt{3}$
- (C) $50\sqrt{3}$
- (D) $200\sqrt{3}$

---

100 HARD

Two ships leave port at the same time. One travels 40 km at N 30° E, the other 60 km at S 60° E. What is the angle between their paths?

**HINT & CONCEPT**

📐 N30°E and S60°E: from North, first is 30°; from South, second is 60° = from North, 120°. Angle between = 120°−30°=90°.

- (A) 30°
- (B) 60°
- (C) 90°
- (D) 120°

---

# Answer Key

| | | | | |
|---|---|---|---|---|
| 01. D | 02. B | 03. B | 04. C | 05. C |
| 06. C | 07. B | 08. B | 09. B | 10. A |
| 11. B | 12. B | 13. C | 14. B | 15. B |
| 16. C | 17. B | 18. A | 19. D | 20. B |
| 21. B | 22. C | 23. A | 24. C | 25. B |
| 26. B | 27. A | 28. B | 29. A | 30. B |
| 31. B | 32. C | 33. B | 34. B | 35. B |
| 36. A | 37. C | 38. B | 39. B | 40. A |
| 41. C | 42. B | 43. B | 44. B | 45. B |
| 46. A | 47. C | 48. B | 49. B | 50. A |
| 51. B | 52. A | 53. B | 54. B | 55. B |
| 56. B | 57. B | 58. B | 59. B | 60. C |
| 61. B | 62. B | 63. C | 64. B | 65. A |
| 66. B | 67. B | 68. B | 69. A | 70. B |
| 71. A | 72. C | 73. B | 74. C | 75. C |
| 76. C | 77. C | 78. A | 79. C | 80. B |
| 81. B | 82. A | 83. C | 84. B | 85. B |
| 86. A | 87. B | 88. A | 89. B | 90. B |
| 91. C | 92. D | 93. A | 94. B | 95. B |
| 96. D | 97. A | 98. C | 99. B | 100. C |